# Chapter 5: Valuing bonds

When governments and companies borrow money, they often do so by issuing bonds. A bond is simply a long-term dept. The borrower issues (i.e., sells) a bond to the lender for some amount of cash. The arrangement obligates the issuer to make specified payments to the bondholder on specified dates. These payments are called **coupon payments** (in Europe these coupons are usually paid annually, while in the United States they are usually paid semi-annually). When the bond matures, the issuer repays the dept by paying the bondholder the bond's **face value (principal)**. The **coupon rate** of the bond serves to determine the interest payment: the annual payment is the coupon rate times the bond face value. The coupon rate, the maturity date and the face value of the bond are part of the **bond indenture**, which is the contract between the issuer and the bondholder.

## 5.1. Valuing bonds using the present value formula

Assume that a bond is issued in January 2006 by a government/company with the following characteristics: face value = $1,000, coupon rate = 5% and matures in January 2010. The buyer of this security will receive each year until 2010 an interest payment of $0.05 \times 1,000 = \$50$. This is the bond coupon payment. When the bond matures in 2010 the government/company pays the final $50 interest, plus $1,000 face value. So the cash flows from owning the bond are as follows.

Cash flows

| 2007 | 2008 | 2009 | 2010 |
|---|---|---|---|
| $50 | $50 | $50 | $1,050 |

What is the current price of this bond? To answer this question we must determine the present value of this stream of cash flows. However, in order to use the present value rule we need an appropriate discount rate. We observe that in January 2006 other medium-term bonds (with the same exposure to risk) offered a return of about 3%. That is what you are giving up when you bought the bond. Therefore, to value this 5% bond you need to discount the cash flows at 3%:

$$\text{PV} = \frac{50}{1.03} + \frac{50}{1.03^2} + \frac{50}{1.03^3} + \frac{1,050}{1.03^4} = \$1,074.3$$

This is price that the bond is sold in January 2006. Bond prices are usually expressed as a percentage of face value. Thus we can say that the 5% bond is worth 107.43%.

You may have noticed a shortcut way to value this bond. The present value is like a package of two investments. The first pays off the four annual coupon payments of $50 each like an annuity and the second pays off the $1,000 face value at maturity. Thus,

$$\text{PV} = 50\left(\frac{1}{0.03} - \frac{1}{0.03(1+0.03)^4}\right) + \frac{1,000}{1.03^4} = \$1,074.3$$

In general, any bond can be valued as a package of an annuity (the coupon payments) and a single payment (the face value). In general we can write,

$$P_0 = C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right] + \frac{P_T}{(1+r)^T} \quad (1)$$

where $P_0$ is the current bond price, $P_T$ is the face value, C is the coupon payment, T is the time-to-maturity of the bond and r is an appropriate discount rate.

Rather than asking the value of the bond, we could have phrased our question the other way around: If the price of the bond is $1,074.3, what return do investors expect? In that case we need to find the value of r that solves the following equation:

$$50\left(\frac{1}{r} - \frac{1}{r(1+r)^4}\right) + \frac{1,000}{(1+r)^4} = \$1,074.3$$

This rate r is called the bond's **yield to maturity**. In our case r = 3%. The only general procedure to calculate the yield to maturity is numerically using computer programs, like for example the function YIELD of Microsoft Excel.
The yield to maturity is less than the coupon rate because you are paying $1,074.3 for a bond with face value $1,000. You lose the difference of $74.3 if you hold the bond to maturity. On the other hand you get four cash payments of $50. To **current yield** of your investment is 50/1,074.3 = 4.65%. The yield to maturity blends the return from the coupon payments with the declining value of the bond over its remaining life.
Note that if r = 5% (coupon rate) then the bond price is equal to the face value ($1,000). In our example the bond price is higher compared to the face value because the coupon rate is higher compared to the market interest rate. On the other hand, if the coupon rate is lower to the interest rate the bond will be sold less to the face value. A bond, such as the previous one, that is priced above its face value is said to sell at a *premium*. Investors who buy a bond at a premium face a capital loss over the life of the bond, so the dividend yield of these bonds is always less than the current yield. A bond that is priced below face value sells at a *discount*. Investors in discount bonds face a capital gain over the life of the bond, so the dividend yield on a discount bond is greater than the current yield.

How can we interpret the yield to maturity? This yield is equal to the expected rate of return realized over the life of the bond. However, this feature holds under the assumption that *the future interest rates will be constant and equal to the yield to maturity and that the default risk is not priced.*

**Example**: Assume a two period bond with coupon rate equal to 5% and face value $1,000. Also assume that the current bond price is $964.73. The yield to maturity solves:

$$P_0 = \frac{50}{1+r} + \frac{1,050}{(1+r)^2} = \$964.73$$

The yield to maturity calculated using a computer program is 6.94%.
Consider that the investor will sell this bond at year 1. If the yield to maturity remains at 6.94% then the price would be:

$$P_1 = \frac{1,050}{1+0.0694} = \$981.85$$

The one-year return on this investment is:

$$\frac{981.85+50-964.73}{964.73}=6.94\%$$

which is equal to the yield of maturity.

Now assume that the price of bond after one year is $970. Then, the yield of maturity after one year solves:

$$970=\frac{1,050}{1+r}$$

which gives r = 8.24%. We observe that the yield has increased as the market price is lower than the price predicted by the previous yield of 6.94%. The one-year return of this investment is:

$$\frac{970+50-964.73}{964.73}=5.72\%$$

Now we observe that the one-year return is lower than the yield to maturity as the price after one year is lower than that predicted by the 6.94% yield.

Now suppose that the current price of this bond is $950. The yield to maturity is now 7.79%. We observe that as the price decreases the yield to maturity increases. This price, however, also reflect a possibility of default for the entity that issued the bond. Assumed that there is 20% probability that the entity will default, and that, if default does occur, bondholders receive half of the face value, i.e. $500. In that case the expected payoff will be $50+1,000\times0.8+500\times0.2=950$. We can now estimate the expected return of the bond as the discount rate that makes the expected cash flows of it to be equal to the current price:

$$950=\frac{50}{1+r}+\frac{950}{(1+r)^2}$$

Solving for r we obtain r = 2.26%. We observe that the yield to maturity (which is also called the **promised yield**) overestimate the true expected return. This is due to the fact that the yield to maturity gives the expected return under the assumption that the entity will not default. But the correct expected return is 2.26%, which also takes into account the possibility of a default.

For valuation purposes you can use, either the promised yield with conjunction to the promised face value, or the expected return with conjunction to the expected face value. Both approaches will give you the current value of the bond equal to $950.

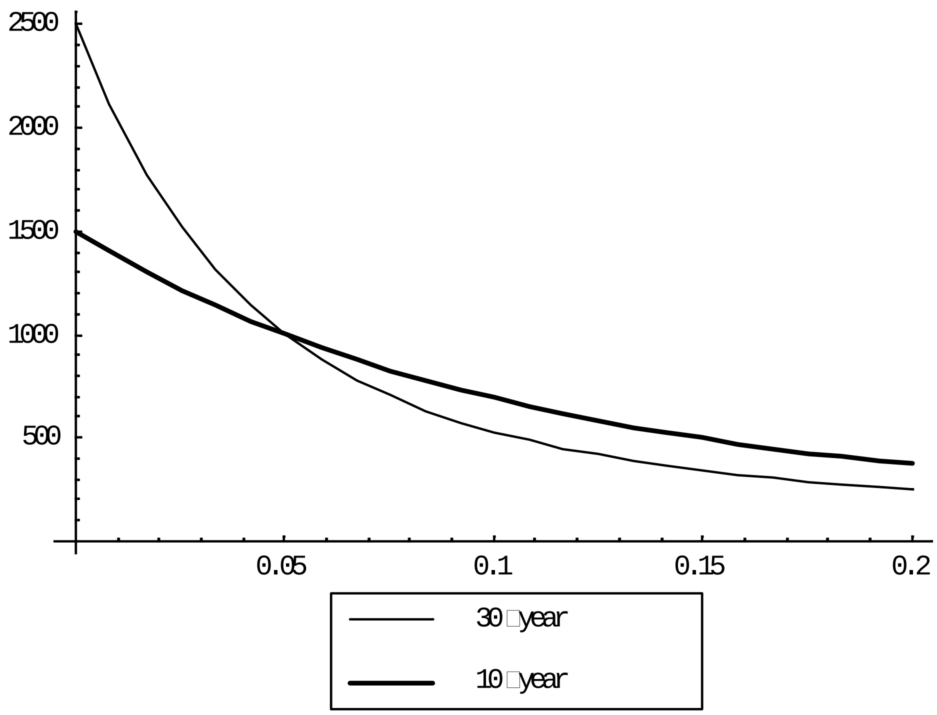
Apart from coupon bonds, already discussed, there is another important category of bonds: **zero-coupon bonds**. These are the simplest assets traded in the market issued by governments. They are bonds that pay no coupons but have a face value that is guaranteed at maturity. In the US they are known as **Treasury strips**. The current price of a zero-coupon bond is given as:

$$\boxed{P_0=\frac{P_T}{(1+r)^T}} \qquad (2)$$

where $P_T$ is the face value, T is the time-to-maturity and r the yield to maturity of this bond. Again, we can inverse formula (2) in order to estimate the yield of a zero-coupon bond given its price $P_0$.

# 5.2. How bond prices vary with interest rates

If interest rates rise the present value of the payments to be received by the bondholder is lower. *Therefore, the bond price will fall as market interest rate rises.* This makes sense. If you buy the bond at the face value with a 5% coupon rate, and market rates subsequently rise, then you suffer a loss: you have tied up your money earning 5% when alternative investments offer higher returns. This will be reflected in a fall in its market price. The following figure shows the price of a 30-year, 5% coupon bond with face value $1,000 for a range of interest rates. The negative slope illustrates the inverse relationship between prices and rates. Note also from the figure that the shape of curve implies that an increase in the interest rate result in price decline that is smaller than the price gain resulting from a decrease of equal magnitude in the interest rate. This property of bond prices is called *convexity*. This feature reflects the fact that progressive increases in the interest rate result in progressively smaller reductions in the bond prices.
One key factor that determines the sensitivity of the bond price with respect to the interest rates fluctuations is the maturity of the bond. The above figure also presents the prices for a 10-year bond. Namely, keeping all other factors the same, the longer the maturity of the bond, the greater the sensitivity of price to fluctuations in the interest rate. This makes sense. If the interest rate subsequently rises, the longer the period for which your money is tied-up, the greater the loss, and correspondingly the greater the drop in bond price. This is why short-term bonds are considered to be safest. They are free of the price risk attributed to the interest rate volatility.




Summarizing the above discussion we conclude the followings:

- Bond prices and yields are inversely related; as the yield increases, bond prices fall and the opposite.

- An increase in a bond's yield to maturity results in a smaller price decline than the price gain associated with decrease of equal magnitude in yield.
- Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds.

We can also demonstrate that:

- The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases.

The above propositions confirm that the maturity is a major determinant of interest rate risk. However, maturity alone is not sufficient to measure interest rate risk. Coupon rates and the yield to maturity at which the bonds currently sell are also significant.

Therefore, we need to define another measure to account for the sensitivity of bond prices to changes in interest rates. This measure is called the **duration** of a bond and it is defined as:

$$\boxed{D = \sum_{t=1}^{T} t \times w_t} \qquad (3)$$

where $w_t = \frac{CF_t / (1+y)^t}{P_0}$ and $CF_t$ is the cash flow of the bond at time t, y is the yield to maturity, $P_0$ is the bond price and T is the time-to-maturity.

The above formula is a weighted average of the times until the receipt of each of the bond's payments. The weight associated with each payment time relates the importance of that payment to the value of the bond. The nominator is just the present value of each payment which is divided by the total present value, equal to the bond's price.

**Example:** Assume a 5.5% 4-year coupon bond with face value $1,000 and yield equal to 2.75%. The initial price of this bond is $1,102.8.

| Year | Payments | PV(payments) | $w_t$ | $t \times w_t$ |
|---|---|---|---|---|
| 1 | 55 | 53.53 | 0.049 | 0.049 |
| 2 | 55 | 52.10 | 0.047 | 0.094 |
| 3 | 55 | 50.70 | 0.046 | 0.138 |
| 4 | 1,055 | 946.51 | 0.858 | 3.433 |
| | | | | D = 3.714 |

Thus, a 4-year zero-coupon bond has the same time to maturity as the 5.5% 4-year coupon bond, but the first three years' coupon payments account for smaller fraction of the bond's value. In this sense the zero-coupon bond is longer than the 5.5% one.

Duration is a key concept in fixed-income portfolio management because is a measure of the interest rate sensitivity of a bond. Specifically, it can be shown that when interest rates change, the proportional change in a bond's price can be related to the change in its yield according to the rule:

$$\boxed{\frac{\Delta P}{P} = -D^* \Delta y} \qquad (4)$$

where $D^* = \frac{D}{1+y}$ defined the **modified duration** of the bond.

The last equation implies that modified duration is a natural measure of the bond's exposure to changes in interest rates. It is the volatility of the bond price related to changes in interest rates.

**Example:** Assume the 5.5% coupon bond with time to maturity 4 years. When the yield to maturity is 2.75% the price is $1,102.8. If the yield increases by 1 point basis to 2.76% its price will fall to $1,102.4, a percentage decline of 0.036%.
The duration of this bond was found equal to 3.714 years. Therefore, the modified duration is: $\frac{3.714}{1+0.0275}=3.6146$.
The percentage change given by formula (4) is:

$$\frac{\Delta P}{P}=-3.6146\times 0.01\%=-0.036\%.$$

The negative sign signifies the decline of the prices with respect to the increase in interest rates.

# 5.3. The term structure of interest rates

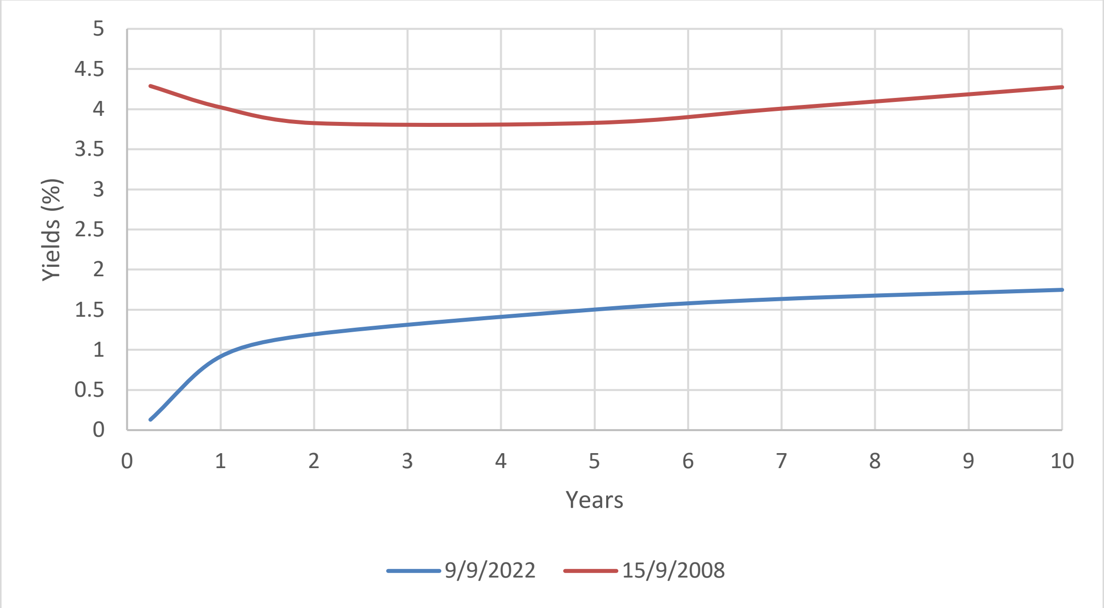
## 5.3.1. Measuring the term structure

In the previous sections we value bonds using a single yield to maturity. For many purposes, using a single discount rate is a perfectly acceptable approximation, but in reality short-term and long-term interest rates are not same.
The relation between short- and long-term interest rates is called the **term structure of interest rates**. It is the relationship between maturity and the respective interest rate. To measure the term structure of interest rates we use the yield to maturity of zero-coupon bonds, for example Treasury strips. Given the prices of these bonds we solve formula (2) with respect to r. These are also known as the **spot rates**.

The following table presents the Euro area spot rates for different maturities on two different days, 15/9/2008 and 9/9/2022.

| Time to Maturity | 15/9/2008 | 9/9/2022 |
|---|---|---|
| 3 months | 4.29% | 0.13% |
| 1 year | 4.02% | 0.92% |
| 2 years | 3.83% | 1.19% |
| 5 years | 3.83% | 1.50% |
| 7 years | 4.01% | 1.63% |
| 10 years | 4.27% | 1.75% |

The data of the table generate the term structure of interest rates for these two different days presented in the following graph. The most common relationship is term structure in which the longer the maturity, the higher the yield (see blue line). That is investors are rewarded for holding longer maturity Treasuries in the form of a higher potential yield. This shape is referred to as a **normal** or **positively sloped term structure**. A **flat term structure** is one in which the yield for all maturities is approximately equal (see red line). However, there have been times when the relationship between maturities and yields were such that the longer the maturity the lower the yield. Such a downward behavior is referred to as an **inverted** or **negatively sloped term structure**.




The term structure of interest rates can be used to price a coupon bond. This is shown in the following example.

**Example**: Consider that the 1-year, 2-year, 3-year and 4-year spot rates are $r_1$ = 1%, $r_2$ = 1.5%, $r_3$ = 2% and $r_4$ = 3%. Consider two coupon bonds. The first has a coupon rate of 3% and matures in 3 years, while the second has the same coupon rate but matures in 4 years. The face value is equal to \$1,000 for both bonds. Using the spot rates we can price the two bonds and then calculate the time to maturity. We have that,

$$P_0 = \frac{30}{1+0.01} + \frac{30}{(1+0.015)^2} + \frac{1,030}{(1+0.02)^3} = 1,029.4$$

The yield to maturity of this bond, denoted as y, solves the following equation:

$$1,029.4 = \frac{30}{1+y} + \frac{30}{(1+y)^2} + \frac{1,030}{(1+y)^3}$$

and it is equal to 1.98%. Observe that the yield to maturity lies somewhere between the three spot rates used in the valuation of this bond. For the second bond we have that:

$$P_0 = \frac{30}{1+0.01} + \frac{30}{(1+0.015)^2} + \frac{30}{(1+0.02)^3} + \frac{1,030}{(1+0.03)^4} = 1,002.2$$

The yield to maturity is equal to 2.94%. Notice now that the yield to maturity increases as bond maturity increases. The yield increases with maturity because the term structure is upward-sloping.

As we can define the term structure of interest rates as a broad image of spot rates we can define the **yield curve** as a broad image of yields to maturity. In the previous example we have calculated two points in the yield curve. Note however, that as the previous example has shown, these two curves are tightly related

### 5.3.2. Explaining the term structure

The term structure in 24/10/1994 (as shown in the previous figure) was upward-sloping. Does this imply that we should invest in long- rather in short-term zero-coupon bonds? And why some investors prefer the low-return short-term bonds? Here are three possible reasons why you might decide to hold short-term bonds, despite their low return:

1. You believe that short-term interest rates will be higher in future
2. You worry about the greater exposure of long-term bonds to changes in interest rates
3. You worry about the risk of higher future inflation

Suppose that you own a 1-year Treasury strip. A year from now, when the bond matures, you can reinvest the proceeds for another 1-year period and get the rate of return of that period, which is not known from today. If you expect that the interest rate of the second year will be high enough to offset the low return of the first year, then you will be willing to buy the 1-year bond. This is known as the **expectations theory**. This argument implies that an upward-sloping term structure indicates that the market expected future short-term rates to rise.

The second reason relates the term structure to risk. If you were confident about the future level of interest rates, you will simply choose the strategy that offers the highest return. But if you are not sure, you may also choose a less risky strategy even if it means giving up some return. As we noted previously longer-term bonds are more sensitive to changes in interest rates. Thus, investors will be prepared to hold long-term bonds only if they offer the compensation of a higher return. In this case the term structure will be upward-sloping more often than not.

Note that the two previous reasons can work together. For example, if the market expects short-term rates to fall, the exposure to risk of long-term bonds, will make the term structure less downward sloped.

The third reason is related to future inflation. Bonds guarantee a nominal payment after a period of time. However, they did not guarantee what you can buy with this amount of money. Stated alternatively, they don’t guarantee the real value of this payment. The risk of inflation makes investors reluctant to invest in long-term bonds. You can reduce the exposure to inflation risk by investing short-term and rolling over the investment. Even if you don’t know future interest rates you know that they will adapt to inflation. If inflation increases, you will be able to roll over your investment at higher interest rates. If inflation is an important source of risk for long-term investors, borrowers must offer some extra incentive if they want investors to lend in long-term. That is why we see a steeply upward-sloping term structure when inflation is particularly uncertain.

## 5.4. The risk of default

The following table presents the yield to maturity (promised yield) of several corporate bonds which matures in 2019 and 2020. Notice that the yields differ substantially and they are related to the bond ratings provided by Moody’s.

| Name | Coupon rate | Maturity | Moody's Rating | Price | Yield (%) |
|---|---|---|---|---|---|
| LYONDELLBASELL INDS N V | 5.00% | 04/15/2019 | Baa1 | 111.964 | 2.26 |
| Reynolds Group Issuer LLC | 5.75% | 10/15/2020 | B1 | 104 | 4.72 |
| SCIENTIFIC GAMES INTL INC | 9.25% | 06/15/2019 | B3 | 105.33 | 6.80 |
| CAESARS ENTMT OPER CO INC | 8.50% | 02/15/2020 | Caa1 | 79.58 | 13.76 |

Data provided by http://finramarkets.morningstar.com/ , May 2014.

The difference to these yields associated to bond ratings can be attributed to the risk of default. For example, Caesars Entertainments bond is considered by Moody's a highly risky investment (referred to as high yield or junk bond) because the agency considers that there is a high probability of default for this firm. On other hand, LyondellBasell, one of the largest plastics, chemicals and refining companies, is considered as a confident firm which can service its debt, and this is reflected in the low yield on its bond.
As argued earlier the safety of most corporate bonds can be judged from bond ratings provided by Moody's, Standard & Poor, and Fitch. The next table reports the possible bond ratings in declining order of quality.

| Moody's | Standard and Poor's and Fitch |
|---|---|
| Investment-grade bonds | |
| Aaa | AAA |
| Aa | AA |
| A | A |
| Baa | BBB |
| Junk bonds | |
| Ba | BB |
| B | B |
| Caa | CCC |
| Ca | CC |
| C | C |

Because of risk of default yields of corporate bonds are higher than those of government bonds. The difference between those two are known as the **yield spread**. We can interpret this yield as the annual premium that would be needed to insure the bond against default. For example, if the difference between a default-free government bond and a comparable corporate bond is 5%, then we can interpret this 5% as an insurance premium that we should pay every year in order to be compensated for any loss in the bond's value in the event of a default.

Credit rating agencies base their quality ratings largely on the level and trend of the some of the issuer's financial ratios. The key ratios used to evaluate safety are:

- Coverage ratios: Times-interest-earned (T-I-E) ratio. A high T-I-E means that the firm's earnings can easily cover its interest obligations.
- Leverage ratios: Debt-to-equity (D/E) ratio. A high D/E ratio means that the firm has accumulated a large amount of debt relative to equity.

- Liquidity ratios: Current ratio, quick ratio. A high liquidity ratio means that the firm has enough liquid assets to pay for its bills.
- Profitability ratios: ROA, ROE. A high ROA (or ROE) means that the firm offers good prospects on the firm's investments.
- Cash flow-to-debt (CF/D) ratio. A high CF/D ratio means that the firm generates enough cash with respect to its outstanding debt.

Based on these ratios we create **credit scoring systems** estimating the probability of default. The next table shows the average financial ratios across firms with different ratings. You can observe that Aaa firms are more profitable, they have a lower debt ratio, and they generate enough cash to repay their debts.

| | Aaa | Aa | A | Baa | Ba | B | C |
|---|---|---|---|---|---|---|---|
| EBITA/Assets (%) | 12.3% | 10.2% | 10.8% | 8.7% | 8.5% | 6.7% | 4.1% |
| Operating profit margin (%) | 25.4% | 17.4% | 14.9% | 12.0% | 11.5% | 9.0% | 4.6% |
| EBITA to interest coverage (multiple) | 11.5 | 13.9 | 10.7 | 6.3 | 3.7 | 1.9 | 0.7 |
| Debt/EBITDA (multiple) | 1.9 | 1.8 | 2.3 | 2.9 | 3.7 | 5.2 | 8.1 |
| Debt/(Debt + Equity) (%) | 35.1% | 31.0% | 40.7% | 46.4% | 55.7% | 65.8% | 89.3% |
| Funds from operations/Total debt (%) | 41.5% | 43.4% | 34.1% | 27.1% | 19.9% | 11.7% | 4.6% |
| Retained cash flow/net Debt (%) | 31.4% | 30.1% | 27.3% | 25.3% | 19.7% | 11.5% | 5.1% |

Note: EBITA is earnings before interest, taxes, and amortization.
Source: Moody's Financial Metrics, *Key Ratios by Rating and Industry for Global Non-Financial Corporations*, December 2013.

# EXERCISES – 5

**1.** Suppose that 5-year government bonds are selling on a yield of 4%. Value a 5-year bond with a 6% coupon. The bond makes annual payments. Now suppose that interest rates fall to 3%. How would the bond value change?

**2.** A newly issued bond has maturity of 10 years and pays 7% coupon rate annually. The face value is $1,000. The bond sells at face value.

a. What is the duration of the bond?
b. Assume that the yield to maturity immediately increases by 1%. What is the percentage price change predicted by the duration rule?

**3.** You have estimated spot rates as follows:

| Year | Spot rate |
|---|---|
| 1 | $r_1 = 5\%$ |
| 2 | $r_2 = 5.4\%$ |
| 3 | $r_3 = 5.7\%$ |
| 4 | $r_4 = 5.9\%$ |
| 5 | $r_5 = 6\%$ |

a. Calculate the prices of the following bonds. Assume annual compounding and face value $1,000.
   - 5% coupon, 2-year maturity
   - 5% coupon, 5-year maturity
   - 10% coupon, 5-year maturity

b. Explain intuitively why the yield to maturity on the 10% 5-year bond is less than the 5% 5-year bond.

c. What should be the yield to maturity on the 5-year zero-coupon bond?

**4.** The following table presents financial data for two companies (in thousands of US dollars):

| Companies | X | Y |
|---|---|---|
| Total assets | 1,600 | 1,200 |
| EBITA | 150 | 70 |
| Interests | 25 | 35 |
| Debt | 700 | 800 |

Calculate the following financial ratios: EBITA/Assets, EBITA/Interests and Debt/Assets for the two firms. Using the last table in Section 5.4 estimate their credit rating. Which firm is considered as safer?