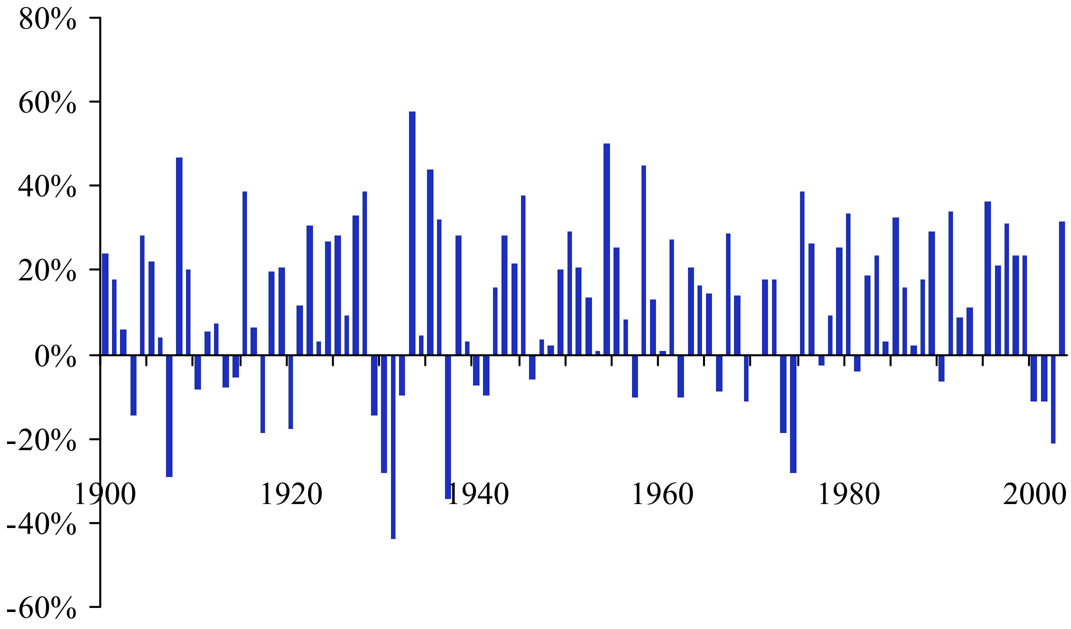
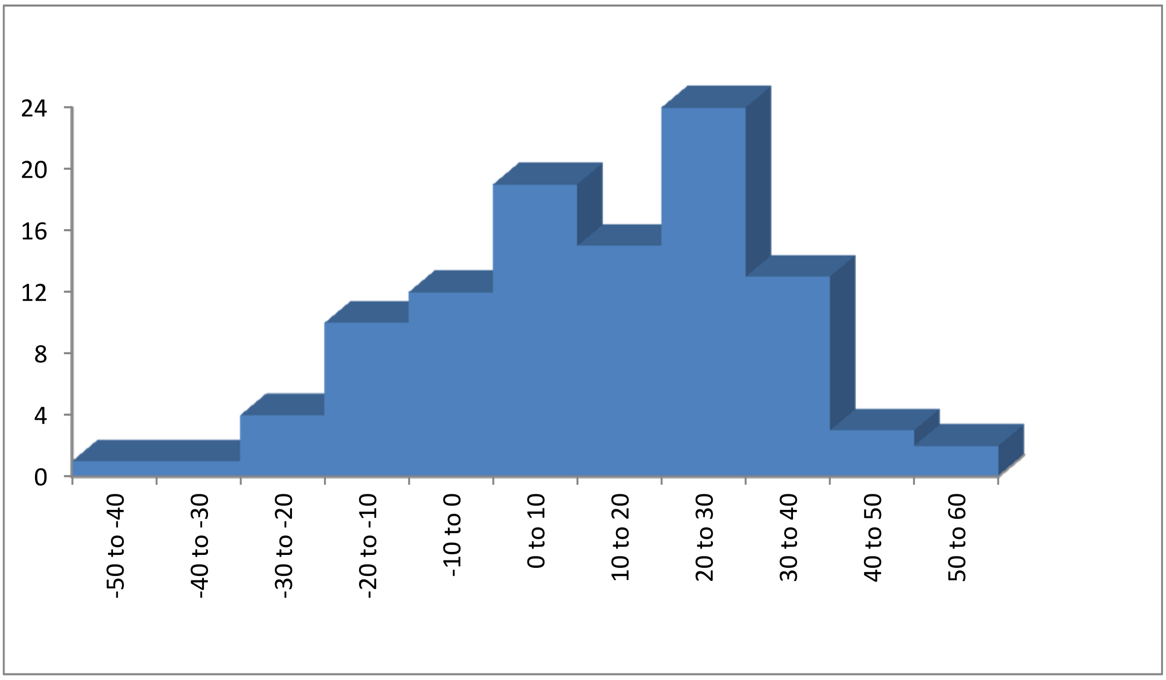
Chapter 10: Risk – Return

10.1. Measuring the variability of an investment

First, what do we mean when we refer to the risk of an investment? Risk is related to the *uncertainty* of future cash flows of an investment. Some of the probable outcomes are not desirable because they will cause the decrease in the value of this investment. The possibility of appearance of these bad outcomes generates risk.
For example the following graph presents the annual return of a portfolio of US common stocks from 1900 to 2003. The fluctuations in year-to-year return are remarkably wide. The highest annual return was 57.6% in 1933 (a partial rebound from the stock market crash of 1929-1232). However, they were losses exceeding 25% in 5 years, the worst being the -43.9% in 1931.



Another way of presenting these data is by a histogram or frequency distribution. This is done by the next figure where the variability of year-to-year returns shows up in wide "spread" of outcomes. This graph implies that an investor who bought this portfolio of US common stocks at the beginning of the year could end up at the end of the year with a return 20-30% with a probability of 24%. However, there is small probability (close to 6%) to end up with a return lower than -20%. How we can measure the degree of this variability?



The standard statistical measures of variability are **variance** and **standard deviation**. The variance of the investment return, denoted as R, is defined as the expected squared deviation from the expected return E(R). In other words,

$$\mathrm{Var(R)} = \mathrm{E}\left[(\mathrm{R}-\mathrm{E(R)})^2\right]$$

The standard deviation is simply the squared root of variance:

$$\mathrm{SD(R)} = \sqrt{\mathrm{Var(R)}}$$

These definitions imply that the greater the variance and the standard deviation are, the more probable is for the investment to have extreme outcomes (including both negative and positive returns). If for example the variance is equal to zero, then the investment return has only one possible outcome equal to the expected return. In that case we have a *certain* investment.

We prefer to measure variability using the standard deviation instead of the variance because the former is measured in the same units as R. Therefore, since R is measured in percent, SD(R) is also quoted in percent. Standard deviation is often denoted by σ and variance by σ^2.

Strictly speaking in order to measure the variance and standard deviation of any future investment you need to identify the possible outcomes, assign a probability to each outcome and do the calculations. But how can you identify the possible outcomes and the respective probabilities? To do so, you need to impose a model (in other words a mathematical formation) which will show how the investment would evolve in the future.

The simplest model that we can impose is to assume that the variability of the returns in the future should be similar to the variability of the investment returns in the past. If we denote as R_i, i = 1, 2, …, N the annual observations of the investment return then the variance of this return can be estimated by:

$$\text{Var}(R) = \frac{1}{N-1}\sum_{i=1}^{N}\left(R_i - \overline{R}\right)^2$$

where $\overline{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$ is the sample mean. This is called the **sample variance**. The square root of the sample variance defines the **sample standard deviation**. If we use the data of the previous graphs we can estimate a sample standard deviation equal to 20.1%. If we repeat the same calculations for a portfolio of short-term US bonds (Treasury bills) and long-term bonds we will get a sample standard deviation of 2.8% and 8.2%. These numbers reflect the fact that for the past century US common stocks were much more volatile than long-term bonds, and those were more volatile than short-term ones.

The selection of the return period (annually in the previous example) and the number of observations (i.e, N) is left to the researcher. Recent research studies demonstrated that using intra-day data produces more accurate estimates for the variability of returns.

10.2. Calculating portfolio risk

In the previous section we have presented how we can measure the variability of a single investment using the variance or the standard deviation. However, most of the times investors hold a basket of different investments, in other words they create *portfolios*. How we can measure the variability of these portfolios?

Assume that the portfolio consists of two common stocks. Let denote as $E(R_1)$, $E(R_2)$ and σ_1, σ_2 the expected returns and the standard deviations of these two stocks, respectively. Let also denote as x_1, x_2 the fraction of the total wealth in the two stocks, respectively. If for example we have a total of \$100 and we have used \$60 to buy shares of the first stock, then $x_1 = 60\%$ and thus $x_2 = 40\%$. Always remember that $x_1 + x_2 = 1$. These numbers define the **weights** of the portfolio.

The expected return of the portfolio is the weighted average of the expected returns of the two stocks, i.e.,

$$\boxed{E(R) = x_1 E(R_1) + x_2 E(R_2)}$$

If $E(R_1) = 10\%$ and $E(R_2) = 15\%$, then

$$E(R) = x_1 E(R_1) + x_2 E(R_2) = 0.6 \times 0.1 + 0.4 \times 0.15 = 0.12$$

Thus the expected return of the portfolio is equal to 12%.

In order to measure the variance and standard deviation of the portfolio we need to introduce a new parameter. This is the **covariance** of the returns of the two common stocks. The covariance measures the linear association of the two returns. It is defined as:

$$\text{Cov}\left(R_1, R_2\right) = E\left[\left(R_1 - E(R_1)\right)\left(R_2 - E(R_2)\right)\right]$$

It can also be written as:

$$\mathrm{Cov}\left(R_1,R_2\right)=\rho_{12}\sigma_1\sigma_2$$

The parameter ρ_{12} is known as the **correlation coefficient** of two returns. This parameter is always between -1 and 1 and accounts for the sign and magnitude of the covariance. We can distinguish three cases:

1. $\rho_{12} = 0$. The returns of the two stocks are uncorrelated. There is no linear association between them.
2. $\rho_{12} > 0$. The returns are positive correlated. This means that as the return R_1 increases, the return R_2 also tends to increase. If $\rho_{12} = 1$, then we have a perfect linear positive dependence between the two returns, i.e., we can write that $R_1 = a + bR_2$ with $b > 0$.
3. $\rho_{12} < 0$. The returns are negatively correlated. This means that as the return R_1 increases, the return R_2 tends to decrease. If $\rho_{12} = -1$, then we have a perfect linear negative dependence between the two returns, i.e., we can write that $R_1 = a + bR_2$ with $b < 0$.

The following graph presents four different cases for the value of the correlation coefficient. As in the case of the variance we can use sample observations to estimate the covariance. If we have a sample of the returns of the two stocks, $R_{1,i}$, $R_{2,i}$, i = 1, 2, …, N, then the **sample covariance** is defined as:

$$\mathrm{Cov}(R_1,R_2)=\frac{1}{N-1}\sum_{i=1}^{N}\left(R_{1,i}-\overline{R}_1\right)\left(R_{2,i}-\overline{R}_2\right)$$

The variance of the portfolio is defined as:

$$\boxed{\begin{aligned}\mathrm{Var}(R) &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\mathrm{Cov}(R_1,R_2) = \\ &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2\end{aligned}} \quad (1)$$

Suppose that the standard deviation of the two stocks is 18.2% and 27.3%, respectively and the correlation coefficient is 0.4. Then the variance of the portfolio equals:

$$\begin{aligned}\mathrm{Var}(R) &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2 = \\ &= 0.6^2\times 18.2^2 + 0.4^2\times 27.3^2 + 2\times 0.6\times 0.4\times 0.4\times 18.2\times 27.3 = 333.9\end{aligned}$$

and the standard deviation $\mathrm{SD}(R)=\sqrt{333.9}=18.3\%$.

Suppose now that the two stocks are perfectly positively correlated, i.e., $\rho_{12} = 1$. Then formula (1) can be written as:

$$\mathrm{Var}(R) = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2 = \left(x_1\sigma_1 + x_2\sigma_2\right)^2$$

and the standard deviation equals:

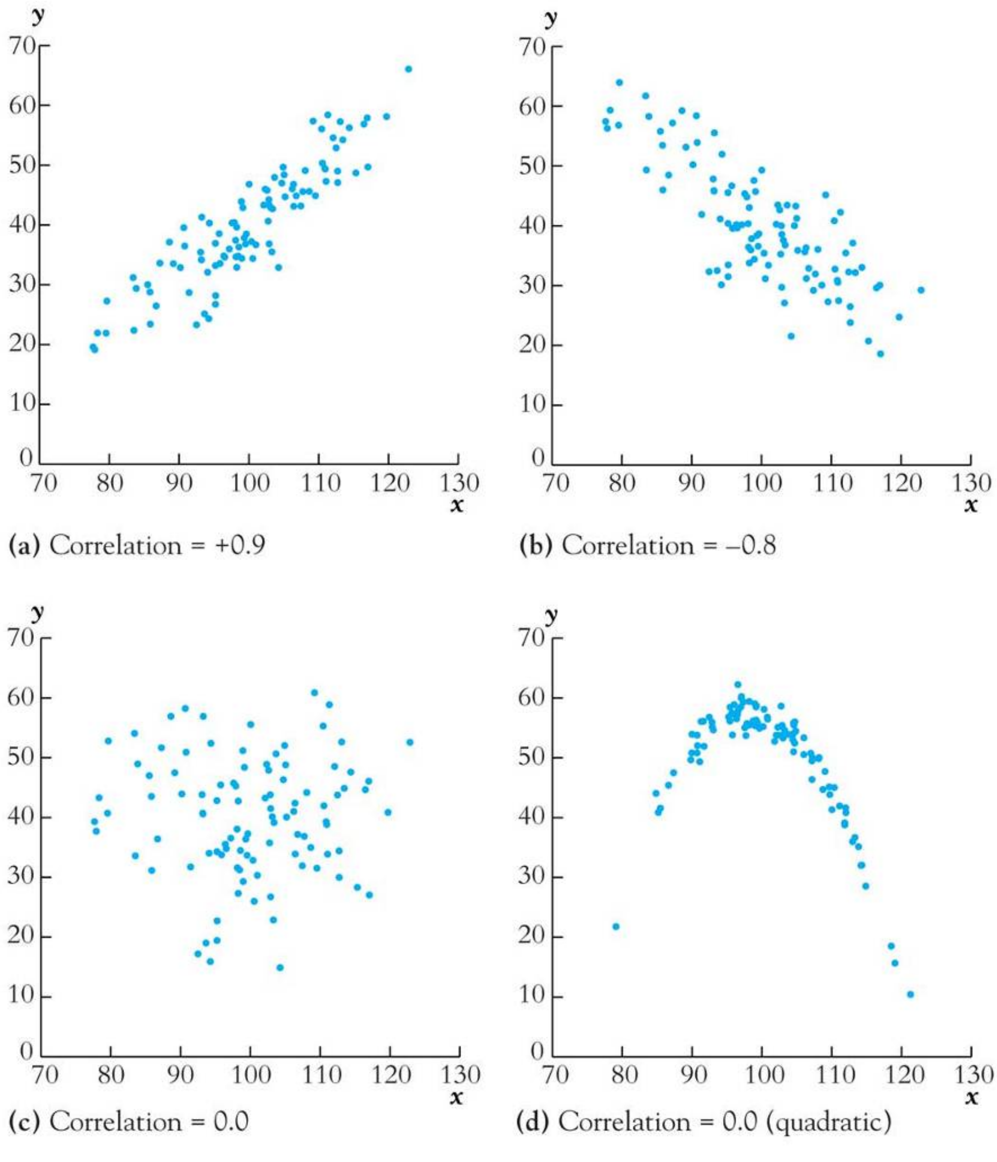
$$\mathrm{SD}(R) = x_1\sigma_1 + x_2\sigma_2$$

In that particular case the standard deviation is a weighted average of the standard deviations of the two stocks. If we fill in the numbers we find that:

$$\mathrm{SD}(R) = x_1\sigma_1 + x_2\sigma_2 = 0.6\times 18.2 + 0.4\times 27.3 = 21.8\%$$

Inspecting the different results we conclude that:

1. A correlation coefficient less than 1 decreases the standard deviation of the portfolio, in other words it decreases the variability of your investment. A correlation coefficient of 0.4 implied a standard deviation of 18.3% compared to 21.8% when $\rho_{12} = 1$.
2. When $\rho_{12} = 0.4$ the portfolio expected return and standard deviation was equal to 12% and 18.3%, respectively. At the same time the expected return and standard deviation of the first stock was 10% and 18.2%, respectively. Thus, it worth investing in the portfolio for gaining an extra 2% of expected return for a 0.1% increase in variability. If you try with a correlation coefficient less than 0.4 you will get a portfolio standard deviation even less than 18.2%.



(a) Correlation = +0.9

(b) Correlation = –0.8

(c) Correlation = 0.0

(d) Correlation = 0.0 (quadratic)

Actually the smaller the correlation coefficient, the lower the portfolio standard deviation would be. The extreme case is to assume $\rho_{12} = -1$. In that case you can write formula (1) as:

$$Var(R) = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2x_1x_2\sigma_1\sigma_2 = \left(x_1\sigma_1 - x_2\sigma_2\right)^2$$

The standard deviation equals:

$$SD(R) = x_1\sigma_1 - x_2\sigma_2$$

In that case we can construct a portfolio so that it has a standard deviation equal to zero. This portfolio would have zero variability and would provide to the investor a certain return of 12% based on the numbers we use. To find this portfolio we set the previous equation equal to zero and we solve for x_1 and x_2. We have that:

$$SD(R) = x_1\sigma_1 - x_2\sigma_2 = 0 \Rightarrow x_1 = x_2\frac{\sigma_2}{\sigma_1}$$

Remember that $x_1 + x_2 = 1$, so:

$$x_1 = x_2\frac{\sigma_2}{\sigma_1} \Rightarrow x_1 = (1 - x_1)\frac{\sigma_2}{\sigma_1} \Rightarrow x_1 = \frac{\sigma_2/\sigma_1}{1+\sigma_2/\sigma_1}$$

In our case

$$x_1 = \frac{\sigma_2/\sigma_1}{1+\sigma_2/\sigma_1} = \frac{27.3/18.2}{1+27.3/18.2} = 0.6$$

and $x_2 = 1 - x_1 = 1 - 0.6 = 0.4$.

Of course in real life there are no common stocks with perfect negative correlation. However, the basic conclusion from the above discussion is that creating portfolios provides to investors the average expected return of individual securities but with variability lower to the average one as far as the correlation coefficient is not 1. The reduction of variability by investing in a variety of assets is called **diversification**. If the asset values do not move up and down in perfect synchrony, a diversified portfolio will have less variability than the weighted average variability of its constituent assets. The loss in one asset is not followed by a loss in the second asset and that reduces the variability of the total position. In the extreme case where the $\rho_{12} = -1$ the loss in the first asset is followed, with certainty, by a gain in the second asset which eliminates the variability of the total position. *Financial theory dictates that because of the diversification any rational investor should invest in portfolios of assets rather than in individual securities. For this rational investor the risk of his/her position can be measured by the standard deviation of the portfolio.*

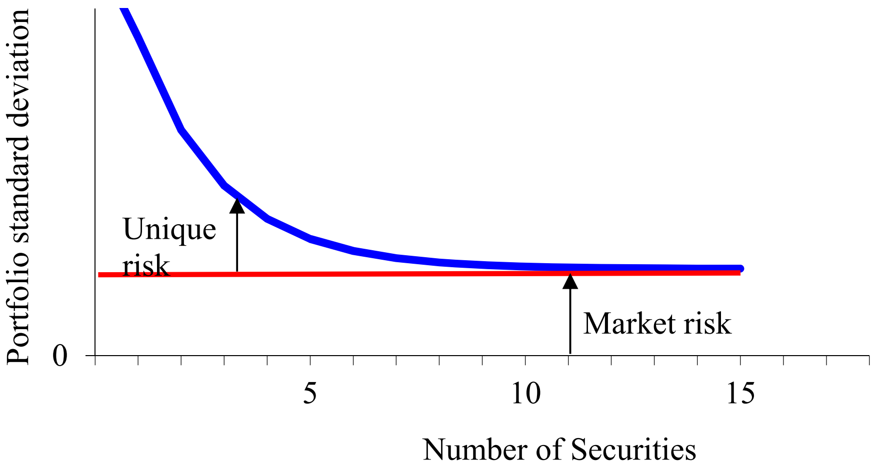
Most of times we create portfolios that contain for than two assets. In that case formula (1) can be written as:

$$\boxed{Var(R) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}}$$

where N is the number of assets in the portfolio. Note that when i = j, then $\sigma_{ij} = \sigma_i^2$, the variance of stock i.

The following graph presents the relation between the number of stocks in the portfolio and its standard deviation. Even a little diversification can provide a substantial reduction in variability. However, as you can also observe we cannot eliminate the total variability even if N becomes very large. The risk that potentially can be eliminated by diversification is called **unique risk** (or **unsystematic risk** or

diversifiable risk). Unique risk stems from the fact that much of the uncertainties that surround an individual company are peculiar to that company and perhaps its competitors. But there is also a risk that you can't avoid, regardless of how much you diversify. This risk is generally known as the **market risk** (or **systematic risk** or **undiversifiable risk**). Market risk stems from the fact that there are other economy-wide uncertainties that threaten all businesses. That is why stocks have a tendency to move together. And that is why investors are exposed to market uncertainties, no matter how many stocks they hold. Based on this analysis, a *well-diversified* portfolio is a portfolio in which the unique risk of the individual securities has disappeared.



10.3. How individual securities affect portfolio risk

In the previous paragraph we have shown that a rational investor reduces the risk by diversification. Thus he/she is not interested in the risk of an individual security but in the effect that each security will have on the risk of their portfolios.
In order to measure this effect we should rewrite formula (1) as:

$$\begin{aligned} Var(R) &= x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2 = \\ &= x_1\left(x_1\sigma_1^2 + x_2\rho_{12}\sigma_1\sigma_2\right) + x_2\left(x_2\sigma_2^2 + x_1\rho_{12}\sigma_1\sigma_2\right) = \\ &= x_1\sigma_{1p} + x_2\sigma_{2p} \end{aligned}$$

The term σ_{1p} is the covariance of the first stock with the portfolio. The term σ_{2p} is the covariance of the second stock with the portfolio. If you denote $\sigma_p^2 \equiv Var(R)$ you can write the previous formula as:

$$x_1\frac{\sigma_{1p}}{\sigma_p^2} + x_2\frac{\sigma_{2p}}{\sigma_p^2} = 1$$

The last formula describes the fraction of the portfolio risk which is due to each stock. Thus, $x_1\frac{\sigma_{1p}}{\sigma_p^2}$% of the total risk is due to the first stock and $x_2\frac{\sigma_{2p}}{\sigma_p^2}$% is due to the second stock. These terms depend on two parameters. The first is the fraction of wealth invested in the security (the x_i) and the second is $\frac{\sigma_{ip}}{\sigma_p^2}$. This term is known as the **beta** of the individual security *relative to the portfolio*. This parameter captures the effect of the stock to the risk of the portfolio. The larger the beta of a stock, the more this stock contributes to the overall risk of the investment.

For the previous example (with $\rho_{12} = 0.4$) we can show that the beta of the first stock equals:

$$\beta_1 = \frac{\sigma_{1p}}{\sigma_p^2} = \frac{278.2}{333.9} = 0.83$$

while the beta of the second stock is:

$$\beta_2 = \frac{\sigma_{2p}}{\sigma_p^2} = \frac{417.4}{333.9} = 1.25$$

Thus $x_1\beta_1 = 0.6 \times 0.83 = 0.5$ of the total risk is due to the first stock, and $x_2\beta_2 = 0.4 \times 1.25 = 0.5$ is due to the second one.

We can also demonstrate that a small increase in the fraction of wealth in the first (second) stock would increase the relative risk of the portfolio by $\beta_1(\beta_2)$. So, beta measures the sensitivity of the portfolio risk to a change in the position held in the respective security.

The definition of beta implies that this parameter is measured with respect to a specific portfolio. However, a rational investor would hold a portfolio that eliminates all unique risk of the individual securities. Among these well-diversified portfolios it exists one with special interest. It is the **market portfolio** that contains all the possible investments in the economy. Beta is then measured relative to this market portfolio, i.e.,

$$\boxed{\beta_i = \frac{\sigma_{im}}{\sigma_m^2}}$$

where σ_{im} is the covariance between the stock returns and the market portfolio returns and σ_m^2 is the variance of the market portfolio returns.

We have shown that beta represents the effect of each stock to the risk of the portfolio. In the context of the market portfolio, where only market risk exists, beta should capture the market risk of the respective security. We can decompose the total variability of the stock in two terms. The first is related to the market risk and the second is related to the unique risk. Mathematically we can write:

$$\boxed{\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_\varepsilon^2}$$

The first term, which accounts for the market risk, depends on the beta of the particular security.

Any investor could choose any well-diversified portfolio he/she prefers. In the context of the previous formula this means $\sigma_\varepsilon^2 = 0$. So,

$$\sigma_p^2 = \beta_p^2\sigma_m^2$$

where β_p is the beta of the portfolio. We can demonstrate that the beta of the portfolio is given as the weighted average of the betas of the securities that form this portfolio. For the previous example this implies that:

$$\beta_p = x_1\beta_1 + x_2\beta_2 = 1$$

The previous formula implies that a rational investor would choose a portfolio with an exposure to the market risk (measured by σ_m^2) that depends on the beta of this portfolio. When $\beta_p = 1$ (like the example) then, $\sigma_p^2 = \sigma_m^2$, in other words we have invested in a portfolio with a risk equal to the market risk. If now the portfolio beta where 1.5 then the risk of this portfolio would have been 1.5 times that of the market. A well-diversified portfolio with a beta of 1.5 would amplify every market move by 50% and end up with 150% of the market risk. If the portfolio beta were 0.5, it would be half as risky as the market portfolio. Thus the risk of a well-diversified portfolio is proportional to the portfolio beta, which equals the average beta of the securities. But the beta of a security captures the market risk of this asset. This brings us to a fundamental concept in financial theory. *The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio. The market risk of the security is measured by its beta.*

10.4. The capital asset pricing model

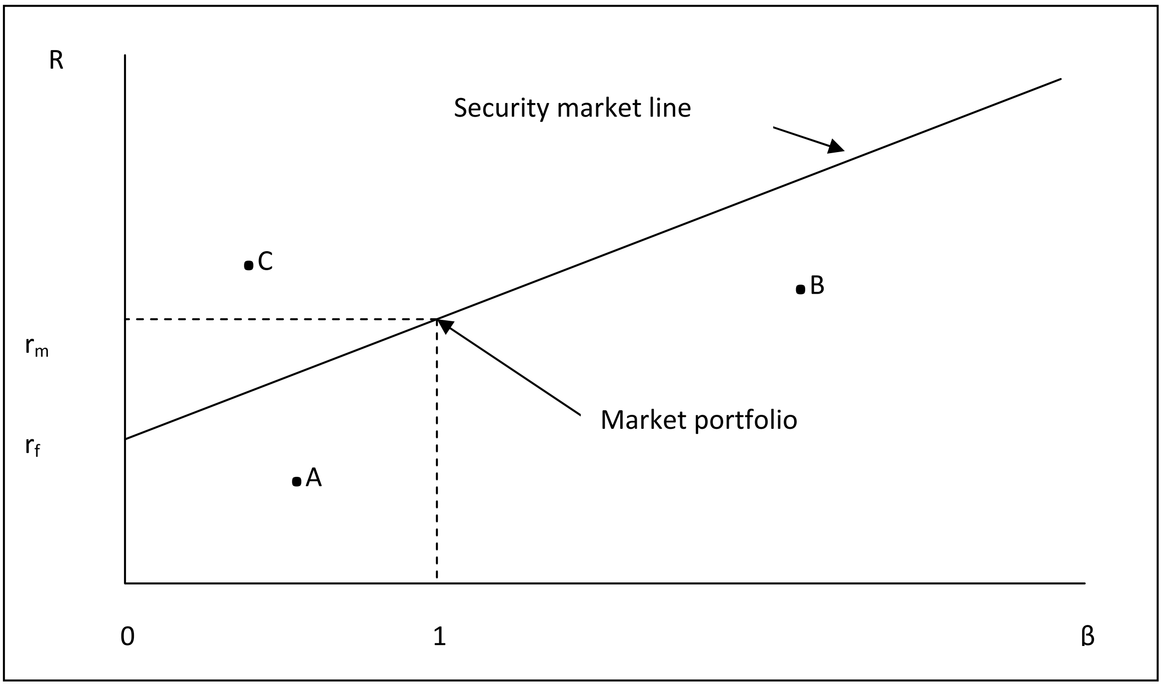
Consider an investment with a fixed return. For example a zero-coupon bond, like the US Treasury bills. Since the return of these securities is fixed they should have beta equal to 0. The return of this security is known as the **risk-free rate**, denoted as r_f. We have also introduced the market portfolio, which by construction, has a beta equal to 1. The difference between the expected return of the market portfolio, denoted as r_m, and the risk free rate is known as the **market risk premium**. For example in the US since 1900 the market risk premium has averaged 7.6% a year.

The next figure plots the risk (measured by the beta) and expected return from Treasury bills and the market portfolio. The Treasury bills have a beta equal to 0 and a risk premium equal to 0. The market portfolio has a beta equal to 1 and a risk premium $r_m - r_f$. This gives us two benchmarks for the expected risk premium. But what is the expected risk premium when beta is not 0 or 1?

The answer is provided by the **capital asset pricing model** (**CAPM**). This model states that in a competitive market, the expected risk premium varies in direct proportion to beta. This means that in the figure all investments must plot along the sloping line, known as the **security market line**. The expected risk premium of an investment with a beta of 0.5 is, therefore, half the expected market risk premium; the expected risk premium on an investment with a beta of 2 is twice the expected market risk premium. We can write this relationship as:

$$\boxed{r - r_f = \beta\left(r_m - r_f\right)} \qquad (2)$$

This formula comes from the fact that any investor separates his/her wealth between the market portfolio and the risk-free asset. The fraction of his/her wealth placed in the market portfolio depends on his/her aversion towards risk. A more risk-averse investor would place a larger fraction of his wealth in the risk-free asset than in the market portfolio. So everybody holds the market portfolio. In the previous section we have shown that an investor should care about the effect of an individual security to the overall risk of the portfolio. This effect is measured by the beta. Therefore, the risk premium of an individual investment depends on the beta of this investment and the risk premium of the market portfolio, which the investor partly holds. That is what the CAPM says.



Imagine that you encounter stock A in the previous graph. Would you buy it? The answer is no. If you want an investment with the same beta, you could get a higher expected return by investing part of your money in the risk-free asset and part in the market portfolio. This is what the CAPM implies since you can write formula (1) as:

$$r = (1-\beta)r_f + \beta r_m$$

So by investing (1- β)% of your wealth in the risk-free asset and β% of your wealth in the market portfolio you could get an expected return on the security market line, i.e., higher than the expected return of stock A. So, everybody would sell stock A, the price would drop, which means that the expected return increases until it matches the security market line. What about stock B? Again you would not buy it. You would prefer to invest (1 - β)% in the risk-free asset and buy β% of the market portfolio. But now $\beta > 1$, which means that $1 - \beta < 0$. So you should invest a negative fraction of your wealth in the risk-free asset. This means that you should use the risk-free asset in order to borrow money and place this money to the market portfolio.

So in a competitive capital market nobody would hold a stock that offers an expected risk premium of less than that predicted by the CAPM. But what about other possibility? Are there investments, like stock C lying above of the security market line? If we take all stock together, we have the market portfolio. Therefore, we know that stocks on average lie on the line. Since none lies below the line, then there also can't be any that lie above the line. Thus each and every stock must lie on the security market line and offer an expected risk premium given by the CAPM.

10.5. Company and project cost of capital

The **company cost of capital** is defined as the expected return on a portfolio of all the company's existing securities. It is the opportunity cost of capital for investment in the firm's assets, and therefore the appropriate discount rate for the firm's average-risk projects. Instead of investing in a new project, the firm could return the money to shareholders. The opportunity cost of investing is the return that investors could expect to earn by buying financial assets. This expected return is the company cost of capital. If the firm has no significant amount of debt outstanding, then the company cost of capital is just the expected rate of return on the firm's stock. Therefore, if a firm decides to expand its existing business it could discount the forecasted cash flows at the company cost of capital.

However, the company cost of capital is *not* the correct discount rate if the new projects are more or less risky than the firm's existing business. Each project should in principle be evaluated at its own opportunity cost of capital.

For a firm composed of assets A and B the firm value is:

$$\text{Firm value} = PV(AB) = PV(A) + PV(B)$$

Investors would value A by discounting its forecasted cash flows at a rate reflecting the risk of A, and they would value B, possibly to a different discount rate reflecting the risk of B. Therefore, *the true cost of capital depends on the use to which that capital is put.*

Even if the company cost of capital may not be the correct discount rate its estimation is useful for two reasons:

1. many projects can be treated as average-risk. For these projects the company cost of capital is the right discount rate.
2. the company cost of capital is a useful starting point for setting discount rates for riskier or less risky projects.

We defined the company cost of capital as "the expected return on portfolio of all the company's existing securities". That portfolio usually includes dept as well as equity. Thus the company cost of capital is estimated as a blend of the cost of dept (interest rate) and the cost of equity (the expected return demanded by investors in the firm's stock). Since the company cost of capital is the average of the cost of dept and the cost of equity it is also called the **weighed-average cost of capital (WACC)**. Therefore,

$$\boxed{WACC = r_{assets} = \frac{D}{V} r_D + \frac{E}{V} r_E}$$

We denote D the market value of the dept, E the market value of equity and V the firm's value, where V = D + E.

For example if the value of the firm is V = \$100,000 and the D = \$30,000 is the value of a dept with expected return 7.5% and E = \$70,000 is the value of the equities with expected return 15% then,

$$WACC = \frac{30,000}{100,000} 7.5\% + \frac{70,000}{100,000} 15\% = 12.75\% .$$

If the firm is planning to invest to a project that has the same risk as the firm's existing business, the OCC = WACC = 12.75%.

10.6. Measuring the cost of equity

The cost of equity, that is, the expected return of the firm's stock, can be estimated by the CAPM given by formula (2).

However, in order to calculate the expected stock return you must first estimate beta. In practice you estimate beta by running the above regression:

$$r_{E,t} - r_f = \alpha + \beta\left(r_{m,t} - r_f\right) + \varepsilon_t,\ t = 1,2,...,T .$$

where $r_{E,t}$ and $r_{m,t}$ are historical returns of the stock and of the market portfolio (which is duplicated by a large index). The Excel file Chapter 10_beta.xls presents an example for the estimation of the stock beta through the previous regression.

Empirical studies have shown that there is a large margin for error when estimating beta for individual stock. That is why financial managers often use *industry betas*. These betas are estimated using a portfolio of the same sector stocks.

Suppose that in mid-2004 you were asked to estimate the cost of equity for Union Pacific Corporation. The beta of the Union's Pacific stock was estimated at 0.47, whereas the industry beta was estimated at 0.49. Taking into consideration the previous statement we will use the industry beta. In May 2004 the risk-free interest rate was 3.3%. Suppose you decide to use a market risk premium of 8%. Then, the resulting estimate for Union's Pacific cost of equity is:

$$r_E = r_f + \beta\left(r_m - r_f\right) = 3.3\% + 0.49 \times 8\% = 7.2\%$$

It is always useful to check on such estimates. To this end, we can get the cost of equity estimate based on the constant-growth discounted cash flow (DCF) formula

$$r_E = \frac{D_1}{P_0} + g$$

where P_0 is the current stock's price, D_1 is the dividend paid at the end of year 1 and g is the constant growth rate of dividends. Based on the average monthly dividend yield equal to 1.44% and a forecasted growth rate of 12.05% this analysis gives for the Union Pacific a cost of equity of 13.5%, much more high than the CAPM estimates. You could look to further checks, using more elaborated pricing models (for example a multi stage DCF model or an extension of the CAPM, incorporating more variables).

10.7. Setting discount rates when the beta is unknown

In many cases the previous analysis to calculate the discount rate using the stock or industry beta cannot be used. For example, if the project is not average-risk, or your company is the first that runs this project.

In some cases an asset is publicly traded. If so we can estimate risk from past prices. Suppose your company wants to assess the risk of investing in commercial real estate. Here, it can turn into indexes of real estate prices and returns derived from sales and appraisals of commercial properties.

But there are also many cases where the assets involved in the project have not a price record or the investment is not close enough to the existing business of the company to justify using the WACC. In such cases three pieces of advice can be offered:

1. **Avoid fudge factors. Adjust cash flow first.**
   We have defined risk, from the investor's viewpoint, as the standard deviation of portfolio return or the beta of common stock or other security. But investors and companies have in mind "bad outcomes" when they talk about risk. The risks of a project reflect things that can go wrong, like for example a geologist that looks for oil worries about the risk of a dry hole.
   Managers often add fudge factors to discount rates to offset such worries. However, this sort of adjustment is not the right method to treat risk. First, the bad outcomes, like the one we cited, appear to reflect unique (i.e. diversifiable) risks that would not affect the expected return demanded by investors. Second, the need for a discount rate adjustment usually arises because managers fail to give bad outcomes their due weight in cash flow forecasts. The managers then try to offset that mistake by adding a fudge factor to the discount rate.
   What managers ought to do is first try to estimate as good as possible the expected cash flows of the project. Most of the times the adjustment for the possible "bad outcomes" should be made in the forecasted cash flows of the project than in the cost of capital. Managers often work out a range of possible outcomes for the projects, sometimes which explicit probabilities attached. We will give more

elaborated examples and further discussion in the following sections. So the first step to perform is to provide a precise estimate of the cash flows. The second step is to consider whether investors would regard the project as more or less risky than typical for a company. Here you must search for characteristics of the project that are associated with high or low betas. This brings us to our second point.

2. **Often the characteristics of high and low beta assets can be observed when the beta itself cannot.**
   a. **Cyclicality**. The beta is a measure of the market risk of an investment. It depends on the correlation of this investment to the market portfolio. For example, the search of gold can be thought as a project with extremely uncertain future earnings, but it is not likely to depend on the performance of the market portfolio. Even if they do find gold, they do not bear much market risk. Therefore, an investment in gold has a high standard deviation but a relatively low beta.
   What really counts is the correlation between the firm's earnings and the aggregate earnings an all real assets. We can measure this either by the *accounting beta* or by the *cash flow beta*. These are just like a stock beta except that changes in book earnings or cash flow are used in place of rates of return on securities. We would predict that firms with high accounting or cash flow betas should also have high stock betas.
   This means that cyclical firms (firms whose revenues and earnings are strongly dependent on the state of the business cycle) tend to be high beta firms. Thus you should demand a higher rate of returns from investments whose performance is strongly tied to the performance of the economy.
   b. **Operating leverage**. A production facility with high fixed costs, relative to variable costs, is said to have high *operating leverage*. High operating leverage means high risk.
   The cash flow generated by any project can be broken down into revenues, fixed costs and variable costs:
   $$CF = Rev - FC - VC$$
   We can break down the project's present value in the same way:
   $$PV = PV(Rev) - PV(FC) - PV(VC)$$
   or equivalently
   $$PV(Rev) = PV(FC) + PV(VC) + PV$$
   We can think of the last relationship as determining the present value of a portfolio, which is equal to the sum of the securities values. Thus, the beta of the portfolio (the revenues) is a weight average of the betas of the component part:
   $$\beta_{Rev} = \beta_{FC}\frac{PV(FC)}{PV(Rev)} + \beta_{VC}\frac{PV(VC)}{PV(Rev)} + \beta\frac{PV}{PV(Rev)}$$
   The fixed cost beta should be about zero; whoever receives the fixed costs receives a fixed stream of cash flows. The beta of the revenues and variable costs should be approximately the same, because they respond to the same

underlying variable, the rate of output. Therefore, if we set in the last equation $\beta_{FC} = 0$, $\beta_{Rev} = \beta_{VC}$ and we solve for the project beta we obtain:

$$\beta = \beta_{Rev} \frac{PV(Rev) - PV(VC)}{PV} = \beta_{Rev} \left(1 + \frac{PV(FC)}{PV} \right)$$

Thus, given the cyclicality of revenues (reflected in β_{Rev}), the project beta is proportional to the ratio of the PV of the fixed costs to the PV of the project. Therefore, other things be equal the alternative with the higher ratio of fixed costs to project PV will have the higher project beta. Empirical tests confirm that companies with high operating leverage do have high betas.

3. Do not confuse beta with diversifiable risk. A project may look extra risky but if the projects uncertainties are not correlated with market or other macroeconomic risks, then the project is only average-risk to a diversifiable investor.

EXERCISES-10

1. Suppose the standard deviation of the market return is 20%.

a. What is the standard deviation of returns on a well-diversified portfolio with a beta of 1.3?

b. What is the standard deviation of returns on a well-diversified portfolio with a beta of 0?

c. A well-diversified portfolio has a standard deviation of 15%. What is its beta?

d. A poorly diversified portfolio has a standard deviation of 20%. What can you say about its beta?

2. Download to a spreadsheet monthly adjusted prices for two stocks with a ticker symbol which is close to your family name from **finance.yahoo.gr** or from **google.com/finance**.

a. Calculate the annual standard deviation of returns for each firm, using the most recent three years of monthly returns. Use the Excel function STDEV.

b. Use the Excel function CORREL to calculate the correlation coefficient between the monthly returns.

c. Calculate the standard deviation of returns for a portfolio with equal investments in each of two stocks.

Download the monthly adjusted prices of the S&P 500 index. This can be considered as a proxy of the market portfolio. Also assume that the risk-free rate is equal to 3%. Beta is calculated by the Excel function SLOPE where “y” range refers to the firm’s risk premium and “x” range refers to the market risk premium.

d. Calculate the beta of the two stocks.

e. Calculate the beta of the portfolio that you have constructed in c.

f. Estimate the expected return of the two stocks and the portfolio assuming an expected market risk premium equal to 8%.

3. There are investments (like some derivative products) with negative betas. Suppose you find such an asset with β = -0.25.

a. How would you expect this asset's rate of return to change if the overall market rose by an extra 5%? What if the market fell by an extra 5%?

b. You have $1 million invested in a well-diversified portfolio of stocks. Now you receive an additional $20,000 bequest. Which of the following actions will yield the safest overall portfolio return? (i) Invest $20,000 in Treasury bills (which have beta equal to 0), (ii) Invest $20,000 in stocks with β = 1, (iii) Invest $20,000 in the asset with β = -0.25. Explain your answer.

4. You have $10 million invested in long-term corporate bonds. This bond portfolio's expected annual rate of return is 9% and the annual standard deviation is 10%. A financial manager advises you to invest in an index fund that closely tracks the S&P 500 index. The fund has an expected return of 14% and its standard deviation is 16%.

a. Suppose that you put all your money in a combination of the index fund and Treasury bills. Can you thereby improve your expected return without changing the risk of the portfolio? The Treasury bill yield is 6%.

b. Could you do even better by investing equal amounts in the corporate bond portfolio and the index fund? The correlation coefficient between the bond portfolio and the index fund is 0.1.

5. The total market value of a common stock of firm A is $6 million and the total value of its debt is $4 million. The treasurer estimates that the beta of the stock is 1.5 and that the expected risk premium of the market is 6%. The Treasury bill rate is 4%.

a. What is the expected return on firm's A stock?

b. Estimate the company cost of capital.

c. What is the discount rate for an expansion of the firm's present business.

d. Suppose the company wants to diversify into a manufacture of a different product than the main product that the firm produces. The beta of a firm's stock that produces this alternative product is 1.2. Estimate the required return on the firm's A new project.

6. You run a perpetual encabulator machine, which generates revenues averaging $20 million per year. Raw material costs are 50% of revenues. These costs are variable, they are always proportional to revenues. There are no other operating costs. The cost of capital is 9%. Your firm's borrowing rate is 6%. Now you are approached by another firm, which proposes a fixed-price contract to supply raw materials at $10 million per year for 10 years.

a. What happens to the operating leverage and business risk of the encabulator machine if you agree to the fixed-price contract?
b. Calculate the present value of the encabulator machine with and without the fixed-price contract.

7. PowerGun Company has just dispatched a year's supply of machineguns to the government of the Central Antarctic Republic. Payment of $250,000 will be made one year hence after the shipment arrives. Unfortunately there is good chance of a coup d'etat, in which case the new government will not pay. PowerGun Company financial manager therefore decides to discount the payment at 40% rather than at the 12% cost of capital.

a. What's wrong with using 40% rate to offset the political risk?
b. How much is the $250,000 payment really worth if the odds of a coup d'etat are 25%?