# CAPITAL BUDGETING WITH DEBT FINANCING

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## Topics covered

- The Adjusted Present Value (APV) Method
- The after-tax Weighted Average Cost of Capital (WACC) method
- Capital structure and the cost of capital
  - the cost of equity
  - the cost of debt
  - Modigliani-Miller propositions
- Effect of personal taxes
- Effect of financial distress

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## Capital Budgeting with debt financing

- Questions:
  - How to do capital budgeting with debt financing?
  - What is a firm's optimal capital structure?
- Assumptions:
  - capital markets are frictionless
  - individuals can borrow and lend at $r_f$.
  - Corp. taxes are the only form of government levy
  - there are no costs to bankruptcy
  - all cash flow streams are perpetuities (can be relaxed)
  - managers always maximize shareholders' wealth (i.e. no agency costs)

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- Notation:
  - $E$, $B$: market value of equity and debt
  - $D$: face value of debt (principal)
  - $A$: market value of assets
  - $r_A$, $r_E$, $r_D$: required rate of return on assets, equity and debt
  - $\tau_c$: corporate tax rate
  - $V$: market value of the firm ($V = E + B$)
  - $V_U$, $V_L$: value of unlevered and levered firm ($V_U = E$, $V_L = E + B$)
  - $\rho$: opportunity cost of capital (correct discount rate for all equity financed project)
  - $r_{TS}$: required rate of return on tax-shield
  - $r$ : interest rate charged on principal ($D$)

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## 1) The Adjusted Present Value (APV) Method

- Principle: calculate the value of the unlevered firm (or project) and add the value of financing effects separately
- How: separate cash flows related to the unlevered firm from those arising from financing effects, and discount each appropriately
- After-tax cash flows are calculated as follows:

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| | |
|---|---|
| NOI | (Net Operating Income) |
| - r D | (interest on debt) |
| EBT | (Earnings Before Taxes) |
| - $\tau_c EBT$ | (corporate taxes) |
| NI | (Net Income) |

- After-tax cash-flows must be split up between debtholders and shareholders:

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| | |
|---|---|
| shareholders receive: | $(NOI - rD)\,(1-\tau_c)$ |
| bondholders receive: | $rD$ |
| total: | $NOI(1-\tau_c) \; + \; \tau_c rD$ |

- Hence the value of the levered firm is:

$$\begin{aligned} V_L &= \frac{NOI(1-\tau_c)}{\rho} + \frac{\tau_c rD}{r_{TS}} \\ &\equiv V_U + \frac{\tau_c r_D B}{r_{TS}} \\ &\equiv V_U + \text{present value of taxshield} \quad (1) \end{aligned}$$

where $B = \frac{rD}{r_D} =$ the market value of debt

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## Case 1: no corporate taxes ($\tau_c = 0$)

- firm value is independent of capital structure as expressed in MM proposition I
- Modigliani-Miller Proposition I: 'In the absence of any market imperfections (including corporate taxes), the market value of any firm is independent of its capital structure and is given by capitalizing its expected cash flows at the rate, $\rho$, appropriate to its risk class.'
- M&M proposition I is based on the 'law of the conservation of value' or the principle of 'value additivity'.

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$\longrightarrow$ 'The value of the pie is independent of how it is sliced'
$\longrightarrow$ Firm value is determined on the *left-hand* side (of the balance sheet) by real assets, *not* by the proportions of debt and equity securities.

- Combining assets and splitting them up will not affect values as long as this does not affect investor's choice.

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## Case 2: corporate taxes ($\tau_c > 0$)

$$\begin{aligned} V_L &= V_U + \frac{\tau_c r_D B}{r_{TS}} \\ &\equiv V_U + \text{present value of taxshield} \qquad (2) \end{aligned}$$

- If $\tau_c > 0 \Longrightarrow$:
  . government 'subsidizes' interest payments
  . optimal capital structure: 100% debt
- Firms are 100% debt financed; possible ways out of this extreme:
  - Perhaps a fuller examination of the system of

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corporate *and personal* taxation will uncover a tax disadvantage of corporate borrowing, offsetting the present value of the corporate tax shield
- Perhaps firms that borrow incur other costs (eg. bankruptcy costs) offsetting the present value of the tax-shields
- Perhaps other market imperfections discourage firms from using debt financing only

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## Required rates of return under corporate taxes

- We know that:

$$V = V_L = \frac{NOI\,(1-\tau)}{\rho} + \frac{\tau_c r_D B}{r_{TS}} \tag{3}$$

$$r_E = \frac{(NOI - r\,D)\,(1-\tau)}{E}$$
$$\Longrightarrow NOI\;(1-\tau) = r_E\,E + r\,D(1-\tau) \tag{4}$$

- Hence:

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$$V \;=\; \frac{r_E E \;+\; r_D B(1-\tau_c)}{\rho} \;+\; \frac{\tau_c r_D B}{r_{TS}} \qquad (5)$$

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**Assumption 1** *The debt is rebalanced in each future period to keep it at a constant fraction of future project value (i.e. constant leverage ratio).*

- Hence, future debt levels shift up or down depending on the success or failure of the project
  $\Longrightarrow$ interest tax shields pick up project's business risk
  $\Longrightarrow r_{TS} = \rho$

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- Substituting $r_{TS}$ by $\rho$ in $V$ gives:

$$\rho \;=\; r_E\,\frac{E}{V} \;+\; r_D\,\frac{B}{V} \qquad (6)$$

- Rearranging gives M&M proposition II:

$$r_E \;=\; \rho \;+\; (\rho - r_D)\frac{B}{E} \qquad (7)$$

- Using CAPM, the equations can be reformulated as:

$$\beta_A \;=\; \beta_E\,\frac{E}{V} \;+\; \beta_D\,\frac{B}{V} \qquad (8)$$

$$\beta_E \;=\; \beta_A \;+\; (\beta_A - \beta_D)\frac{B}{E} \qquad (9)$$

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## 2) An example: Archimedes Levers

- Financed by mixture of debt and equity
- No corporate taxes
- $r_D = 12\%$  $\beta_E = 1.5$  $r_f = 10\%$
- $r_m = 18\%$  $\frac{B}{V} = .5$
- Determine: $r_E$ , $r_A$ , $\beta_D$ , $\beta_A$

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**Solution**

- Using CAPM:

$$\begin{aligned} r_E &= r_f + \beta_E(r_m - r_f) \\ r_E &= .10 + 1.5(.18 - .10) \\ r_E &= .22 \end{aligned} \tag{10}$$

$$\begin{aligned} r_D &= r_f + \beta_D(r_m - r_f) \\ .12 &= .10 + \beta_D(.18 - .10) \\ \beta_D &= .25 \end{aligned} \tag{11}$$

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$$\begin{aligned} r_A &= \frac{B}{B+E}\, r_D + \frac{E}{E+B} r_E \\ r_A &= .5(.12) + .5(.22) \\ r_A &= .17 \end{aligned} \tag{12}$$

$$\begin{aligned} \beta_A &= \frac{B}{B+E}\, \beta_D + \frac{E}{E+B} r_E \\ \beta_A &= .5(.25) + .5(1.5) \\ \beta_A &= .875 \end{aligned} \tag{13}$$

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- since $r_A = \rho$, one can check that:

$$
\begin{aligned}
r_E &= \rho + (\rho - r_D)\frac{B}{E} \\
r_E &= 0.17 + (0.17 - 0.12)1 \\
r_E &= 0.22
\end{aligned} \tag{14}
$$

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## 2) Archimedes Levers (continued)

- Supppose Archimedes repurchases debt and issues equity so that $\frac{B}{V} = 0.3$.

- this causes $r_D$ to fall to $11\%$.
- How do the other variables change?

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Solution:

- $r_A = \frac{expected\ operating\ income}{overall\ cost\ of\ capital} = constant = 0.17$ (MM prop I)
- Hence: $\beta_A = .875$
- Risk and return of equity and debt will change

\- Therefore, for debt:

$$\begin{aligned} r_D &= r_f + \beta_D(r_m - r_f) \\ .11 &= .10 + \beta_D(.18 - .10) \\ \beta_D &= .125 \end{aligned} \tag{15}$$

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\- Also, for equity:

$$\begin{aligned} r_A &= \frac{B}{B+E} r_D + \frac{E}{E+B} r_E \\ .17 &= .3(.11) + .7 r_E \\ r_E &= .196 \end{aligned} \tag{16}$$

$$\begin{aligned} r_E &= r_f + \beta_E(r_m - r_f) \\ .196 &= .10 + \beta_E(.18 - .10) \\ \beta_E &= 1.20 \end{aligned} \tag{17}$$

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- Alternatively, one can check that:

$$\frac{B}{B+E} = 0.3$$

$$\frac{B}{E} = \frac{3}{7} \quad (18)$$

$$r_E = \rho + (\rho - r_D)\frac{B}{E}$$

$$r_E = 0.17 + (0.17 - 0.11)\frac{3}{7}$$

$$r_E = 0.196 \quad (19)$$

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## 2) The after-tax WACC Method

- WACC = 'Weighted Average Cost of Capital'
- Principle: estimate the free cash flows to an unlevered firm (or project) and discount them at the WACC.
- $V = V_L = \frac{NOI(1-\tau)}{WACC}$
- Value for WACC?
- Under assumption of constant leverage ratio:

$$r_E = \frac{(NOI - r\,D)\,(1-\tau_c)}{E} \Longrightarrow \quad (20)$$

$$NOI\,(1-\tau_c) = r_E E + r\,D\,(1-\tau_c) \quad (21)$$

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- Hence,

$$WACC = \frac{NOI(1-\tau_c)}{V} = r_E \frac{E}{V} + r_D(1-\tau_c)\frac{B}{V} \quad (22)$$

- Miles and Ezzell (1980) have shown that the formula can be extended to any cash flow pattern (e.g. non-perpetual)

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## Required rates of return under corporate taxes

- It follows from M&M I and II that:

$$\begin{aligned} \rho &= r_E \frac{E}{V} + r_D \frac{B}{V} \\ &= \textit{opportunity cost of capital} \\ r_E &= \rho + (\rho - r_D)\frac{B}{E} \\ &= \textit{required return on levered equity} \end{aligned} \quad (23)$$

- Substituting $r_E$ into $WACC$ gives:

$$WACC = \rho - \tau_c r_D \frac{B}{V} \quad (24)$$

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## How to calculate WACC in practice

- B: market value if traded debt, book value otherwise
- E: market capitalization of equity
- $r_D$: known if debt has (little or) no default risk
- $r_E$: use CAPM or a constant growth DCF model, e.g.:

$$\begin{aligned} r_E &= \frac{Div}{P_0} + g \\ &= dividend\ yield + growth\ rate \end{aligned} \tag{25}$$

- $\tau_c$: the company's marginal tax rate

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## Cash flows for capital budgeting purposes

- The appropriate definition of *'free cash flow for capital budgeting purposes'* is after-tax cash flow from operations, assuming that the firm has no debt and net of gross investment, $\Delta I$.

$$\begin{aligned} FCF &= NOI - \tau_c (NOI - Dep) - \Delta I \\ &= NOI(1-\tau_c) + \tau_c Dep - \Delta I \\ &= (NOI - Dep)(1-\tau_c) + Dep - \Delta I \end{aligned} \tag{26}$$

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- Interest expenses and their tax shield are not included in the definition (discounting at the WACC implicitly assumes that the project will return the expected interest payments to creditors and the expected dividends to shareholders)

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## Rebalanced Debt versus Fixed Debt

- If we were to assume that the absolute debt level remains fixed at $B$, then the tax shields are tied to fixed interest payments and hence $r_D$ is a reasonable discount rate.
- Recalculating the formulas by substituting $r_{TS}$ by $r_D$ gives:

$$\rho \;=\; r_E \, \frac{E}{E + B(1-\tau)} \;+\; r_D \, \frac{B(1-\tau)}{E + B(1-\tau)} \quad (27)$$

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- Rearranging gives M&M proposition II:

$$r_E \;=\; \rho \;+\; (\rho - r_D)(1-\tau_c)\frac{B}{E} \qquad (28)$$

- Using CAPM, the equations can be reformulated as:

$$\beta_A \;=\; \beta_E \,\frac{E}{E \,+\, B(1-\tau)} + \beta_D \,\frac{B(1-\tau)}{E \,+\, B(1-\tau)} \qquad (29)$$

$$\beta_E \;=\; \beta_A \;+\; (\beta_A - \beta_D)(1-\tau_c)\frac{B}{E} \qquad (30)$$

- In reality, the truth may be somewhere in between, i.e. $r_D \,<\, r_{TS} \,<\, \rho$

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## Rebalanced Debt versus Fixed Debt

- If we were to assume a fixed absolute debt level and hence $r_{TS} \,=\, r_D$ then recalculating gives:

$$WACC \;=\; \rho\left(1 \,-\, \tau_c\frac{B}{V}\right) \qquad (31)$$

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## Consistency between APV and WACC Method

- One can check that the WACC and APV method are consistent (for $r_{TS} = \rho$)

$$
\begin{aligned}
\frac{NOI(1-\tau_c)}{WACC} &= \frac{NOI(1-\tau_c)}{\rho} + \frac{\tau_c\, r_D\, B}{\rho} \\
\Longleftrightarrow \quad NOI(1-\tau_c)\,(\rho - WACC) &= r_D\, B\tau_c WACC \\
\Longleftrightarrow \quad NOI(1-\tau_c)\, r_D\, \tau_c \frac{B}{V} &= r_D\, B\, \tau_c WACC \\
\Longleftrightarrow \quad \frac{NOI(1-\tau_c)}{WACC} &= V \qquad (32)
\end{aligned}
$$

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## Firm Value with both Personal and Corporate Taxes

Notation:

- $\tau_{pB}$ = personal tax rate on interest (i.e. tax on income received from holding bonds)
- $\tau_{pE}$ = effective personal tax rate on equity income (i.e. tax on income received from holding shares: dividends and capital gains)

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| | Operating Income: \$1 | |
|---|---|---|
| | paid out as interest | paid out as equity income |
| Corporate tax | none | $\tau_c$ |
| Income after corporate tax | \$ 1 | \$ $(1 - \tau_c)$ |
| Personal tax | $\tau_{pB}$ | \$ $\tau_{pE}\,(1 - \tau_c)$ |
| Income after all taxes | \$ $(1 - \tau_{pB})$ | \$ $(1 - \tau_c)\,(1 - \tau_{pE})$ |
| | to bondholders | to stockholders |

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- The expected after-tax stream of cash flows to shareholders of an all-equity firm would be: $NOI\,(1 - \tau_c)\,(1 - \tau_{pE})$. Hence,

$$V_U = \frac{NOI\,(1 - \tau_c)\,(1 - \tau_{pE})}{\rho} \qquad (33)$$

- If both bonds and shares outstanding:
  - payments to shareholders:
  $(NOI - rD)\,(1 - \tau_c)\,(1 - \tau_{pE})$
  - payments to bondholders: $rD\,(1 - \tau_{pB})$

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- The value of the levered firm is then:

$$V_L = \frac{NOI\,(1-\tau_c)\,(1-\tau_{pE})}{\rho} + \frac{rD\,[(1-\tau_{pB})-(1-\tau_c)\,(1-\tau_{pE})]}{r_{TS}}$$
$$= V_U + \left[1 - \frac{(1-\tau_c)\,(1-\tau_{pE})}{1-\tau_{pB}}\right] B\frac{r_D}{r_{TS}} \quad (34)$$

where $B \equiv \frac{rD(1-\tau_{pB})}{r_D}$ = market value of debt

- Corporate borrowing *increases* the value of the firm if:

$$(1-\tau_{pB}) > (1-\tau_c)\,(1-\tau_{pE}) \quad (35)$$

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- if $(1-\tau_{pB}) = (1-\tau_c)\,(1-\tau_{pE}) \rightarrow$ debt policy is irrelevant
- In the US it is reasonable to assume that the effective tax rate on common stock is lower than that on bonds: $\tau_{pE} < \tau_{pB}$ (the taxes on the capital gains component can be deferred indefinitely). Hence, the gain from leverage is lower than $\tau_c B$ when personal taxes are considered.

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## Costs of Financial Distress

- Financial distress, if it occurs, is costly and will, therefore, be reflected in the current value of the levered firm:

$$\begin{aligned}\text{Value of firm} &= \text{Value if all equity financed} \\ &\quad + \text{Present value of tax shield} \\ &\quad - \text{Present value of financial distress}\end{aligned}$$

- The costs of financial distress depend on:
  - the probability of distress occuring
  - the magnitude of costs encountered if distress occurs

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- The trade-off between the tax benefits and the costs of distress determines optimal capital structure (figure)

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- The present value of the tax shield is not monotonically increasing all the way through. The reason is that if the firm cannot be sure in advance of profiting from the corporate tax shield, the tax advantage is likely to decrease (or at least level off) beyond some leverage level
- The theoretical optimal debt ratio is reached when the present value of tax savings due to additional borrowing is just offset by increases in the present value of costs of distress

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## Costs of Distress vary with the Type of Asset

- Not only the probability of financial distress is important. Also the value that may be lost if default occurs is important.
- In liquidation:
  - some assets, like good commercial real estate, preserve their value
  - intangible assets linked to the health of the firm as a going concern diminish in value (eg. goodwill, human capital, technology, brand image,...)

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- This explains why debt ratios are low in:
  - the pharmaceutical industry (assets = R&D)
  - service industry (assets = human capital)
  - profitable growth companies, such as Microsoft or Hewlett Packard (assets = technological know-how)

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## The Trade-off Theory of Capital Structure

- This theory balances the tax advantages of borrowing against the cost of financial distress.
- Corporations pick a target capital structure that maximizes firm value:
  - Firms with safe, tangible assets and plenty of taxable income ought to have high targets
  - Unprofitable companies with risky, intangible assets ought to rely primarily on equity financing

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- This theory succesfully explains many industry differences in capital structure, but it does not explain why the most profitable firms *within* an industry generally have the most conservative capital structures