Exercises (Chapters 2 – 4)

1. The Capital Asset Pricing Model (CAPM) is a benchmark model in financial theory. It explains variations in the rate of return on a security as a function of the rate of return on a portfolio consisting of all securities traded in the financial markets, which is called the market portfolio. Generally the rate of return of an investment is measured relative to its opportunity cost, which is the return of a risk-free asset. The resulting difference is called the *risk premium*. The CAPM says that the risk premium on a security is proportional to the risk premium on the market portfolio, that is,

$$E(r_j) - r_f = \beta_j\big(E(r_m) - r_f\big)$$

where $E(r_j)$ is the expected (average) return of security j, $E(r_m)$ is the expected (average) return of the market portfolio, r_f is the risk-free rate and β_j is known as the **beta** coefficient of security j. The beta is important to investors because it reveals the exposure of investors to market risk. It measures the amount of risk investors add to their portfolio by investing in security j. A beta less than 1 indicate that the security is "defensive" since its variation is less than the market's. A beta greater than 1 indicates an "aggressive" security. Beta is an unobserved quantity and should be estimated using an econometric approach. The CAPM shown above is an economic model. The econometric model is obtained by removing the mean and including an intercept (even though theory says it should be zero) and an error term,

$$r_j - r_f = \alpha_j + \beta_j\big(r_m - r_f\big) + \varepsilon$$

In the excel file capm.xlsx are data on the monthly returns of two firms (MSFT and GE), the rate of return of the market portfolio (MKT) and the risk-free rate (RF). The observations cover the period January 1998 to December 2008.

   a. Present descriptive statistics on the excess returns of the two stocks and a joint plot commenting on these results.
   b. Estimate the CAPM model for each stock and comment on the results. Which stock appears aggressive and which defensive?
   c. Construct a 95% confidence interval for the betas of the two stocks. Assume that you are a financial advisor. Explain these results to an investor who has come to you for advice.
   d. Test at the 1% level the hypothesis that MSFT is an aggressive stock and GE is a defensive stock.
   e. Test at the 1% level the hypothesis that the intercept term in the CAPM model for each stock is zero, against the alternative that it is not. What do you conclude?

2. One would expect that new house construction and sales depend on mortgage interest rates. If interest rates are high, fewer people would afford to borrow the funds necessary to buy a new house. Builders are aware of this fact, and thus when mortgage interest rates are high, they will be inclined to build new homes. To examine this validity of this economic argument we collect data on 30-year fixed mortgage rates (FIXED_RATE), housing starts

(STARTS, in thousands) and house sold (SOLD, in thousands). These are contained in the file house_starts.xlsx. There are 184 monthly observations from January 1990 to April 2005.

a. Present descriptive statistics of these variables. Comment on these results.
b. Estimate the simple regression model
$$STARTS = \beta_1 + \beta_2 FIXED\_RATE + \varepsilon$$
Comment on the results. Plot the fitted regression line along with the data scatter.
c. Estimate the simple regression model
$$SOLD = \beta_1 + \beta_2 FIXED\_RATE + \varepsilon$$
Comment on the results. Plot the fitted regression line along with the data scatter.
d. It is conjectured that if the 30-year mortgage interest rate increases by 1% then house starts will fall by 150,000. Test this hypothesis at the 5% level of significance using a two-tail test.
e. Construct a 95% confidence interval for the parameter β_2 of the model estimated in (c). State the meaning of this interval estimate.
f. The mortgage interest rate for May and June 2005 were 6.00% and 5.82%, respectively. Predict the number of housing starts in May and June 2005.
g. Construct 95% prediction intervals for the number of housing starts in May and June 2005, based on the sample data. The actual number of housing starts in May and June 2005 were 2,041 and 2,065 (in thousands). Did your intervals contain the true values?

3. How much does education affect wage rates? This exercise will explore this issue further. The data file wage_educ.xlsx contains hourly wage rates (WAGE) and number of years of education (EDUC) for 4733 individuals.
   a. Present descriptive statistics for these two variables. Plot wage rates (y-axis) against the years of education (x-axis). Comment on these results.
   b. Estimate the linear regression model
   $$WAGE = \beta_1 + \beta_2 EDUC + \varepsilon$$
   and the log-linear model
   $$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \varepsilon$$
   c. Calculate the percentage increase in wages for the average worker for an additional year of education using both models.
   d. Compare the R^2 of the linear model and the generalized R^2 of the log-linear model. Which model fits the data better?
   e. Plot the least squares residuals of the log-linear model against education. Do you observe any patterns?
   f. Using the log-linear model, test the null hypothesis that the rate of return to education is 12% against the alternative that it is not, using a two-tail test at the 5% significance level.
   g. Using each model, predict the wage of a worker with 16 years of education. Compare these predictions to the actual average wage of all workers in the sample with 16 years of education.