EXERCISES -2 (SOLUTIONS)

1.

a. $PV = \frac{1}{0.10} = \$10$

b. $PV = \frac{1/0.10}{1.10^7} = 5.13 \simeq \$5$

c. PV = 10 – 5 = \$5.

d. $PV = \frac{10,000}{0.10 - 0.05} = \$200,000$

2. The present value of the cash flows is:

$$PV = 170,000\left(\frac{1}{0.14} - \frac{1}{0.14(1+0.14)^{10}}\right) = \$886,739$$

and NPV = -800,000 + 886,739 = \$86,739

At the end of five years, the factory's value will be the present value of the five remaining \$170,000 cash flows:

$$PV = \$170,000\left(\frac{1}{0.14} - \frac{1}{0.14\times(1.14)^5}\right) = \$583,623$$

3. We can break this down into several different cash flows, such that the sum of these separate cash flows is the total cash flow. Then, the sum of the present values of the separate cash flows is the present value of the entire project. (All dollar figures are in millions.)

- Cost of the ship is \$8 million
  PV = –\$8 million
- Revenue is \$5 million per year, operating expenses are \$4 million. Thus, operating cash flow is \$1 million per year for 15 years.

$$PV = \$1 \text{ million } \times\left[\frac{1}{0.08} - \frac{1}{0.08\times(1.08)^{15}}\right] = \$8.559 \text{ million}$$

- Major refits cost \$2 million each, and will occur at times t = 5 and t = 10.
  PV = (–\$2 million)/1.08^5 + (–\$2 million)/1.08^{10} = –\$2.288 million
- Sale for scrap brings in revenue of \$1.5 million at t = 15.
  PV = \$1.5 million/1.08^{15} = \$0.473 million

Adding these present values gives the present value of the entire project:

NPV = –\$8 million + \$8.559 million – \$2.288 million + \$0.473 million = –\$1.256 million

4. One way to approach this problem is to solve for the present value of:

(1) \$100 per year for 10 years, and

(2) \$100 per year in perpetuity, with the first cash flow at year 11.

If this is a fair deal, these present values must be equal, and thus we can solve for the interest rate (r).

The present value of \$100 per year for 10 years is:

$$PV = \$100\left[\frac{1}{r} - \frac{1}{r(1+r)^{10}}\right]$$

The present value, as of year 10, of \$100 per year forever, with the first payment in year 11, is: PV_{10} = \$100/r

At t = 0, the present value of PV_{10} is:

$$PV = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

Equating these two expressions for present value, we have:

$$\$100\left[\frac{1}{r} - \frac{1}{r(1+r)^{10}}\right] = \left[\frac{1}{(1+r)^{10}}\right] \times \left[\frac{\$100}{r}\right]$$

which implies that $(1+r)^{10} = 2$. Solving with respect to r we find that r = 7.18%.

5.

a. This is a growing annuity, so the PV is given as:

$$PV = 40,000 \times \left[\frac{1}{(0.08-0.05)} - \frac{(1.05)^{30}}{(0.08-0.05)\times(1.08)^{30}}\right] = \$760,662.53$$

b. PV(salary) x 0.05 = \$38,018.96

Future value = \$38,018.96 x $(1.08)^{30}$ = \$382,571.75

c. $$\$382,571.75 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08\times(1.08)^{20}}\right] \Rightarrow$$

$$C = \$382,571.75 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08\times(1.08)^{20}}\right] = \$38,965.78$$

6. The present value of the annual payments should be equal to \$20,000. Thus,

$$\$20,000 = C \times \left[\frac{1}{0.08} - \frac{1}{0.08\times(1.08)^{12}}\right] \Rightarrow$$

$$C = \$20,000 \Big/ \left[\frac{1}{0.08} - \frac{1}{0.08\times(1.08)^{12}}\right] = \$2,654$$

7. Assume the Zhangs will put aside the same amount each year. One approach to solving this problem is to find the present value of the cost of the boat and then equate that to the present value of the money saved. From this equation, we can solve for the amount to be put aside each year.

$$PV(boat) = \$20,000/(1.10)^5 = \$12,418$$

$$PV(savings) = \text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10\times(1.10)^5}\right]$$

Because PV(savings) must equal PV(boat):

$$\text{Annual savings} \times \left[\frac{1}{0.10} - \frac{1}{0.10\times(1.10)^5}\right] = \$12,418$$

which implies that,

$$\text{Annual savings} \ = \$12,418 \Bigg/ \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5} \right] = \$3,276$$

8. The fact that company A is offering "free credit" tells us what the cash payments are; it does not change the fact that money has time value. A 10% annual rate of interest is equivalent to a monthly rate of 0.83%:

$$r_{monthly} = r_{annual} /12 = 0.10/12 = 0.0083 = 0.83\%$$

The present value of the payments to company A is:

$$\$1,000 + \$300 \times \left[\frac{1}{0.0083} - \frac{1}{0.0083 \times (1.0083)^{30}} \right] = \$8,938$$

A car from company B costs $9,000 cash. Therefore, company A offers the better deal, i.e., the lower present value of cost.

9.

a. First we must determine the 20-year annuity factor at a 6% interest rate.

$$\text{20-year annuity factor} = [1/.06 – 1/.06(1.06)^{20}] = 11.4699.$$

Once we have the annuity factor, we can determine the mortgage payment.

$$\text{Mortgage payment} = \$200,000/11.4699 = \$17,436.91.$$

b. The table is given as follows (see excel file Chapter 2_Exercise 9.xls):

Year	Total payment	Interest payment	Amortization of loan	End-of-year balance
0				200,000
1	17,437	12,000	5,437	194,563
2	17,437	11,674	5,763	188,800
3	17,437	11,328	6,109	182,691
4	17,437	10,961	6,475	176,216
5	17,437	10,573	6,864	169,352
6	17,437	10,161	7,276	162,076
7	17,437	9,725	7,712	154,363
8	17,437	9,262	8,175	146,188
9	17,437	8,771	8,666	137,523
10	17,437	8,251	9,186	128,337
11	17,437	7,700	9,737	118,601
12	17,437	7,116	10,321	108,280
13	17,437	6,497	10,940	97,339
14	17,437	5,840	11,597	85,743
15	17,437	5,145	12,292	73,451
16	17,437	4,407	13,030	60,421
17	17,437	3,625	13,812	46,609
18	17,437	2,797	14,640	31,969
19	17,437	1,918	15,519	16,450
20	17,437	987	16,450	0

c. Nearly 69% of the initial loan payment goes toward interest ($12,000/$17,436.79 = .6882). Of the last payment, only 6% goes toward interest (987.24/17,436.79 = .06).
After 10 years, $71,661.21 has been paid off ($200,000 – remaining balance of $128,338.79). This represents only 36% of the loan. The reason that less than half of the loan has paid off during half of its life is due to the fact that early payments goes to pay the interest on the loan. Only when a significant amount of the debt is paid, can the annual payments go to pay the amount of the loan.

10.

a. The present value of the store is given by the growing perpetuity formula:

$$\text{PV} = \frac{100,000}{0.1 - 0.05} = 2,000,000$$

b. If the value of the store is $2.5 million, this implies that:

$$2,500,000 = \frac{100,000}{0.1 - g} \Rightarrow g = 0.1 - \frac{100,000}{2,500,000} = 0.06$$