

# ΣΥΝΔΥΑΣΜΟΙ

ΟΡΙΣΜΟΣ:  $X$   $|X| = N$

ΣΥΝΔΥΑΣΜΟΣ (ΜΗ-ΔΙΑΤΕΤΑΓΜΕΝΑ ΕΓΧΙΝΩΡΑ) :  
(COMBINATION)

ΜΗ ΔΙΑΤΕΤΑΓΜΕΝΗ K-ΑΔΑ ΔΙΑΚΡΙΤΩΝ ΕΓΧΙΝΩΡΩΝ

$\Rightarrow k \in \mathbb{N}$

ΠΑΡΑΔΕΙΓΜΑ 2.5

4 ΒΙΒΛΙΑ  
2 ΕΓΧΙΝΩΡΑ

$X = \{A, B, C, D\}$

$\{A, B\}$   $\{A, C\}$ ,  $\{A, D\}$ ,  $\{B, C\}$ ,  $\{B, D\}$ ,  $\{C, D\}$

$$\binom{4}{2} = \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2}{2 \cdot 2} = 6$$

$$P(\text{Βιβλίο } A) = \frac{|\{\langle\langle \text{Βιβλίο } A \rangle\rangle\}|}{|X|} = \frac{3}{6}$$

ΛΗΜΜΑ 2.3

k ΑΝΤΙΒΕΤΩΝΑ ΣΤΟ N

ΠΗΓΕΣ ΣΥΝΔΥΑΣΜΩΝ ΕΙΝΑΙ:

$$y = \binom{N}{k} = \frac{N!}{k!(N-k)!}$$

$$\binom{10}{3} = \frac{10!}{3!7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{3! \cdot \cancel{7!}} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

ΑΠΟΔΕΙΞΗ:

y

$$N(N-1) \dots (N-k+1) = y \cdot k!$$

|         |     |
|---------|-----|
| {1,2,3} | 134 |
| {1,2,4} | 142 |
| {1,2,5} |     |
| 126     |     |
| :       |     |
| 12,10   |     |

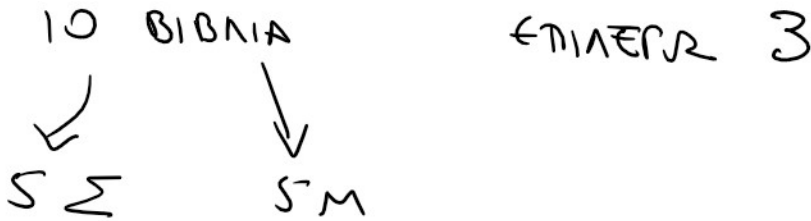
$$\Rightarrow N \cdot \dots \cdot (N-k+1)(N-k)(N-k-1) \dots 3 \cdot 2 \cdot 1$$

ABC

$$\Rightarrow y = \frac{n \cdot \dots \cdot (n-k+1)(n-k)(n-k-1) \dots 3 \cdot 2 \cdot 1}{k! \cdot (n-k)(n-k-1) \dots 3 \cdot 2 \cdot 1 \cdot (n-k)!}$$

- ABC
- A < B
- B < A
- B < C
- C < B
- C < A

ΠΑΡΑΔΕΙΓΜΑ 2.6



A = << 3 ΜΟΝΟ ΜΑΝΟΥΛ >>

B = << 2 ΜΑΝΟΥΛ, 1 ΕΤΙΛΕΡ >>

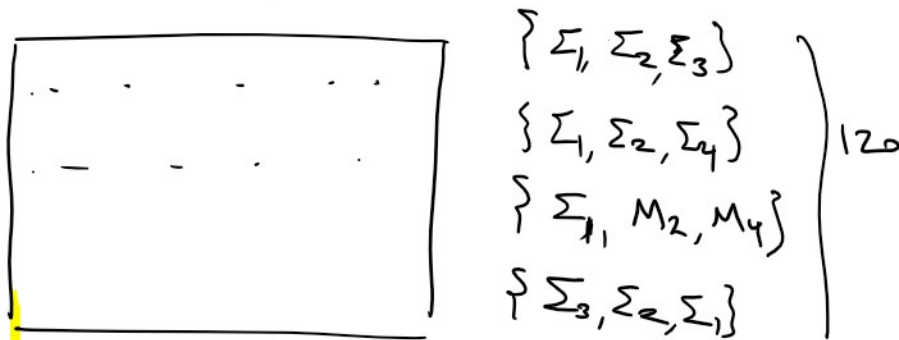
C = << 3 ΜΟΝΟ ΕΤΙΛΕΡ >>

P(A) = ;  
P(B) = ;

ΛΥΣΗ:

Ω: ουσία οι μη διατεταγμένες 3

$|\Omega| = \binom{10}{3} = 120$



- 123    235
- 124    234
- 125    245
- 134    345
- 135
- 145

$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \cdot 4}{2} = 10$

$P(A) = \frac{|A|}{|\Omega|} = \frac{10}{120} = \frac{1}{12}$

~~132~~  
~~231~~

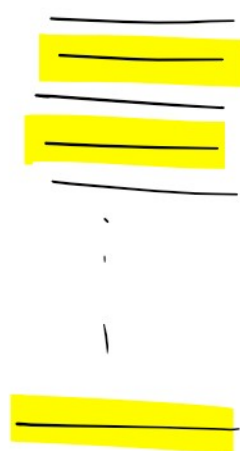
~~132  
231  
213  
312  
321~~

120 120 12

10 BIRNIA  
 5M 5Σ  
 ME A/ATA=GR  
 $P(\text{MONO MANUAL}) = \frac{5 \cdot 4 \cdot 3 / 3!}{10 \cdot 9 \cdot 8 / 3!} = \frac{10}{120}$

2M, 1Σ

$P(B) = \frac{|B|}{120}$   
 $= \frac{50}{120}$



$\binom{5}{2} = \frac{5 \cdot 4}{2} = 10$   
 $\binom{5}{1} = \frac{5!}{1!4!} = 5$   
 $= 50$



$P(A) = P(C)$   
 $P(B) = P(D)$   
 $P(A) + P(B) + P(C) + P(D) = 1$

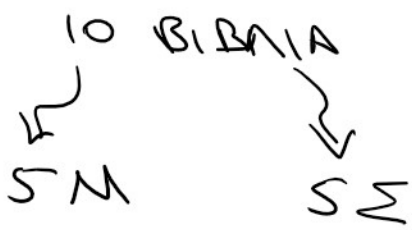
$B = \langle \langle 2M, 1\Sigma \rangle \rangle$

$10 \cdot 9 \cdot 8 = 720$

$|B| = ;$

10 BIRNIA





$$P(\overbrace{2M, 1\Sigma}) = P(B) = \frac{|B|}{|\Omega|}$$

$$|\Omega| = 10 \cdot 9 \cdot 8 = 720$$

1)  $M_1 M_2 M_3$

$1\Sigma, 2M$

2)  $M_2 M_1 M_3$

$$5 \cdot 5 \cdot 4 \cdot 3 = 300$$

3)  $\Sigma_1 M_4 M_5$

4)  $\Sigma_2 M_4 M_5$

$$P(B) = \frac{300}{720} = \frac{30}{72} = \frac{15}{36} = \frac{5}{12}$$

5)

⋮

[Redacted]

$\Sigma M M$

$M \Sigma M$

$M M \Sigma$

$5 \cdot 5 \cdot 4$

$5 \cdot 5 \cdot 4$

$5 \cdot 4 \cdot 5$

$$= 3 \cdot 5 \cdot 5 \cdot 4 = 300$$

$$\frac{3 \binom{5}{2} \binom{5}{1}}{10 \cdot 9 \cdot 8} = \frac{5}{12}$$

ΠΑΡΑΔΕΙΓΜΑ 2.7



ΕΠΙΜΕΡΑ 5

$A = \langle\langle 2A, 3Γ \rangle\rangle$

$P(A) = ;$

$B = \langle\langle 4A, 1Γ \rangle\rangle$

$P(B) = ;$

$C = \langle\langle \dots \rangle\rangle$

$P(C) = .$

$$C = \langle\langle SA \rangle\rangle$$

$$P(C) = ;$$

$$P(A) = \frac{|A|}{|R|}$$

$$|R| = \binom{100}{5} = \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{5 \cdot 4 \cdot 3 \cdot 2} \approx 75 \cdot 10^6$$

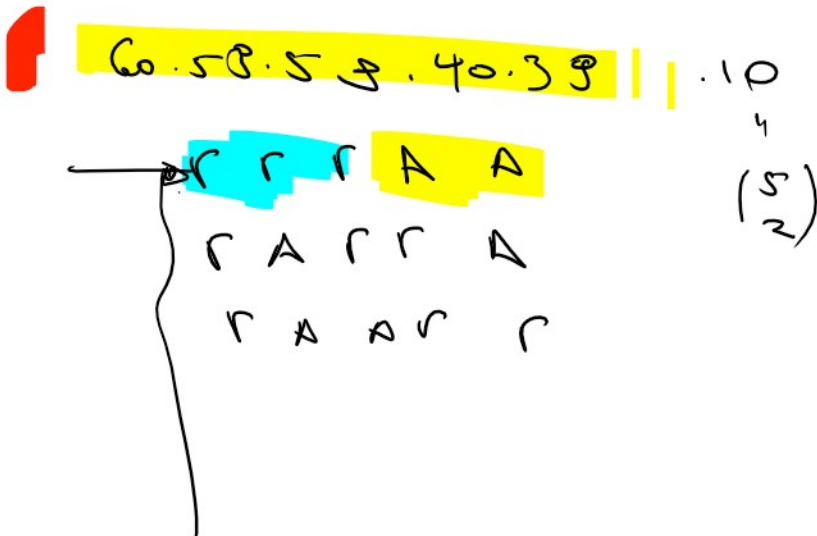
$$|A| = \binom{60}{3} \times \binom{40}{2}$$

$$P(A) = \frac{\binom{60}{3} \binom{40}{2}}{\binom{100}{5}} \approx 35\%$$

$$\binom{100}{5} = \frac{100!}{95!5!} = \frac{100 \cdot 99 \cdot \dots \cdot 96}{5!}$$

$$P(X \text{ AMPER}) = \frac{\binom{40}{2} \binom{60}{5-x}}{\binom{100}{5}}$$

$x = 0, 1, 2, 3, 4, 5$



$$P(B) = P(\langle\langle 4A, 2r \rangle\rangle) = \frac{|B|}{|R|} = \frac{\binom{60}{1} \cdot \binom{40}{4}}{\binom{100}{5}}$$

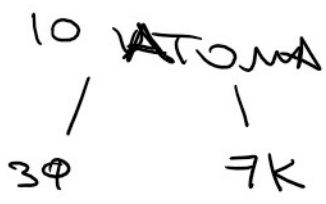
... 7 /

$$\binom{100}{5}$$

1

$$P(C) = P(\ll 5 \text{ \textcircled{A}} \gg) = \frac{|C|}{|U|} = \frac{\binom{40}{5} \cdot \binom{60}{0}}{\binom{100}{5}}$$

ΠΑΡΑΔΕΙΓΜΑ 2.8



$$P(\phi \geq 2, \kappa \geq 2)$$

1ος ΤΡΟΠΟΣ (ΕΡΕΤΟΣ)

ΕΥΛΕΒΟΝΤΑΙ 3

$$P(\phi = 0) = \frac{\binom{7}{3}}{\binom{10}{3}} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = \frac{21}{120}$$

$$P(\kappa = 0) = \frac{\binom{3}{3}}{\binom{10}{3}} = \frac{1}{\frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}} = \frac{1}{120}$$

$$P(A) = 1 - P(A')$$

$$P(\phi \geq 1, \kappa \geq 1) = 1 - P(\phi = 0) - P(\kappa = 0)$$

2ος ΤΡΟΠΟΣ.



$$P(\phi \geq 1, \kappa \geq 1) = P(2\phi, 1\kappa) + P(1\phi, 2\kappa)$$

$$= \frac{\binom{3}{2} \binom{7}{1}}{\binom{10}{3}} + \frac{\binom{3}{1} \binom{7}{2}}{\binom{10}{3}} = \frac{28}{40}$$

$$P(\phi \geq 1, \kappa \geq 1) = \frac{\binom{3}{1} \cdot \binom{7}{1} \cdot 8}{\binom{10}{3}}$$

$\phi_1, \kappa_1, \phi_3$   
 $\phi_3, \kappa_1, \phi_1$

$$\binom{10}{3}$$

$\varphi_3 k_1 \varphi_1$

ΜΑΡΚΟΣ  
ΚΟΥΤΡΑΣ

SHELDON ROSS

FIRST COURSE IN PROBABILITY