Chapter 2: The Present Value Rule

2.1. Introduction to present value

If you find an investment opportunity how do you decide whether it is worthwhile?

To answer this question we must develop a theory that will determine the value of this investment across time. Since we must take the decision today, we must determine the value of the future payoff of the investment today.

For example assume that we have £370,000 and with that amount of money we can buy a land and construct buildings. Then, we can sell these buildings a year from now and earn £420,000 with certainty. Shall we do this investment?
You should go ahead if the **Present Value** of the £420,000 payoff is greater than the investment of £370,000. Therefore, you need to ask, what is the value today of £420,000 to be received one year from now, and is that present value greater than £370,000?

The alternative investment is to put an amount of money in the bank. If the interest rate is $r = 5\%$ per year then in order to gain £420,000 a year from now we must invest:

$$£420,000 \frac{1}{1+5\%} = £400,000$$

The £400,000 is the **Present Value (PV)** of the £420,000 payoff a year from now. On the other hand, the **Future Value** of £400,000 today is the £420,000.

In our example we know the future value of the office building is equal to £420,000. But we need to find the present value of this future cash flow. The answer is £400,000, since the two investments (bank deposit and office building) are equivalent (concerning the risk). Remember, that in both cases we earn £420,000 with certainty.
Thus, the present value of the investment is £400,000. This means that our investment today worth £400,000. A year from now it will worth £420,000. Thus, £400,000 today is worth more than £400,000 tomorrow, it worth £420,000. In other words, £1 today is worth more than £1 tomorrow. This is the first basic principle in finance and we refer to that as the *time value of money*.

Proposition: A pound today worth more than a pound tomorrow, because the pound today can be invested to start earning interest immediately.

The arithmetic of the present value can, in general, be written as

$$\boxed{PV = \frac{C_1}{1+r}}$$

The $DF = \frac{1}{1+r}$, is called **discount factor**, where r is the rate of return of an equivalent investment alternatives in capital market. This rate is called the **discount rate** or **opportunity cost of capital (OCC)**. The C_1 is the (expected) payoff one year hence.

Let us assume that as soon as you have bought the land and begun construction, you decide to sell the project. How much you sell it for? Since the property will be worth £420,000 in a year, investors will be willing to pay £400,000 for it today. That is what it would cost them to get a £420,000 payoff from investing in a bank deposit. Thus, the £400,000 is the only feasible price that satisfies both buyer and seller. Therefore, *the present value of the property is also its market price*.

Back to the example: You invest £370,000 to a project whose present value (the value of the project the time you make the investment) is £400,000.
The **Net Present Value (NPV)** is £400,000 - £370,000 = £30,000, therefore *the project costs less than it worth so it makes a net contribution to value and increases the wealth. Thus, it should be undertaken.*
In general,

$$\boxed{NPV = C_0 + \frac{C_1}{1+r}}$$

where C_0 is the cash flow at time 0 (that is, today). It is usually a negative number, since it represents an investment and therefore a cash outflow. In our example C_0 = - £370,000.

2.2. Risk and present value

We make an unrealistic assumption that the value of the project is known with certainty, but we are not certain about future values.

The £420,000 is the best forecast but it is not a sure thing. If the future value is risky our calculation of the NPV is wrong. Investors could achieve £420,000 with certainty by putting £400,000 in the bank deposit, so they would not buy your building for that amount. You would have to cut the price to attract investors' interest.
Here we can invoke a second basic financial principle: *A safe pound is worth more than a risky one.*

The concepts of present value and the opportunity cost of capital still make sense for risky investments. It is still proper to discount the payoff by the rate of return offered by an equivalent investment. Thus, we must use the (expected) rate of return of another investment with equivalent risk as the opportunity cost of capital.
Not all investments are equally risky. The office development is more risky than a bank deposit but less risky than a new biotech project. Suppose that you believe that the project is as risky as investment in the stock market, which is forecasted to return 12%. Then the

proper OCC should be 12%. That is what you are giving up by not investing in equally risky stocks. The NPV is now:

$$PV = \frac{£420,000}{1+12\%} = £375,000$$

$$NPV = PV - 370,000 = £5,000$$

The office building still makes a net contribution to value, but the increase in the wealth is smaller than in our first calculation, which assumed that the future value of the office building was risk-free. Like before, the present value is also the market value of the project.

The future value of the project is £420,000. The one-year delay of this payoff reduces value to £400,000 (time value of money). If the project is as risky as investment in the stock market, then uncertainty further reduces value to £375,000 (a safe pound is worth more than a risky one).

Confusion sometimes sneaks into discussion about the cost of capital. Suppose a banker approaches you and he is willing to give you a loan of £370,000 at a rate of 8% in order to construct the office building. Does this mean that the cost of capital is 8%? No, this is not right. First, the interest rate of the loan has nothing to do with the risk of the project: it may reflect the good health of your existing business. Second, whether you take the loan or not, you still face the choice between the office building and an equally risky investment in the stock market. The stock market investment can generate the same expected payoff as your office building at a lower cost. A financial manager who borrows £370,000 at 8% and invests at an office building does not act in the interests of shareholders, if the latter can borrow at 8% and invest the money at an even higher return. That is why the 12% expected return on the stock market is the opportunity cost of capital.

2.3. Present values and rates of return

In the previous sections we have presented the present value rule as a decision rule to undertake a project. We can however state our decision rule in another way: *Our project is worth undertaking because its rate of return exceeds the opportunity cost of capital*.

The rate of return on the investment in the project is simply the profit as a proportion of the initial outflow:

$$\text{Return} = \frac{\text{profit}}{\text{investment}} = \frac{420,000 - 370,000}{370,000} = 0.135, \text{ or } 13.5\%$$

The OCC is once again the return foregone by not investing in securities. If the project is as risky as investing in the stock market, the return foregone is 12%. Since 13.5% return on the project exceeds the 12% OCC, you should go ahead with the project.

Therefore, we have two equivalent decision rules for capital investment:

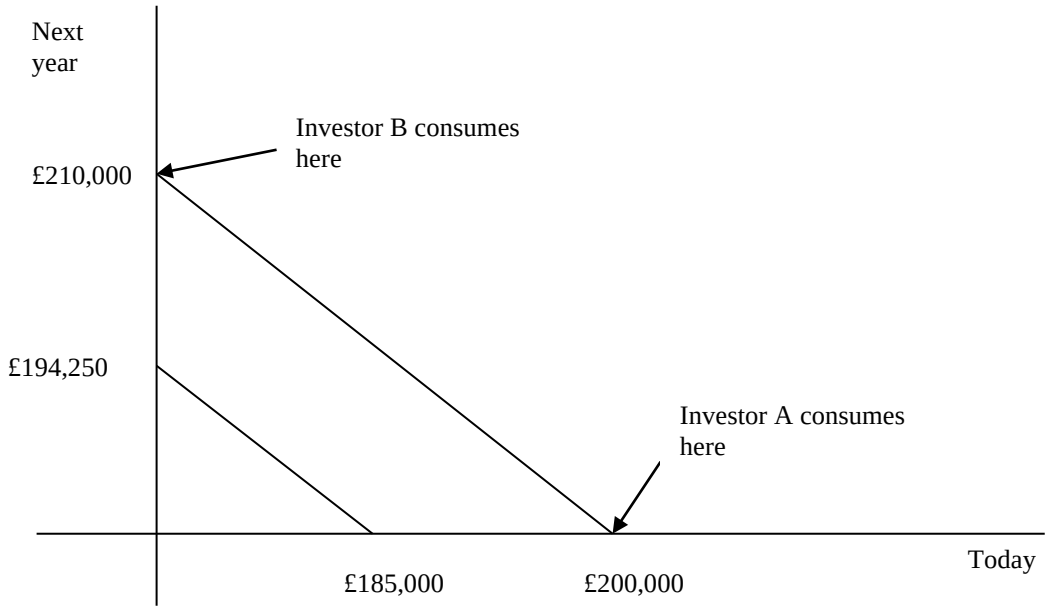
1. **Net present value rule**: Accept investments that have positive net present values.
2. **Rate of return rule**: Accept investments that offer rates of return in excess of their OCC.

Both rules give the same answer, although we will encounter some case later where the rate of return is unreliable. In those cases, you should use the net present value rule.

2.4. The present value rule and the shareholder's preferences

Proposition: The decision taken using the NPV rule is equivalent to the maximization of the wealth of the shareholders of the firm independently of their consumption preferences.

Let investor A and B. Both possess a 50% from the firm but they have different preferences, Investor A wants to consume today than make an investment waiting for the future payoffs, while investor B wishes to save for the future. Investor A would spend his entire share (£370,000/2 = £185,000) now, while investor B would save his amount to the bank and consume $1.05 \times £185,000 = £194,250$ a year from now. However, another investor could select a mixture of these two strategies.



According to the investors preferences should the firm undertake the project?

The answer is **YES**. Both of the investors would be happy since the project will maximize their wealth.

Investor B will earn £210,000 a year from now so he can consume £210,000 – £194,250 = £15,750 more.
Investor A will borrow £200,000, which he can spend today, and return $1.05 \times £200,000 = £210,000$ a year from now. So, he can consume £200,000 - £185,000 = £15,000 more.

Conclusion: The NPV rule is equivalent to the maximization of the wealth of the shareholders independently of their preferences.

The argument we have established is the following:

1. A financial manager should act in the interests of the firm's owners, its stockholders. Each stockholder wants three things: (a) to be rich as possible, that is, to maximize current wealth, (b) to transform that wealth into whatever time pattern of consumption he desires and (c) to choose the risk characteristics of that consumption plan.
2. But stockholders do not need the financial manager's help to achieve the best time pattern of consumption. They can do on their own, providing they have free access to competitive capital markets. They can also choose the risk characteristics of their consumption plan by investing in more or less risky securities.
3. How can the financial manager help the firm's stockholders? There is only one way: by increasing the market value of each stockholder's stake in the firm. The way to do that is to follow all investment opportunities that have a positive NPV.

EXERCISES-2

1. A merchant pays $100,000 for a shipment and is certain that it can be resold at the end of one year for $115,000.
   a. What is the return on this investment?
   b. If the return is lower than the interest rate, does the investment have a positive or a negative NPV?
   c. If the rate of interest is 10%, what is the PV of the investment?
   d. What is the NPV?

2. If the present value of $200 paid at the end of one year is $178.57, what is the one-year discount factor? What is the discount rate?

3. A parcel of land costs $500,000. For an additional $800,000 you can build a motel on the property. The land and motel should be worth $1,500,000 next year. Suppose that common stocks with the same risk as the investment offer a 10% expected return. Would you construct the motel? Why or why not?

4. In Section 2.1, we analyzed the possible construction of an office building on a plot of land appraised at $50,000. We concluded that this investment had a positive NPV of $5,000 at a discount rate of 12%. Suppose a firm of engineers offers to purchase

the land for $58,000, $20,000 paid immediately and $38,000 after one year. U.S. government securities maturing in one year yield 5%.

a. Assume that the firm is sure to pay the second $38,000 cash flow. Should you take its offer or start on the office building?
b. Suppose that you are not sure that the firm will pay. You observe that other investors demand a 10% return on their loans to the firm. Should you accept the offer?

5. Calculate the NPV and the rate of return for each of the following investments. The cost of capital is 20% for all investments.

Investment	C_0	C_1
1	-10,000	18,000
2	-5,000	9,000
3	-5,000	5,700
4	2,000	4,000

a. Which investment is more valuable?
b. Suppose each investment would require the same parcel of land. Therefore you can take only one. Which one?

6. You have just received a $1 million bequest and you are contemplating the following investments:

a. Invest in 5-year government bond yielding 5%.
b. Invest in a local winery. This would involve an outlay of $1 million and is expected to provide an annual income of $1.1 million in the next year. It is about as risky as investment in the stock market.
c. Invest in the stock market. The expected rate of return is 12%.
d. Give a loan to a local restaurant. The owner of the restaurant had arranged a 1-year bank loan for $900,000 at 10%, but asks for $1 million loan from you at 9%.

Which of these investments have positive NPVs? Which would you undertake?

7. Take another look at investment opportunity (b) in exercise 6. Suppose a bank offers you a $600,000 personal loan at 8%. Suppose that you borrow the money, invests $1 million in opportunity (b) and puts the rest of your money in opportunity (c), the stock market. Is this a smart move? Explain.

8. Answer this question by drawing graphs like in Section 2.4. You have $200,000 available to support consumption in periods 0 (now) and 1 (next year). You want to consume exactly the same amount each period. The interest rate is 8% and there is no risk.

a. How much should you invest, and how much can you consume in each period?
b. Suppose that you are given an opportunity to invest up to $200,000 at 10% risk-free. The interest rate stays at 8%. What should you do, and how much can you consume in each period?
c. What is the NPV of opportunity in (b)?