2.2

Monday, March 4, 2024 9:06 AM

ΣΥΝΔΥΑΣΜΟΙ

ΟΡΙΣΜΟΣ: X $|X| = N$

ΣΥΝΔΥΑΣΜΟΣ (ΜΗ-ΔΙΑΤΕΤΑΓΜΕΝΗ ΕΠΙΛΟΓΗ):
(COMBINATION)

ΜΗ ΔΙΑΤΕΤΑΓΜΕΝΗ k-ΑΔΑ ΔΙΑΚΡΙΤΩΝ ΣΤΟΙΧΕΙΩΝ

$\Rightarrow$ $k \leq N$

ΠΑΡΑΔΕΙΓΜΑ 2.5 4 ΒΙΒΛΙΑ $X = \{A, B, C, D\}$
2 ΕΠΙΛΟΓΕΣ

$\{A,B\}$ $\{A,C\}$, $\{A,D\}$, $\{B,C\}$, $\{B,D\}$, $\{C,D\}$

$\binom{4}{2} = \frac{4!}{2!\,2!} = \frac{4\cdot 3\cdot 2}{2\cdot 2} = 6$

$P(\text{ΒΙΒΛΙΟ } A) = \frac{|\text{«ΒΙΒΛΙΟ } A\text{»}|}{|X|} = \frac{3}{6}$

ΛΗΜΜΑ 2.3 k ΑΝΤΙΚΕΙΜΕΝΑ ΑΠΟ N

ΠΛΗΘΟΣ ΣΥΝΔΥΑΣΜΩΝ ΕΙΝΑΙ:

$$y = \binom{N}{k} \overset{\Delta}{=} \frac{N!}{k!\,(N-k)!}$$

$\binom{10}{3} = \frac{10!}{3!\,7!} =$

$\frac{10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}\cdot 7\cdot 6 \cdots 1}{1\cdot \cancel{2}\cdot \cancel{3}\cdot 1\cdot 2\cdot 3 \cdots 7} = 120$

$\{1\,2\,3\}$ 134
$\{1,2,4\}$ 142
$\{1,2,5\}$
126
⋮
12,10

ΑΠΟΔΕΙΞΗ:

y

$N(N-1)\ldots(N-k+1) = y \cdot k!$

$N!$

$\Rightarrow$ $N\ldots(N-k+1)(N-k)(N-k-1)\ldots 3\cdot 2\cdot 1$

ABC

$$\Rightarrow \quad y = \frac{\overbrace{N\cdots(N-k+1)(N-k)(N-k-1)\cdots 3\cdot 2\cdot 1}^{N!}}{\underbrace{k!}_{k!}\;\underbrace{(N-k)(N-k-1)\cdots 3\cdot 2\cdot 1}_{(N-k)!}}$$

ABC
ACB
BCA $\quad 3\cdot 2$
BAC
CAB
CBA

ΠΑΡΑΔΕΙΓΜΑ 2.6

10 ΒΙΒΛΙΑ $\qquad$ ΕΠΙΛΕΓΩ 3

$5\ \Sigma \qquad 5\ M$

A = «3 ΜΟΝΟ MANUAL» $\qquad P(A) = ;$

B = «2 MANUAL, 1 ΣΥΓΓΡΑΜΜΑ» $\qquad P(B) = ;$

C = «3 ΜΟΝΟ ΣΥΓΓΡΑΜΜΑΤΑ»

ΛΥΣΗ:

Ω: ΟΛΕΣ ΟΙ ΜΗ ΔΙΑΤΕΤΑΓΜΕΝΕΣ 3

$$|\Omega| = \binom{10}{3} = 120$$

$$\left.\begin{array}{l}\{\Sigma_1, \Sigma_2, \Sigma_3\}\\ \{\Sigma_1, \Sigma_2, \Sigma_4\}\\ \{\Sigma_1, M_2, M_4\}\\ \{\Sigma_3, \Sigma_2, \Sigma_1\}\end{array}\right\} 120$$

123 $\quad$ 235
124 $\quad$ 234
125 $\quad$ 245
134 $\quad$ 345
135
145

~~132~~
~~231~~

$$\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2} = 10$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{10}{120} = \frac{1}{12}$$

~~132~~
~~231~~
~~213~~
312
321

$|\Omega|$ 120 12

ΝΕ ΔΙΑΤΑΞΕΙΣ

10 ΒΙΒΛΙΑ

5 Μ 5 Σ

$P(\text{ΜΟΝΟ ΜΑΘΗΜ}) = \dfrac{5\cdot4\cdot3/3!}{10\cdot9\cdot8/3!} = \dfrac{10}{120}$

2M, 1Σ

$P(B) = \dfrac{|B|}{|\Omega|}$

120

$= \dfrac{50}{120}$

$\binom{5}{2} = \dfrac{5\cdot4}{2} = 10$

$\binom{5}{1} = \dfrac{5!}{1!\,4!} = 5$

$= 50$

Σ_1 M_{12}, M_{13}, M_{14}, …, M_{45}

Σ_2

120

A	B	D	C
0Σ	100		
10	50	50	0
	2M	2M	10
	1Σ	2Σ	

$P(A) = P(C)$

$P(B) = P(D)$

$P(A) + P(B) + P(C) + P(D) = 1$

$B = $ «2M, 1Σ»

$10\cdot9\cdot8 = 720$

$|B| = ;$

B

10 ΒΙΒΛΙΑ

$P(\overbrace{2M\ \ 1\Sigma}^{B})$ $P(B)$ $|B|$

10 ΒΙΒΛΙΑ

5Μ   5Σ

$P(\overbrace{2M, 1\Sigma}) = P(B) = \frac{|B|}{|\Omega|}$

$|\Omega| = 10 \cdot 9 \cdot 8 = 720$

1) $M_1 M_2 M_3$
2) $M_2 M_1 M_3$
3) $\Sigma_1 M_4 M_5$
4) $\Sigma_2 M_4 M_5$
5)

⋮

$1\Sigma, \ 2M$

$5 \cdot 5 \cdot 4 \cdot 3 = 300$

$P(B) = \frac{300}{720} = \frac{30}{72} = \frac{15}{36} = \frac{5}{12}$

$\underline{\Sigma M M} \qquad \underline{M \Sigma M} \qquad M M \Sigma$

$5 \cdot 5 \cdot 4 \qquad 5 \cdot 5 \cdot 4 \qquad 5 \cdot 4 \cdot 5$

$= 3 \cdot 5 \cdot 5 \cdot 4 = 300$

$$\frac{3\binom{5}{2} \cdot \binom{5}{1}}{10 \cdot 9 \cdot 8} = \frac{5}{12}$$

ΠΑΡΑΔΕΙΓΜΑ 2.7

100 ΑΤΟΜΑ   ΕΠΙΛΕΓΩ 5

40Α   60Γ

A = « 2Α, 3Γ »   P(A) = ;

B = « 4Α, 1Γ »   P(B) = ;

C = « ... »   P(C) = .

$C = \ll 5A \gg$ $\qquad P(C) = ;$

$$P(A) = \frac{|A|}{|\Omega|}$$

$$|\Omega| = \binom{100}{5} = \frac{100\cdot 99\cdot 98\cdot 97\cdot 96}{5\cdot 4\cdot 3\cdot 2} \simeq 75\cdot 10^6$$

$$|A| = \binom{60}{3} \times \binom{40}{2}$$

$$\binom{100}{5} = \frac{100!}{95!\,5!} = \frac{100\cdot 99 \cdots 96}{5!}$$

$$\boxed{\underset{x=2}{P(A)} = \frac{\binom{60}{3}\binom{40}{2}}{\binom{100}{5}}} \simeq 35\%$$

$$P(x \text{ ΑΝΤΡ} = 2) = \frac{\binom{40}{2}\binom{60}{5-x}}{\binom{100}{5}}$$

$x = 0,1,2,3,4,5$

$60\cdot 58\cdot 58\cdot 40\cdot 39 \quad \cdot 10$

$\binom{5}{2}$

Γ Γ Γ Α Α

Γ Α Γ Γ Α

Γ Α Α Γ Γ

$$P(B) = P(\ll 4Α, 1Γ \gg) = \frac{|B|}{|\Omega|}$$

$$= \frac{\binom{60}{1}\cdot\binom{40}{4}}{\binom{100}{5}}$$

$$\frac{\cdots}{\binom{100}{5}}$$

1

$\|$

$$P(C) = P(\text{«5 ΑΜΠΕΣ»}) = \frac{|C|}{|\Omega|} = \frac{\binom{40}{5}\cdot\binom{60}{0}}{\binom{100}{5}}$$

ΠΑΡΑΔΕΙΓΜΑ 2.8

10 ΑΤΟΜΑ

3Φ 7Κ

$$P(\Phi \geq 1, K \geq 1)$$

1ος ΤΡΟΠΟΣ (ΣΚΕΤΟΣ)

ΕΠΙΛΕΓΟΝΤΑΙ 3

$$P(\Phi = 0) = \frac{\binom{7}{3}}{\binom{10}{3}} = \frac{\frac{7\cdot\cancel{6}^{3}\cdot\cancel{5}}{\cancel{3!}}}{\frac{{}^{2}\cancel{10}\cdot 9\cdot\cancel{8}^{4}}{\cancel{3!}}} = \frac{21}{72}$$

$$P(K = 0) = \frac{\binom{3}{3}}{\binom{10}{3}} = \frac{1}{\binom{10}{3}} = \frac{\cancel{3}\cdot\cancel{2}}{\cancel{10}_{5}\cdot\cancel{9}_{3}\cdot 8} = \frac{1}{120}$$

$$P(A) = 1 - P(A')$$

$$P(\Phi \geq 1, K \geq 1) = 1 - P(\Phi = 0) - P(K = 0)$$

2ος ΤΡΟΠΟΣ.

Φ=0 Φ=1 Φ=2 Φ=3

$$P(\Phi \geq 1, K \geq 1) = P(2\Phi, 1K) + P(1\Phi, 2K)$$

$$= \frac{\binom{3}{2}\binom{7}{1}}{\binom{10}{3}} + \frac{\binom{3}{1}\cdot\binom{7}{2}}{\binom{10}{3}} = \frac{28}{40}$$

$$P(\Phi \geq 1, K \geq 1) = \frac{\binom{3}{1}\cdot\binom{7}{1}\cdot 8}{\binom{10}{\ }}$$

$\Phi_1 K_1 \Phi_3$

$\Phi_3 K_1 \Phi_1$

$$\frac{\quad}{\binom{10}{3}}$$

$\varphi_3 \kappa_1 \varphi_1$

ΜΑΡΚΟΣ
ΚΟΥΤΡΑΣ

SHELDON ROSS

FIRST COURSE IN PROBABILITY