1.3

Monday, February 26, 2024 1:22 PM

$\forall \epsilon > 0 \ \exists N_0 : \forall n \geq N_0$

$|a_n - L| < \epsilon$

Ω, A

$$P(A) \triangleq \lim_{n \to \infty} \frac{\#(A)}{n}$$

1) $P(A) \geq 0$

2) $P(\Omega) = 1$

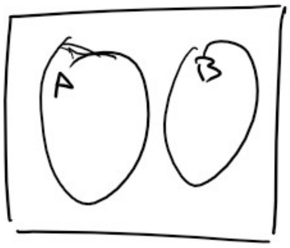
$A = \Omega$

$$P(\Omega) = \lim_{n \to \infty} \frac{\#(\Omega)}{n} = 1$$

3) A, B ΞΕΝΑ $\Leftrightarrow$ $A \cap B = \emptyset$

$$P(A) + P(B) = P(A \cup B)$$

$$\left.\begin{aligned} P(A) &= \lim_{n \to \infty} \frac{\#(A)}{n} \\ P(B) &= \lim_{n \to \infty} \frac{\#(B)}{n} \end{aligned}\right\} \Rightarrow$$



$$P(A) + P(B) = \lim_{n \to \infty} \frac{\#(A) + \#(B)}{n}$$

$$= \lim_{n \to \infty} = \frac{\#(A \cup B)}{n} = P(A \cup B)$$

KOLMOGOROV

ΟΡΙΣΜΟΣ: Ω $\quad \mathcal{F} = \mathcal{P}(\Omega)$

ΜΕΤΡΟ ΠΙΘΑΝΟΤΗΤΑΣ $\quad P : \mathcal{F} \to \mathbb{R}$

ΠΡΩΤΟ ΑΞΙΩΜΑ: $P(A) \geq 0 \quad \forall A \in \mathcal{F}$

ΔΕΥΤΕΡΟ ΑΞΙΩΜΑ: $P(\Omega) = 1$

ΔΕΥΤΕΡΟ ΑΞΙΩΜΑ: $P(\Omega) = 1$

ΤΡΙΤΟ ΑΞΙΩΜΑ:

$A_1, A_2, A_3, \ldots$ ΕΝΔΕΧΟΜΕΝΑ ΞΕΝΑ $\left(A_i \cap A_j = \emptyset,\ i \neq j \right)$

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + \ldots$$

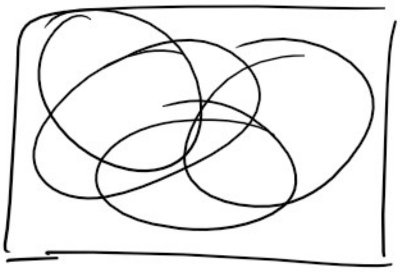
$$= P\left(\bigcup_{i=1}^{n} A_i \right) = \sum_{i=1}^{\infty} P(A_i)$$

ΕΙΔΙΚΗ ΠΕΡΙΠΤΩΣΗ

$$P(A_1 \cup A_2) = P(A_1) + P(A_2)$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\#(A_1 \cup A_2 \cup A_3) = \#A_1 + \#A_2 + \#A_3 - \#(A_1 \cap A_2) \ldots + \#(A_1 \cap A_2 \cap A_3)$$

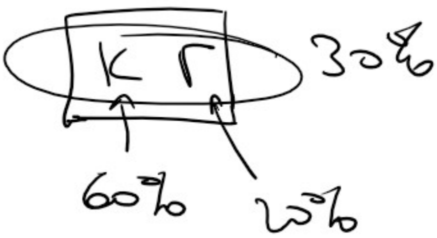


Ω

$P(A) \geq 0$

$P(\Omega) = 1$

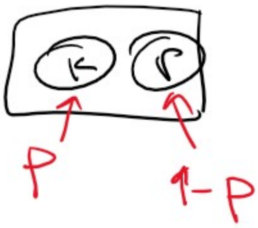
$P(A \cup B) = P(A) + P(B)$



$P(\{K, \Gamma\}) = 1$

$P(\{K\}) + P(\{\Gamma\}) = P(\{K, \Gamma\}) = 1$

$P(K) \geq 0, \quad P(\Gamma) \geq 0$



$0 \leq p \leq 1$

T $1-p$ | $P(A)\geq 0$, $P(A')\geq 0$

$$\begin{cases} P(A)\geq 0 \quad \forall A & \text{ΑΞΙΩΜΑ 1} \\ P(\Omega)=1 & \text{ΑΞΙΩΜΑ 2} \\ P(A\cup B)=P(A)+P(B) & \text{ΑΞΙΩΜΑ 3} \end{cases}$$

ΛΗΜΜΑ 1.4 $\quad \Omega,\ P$

1) $P(A') = 1 - P(A)$

$$1 \overset{A2}{=} P(\Omega) = P(A\cup A') \overset{A3}{=} P(A) + P(A')$$

$$\Rightarrow P(A') = 1 - P(A)$$

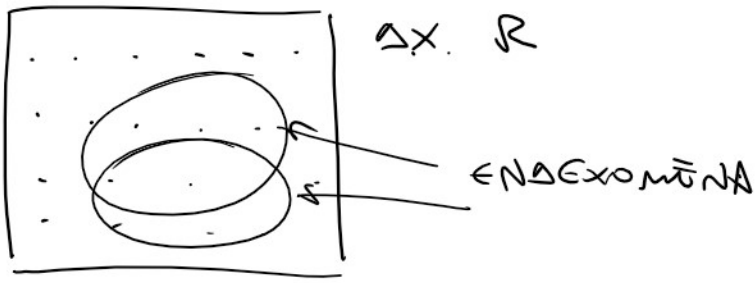
2) $P(\emptyset) = 0$ $\qquad$ $P(\Omega)=1$, $P(\emptyset)=0$

$$P(\emptyset) = 1 - P(\emptyset') = 1 - P(\Omega) = 1-1 = 0$$

3) $P(A) \leq 1 \quad \forall A$

$$P(A) + P(A') = 1 \Rightarrow \qquad P(A')\geq 0 \ \text{A2}$$

$$P(A) = 1 - P(A') \overset{A2}{\leq} 1$$

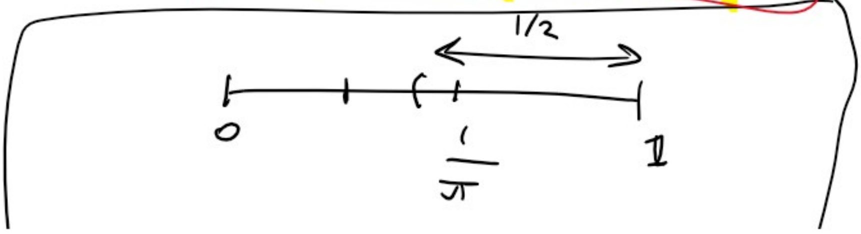


A1. $P(A)\geq 0 \quad \forall A\in\Omega$

A2. $P(\Omega)=1$

A3. $P(A\cup B) = P(A)+P(B)$

$P(A)$



1) $P(A') = 1 - P(A)$

2) $P(\emptyset) = 0$

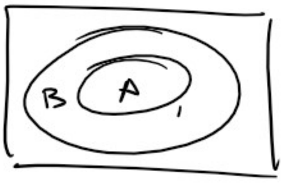
2) $P(\emptyset) = 0$

3) $P(A) \leq 1$

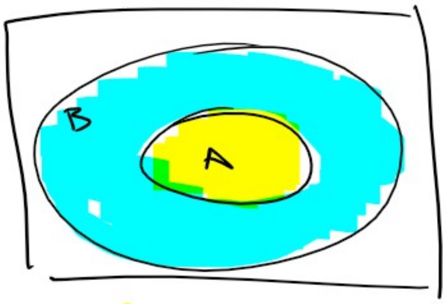
$\frac{1}{\pi}$ $\quad 1$

$P\left(\frac{1}{\pi}\right) = \quad P(E) = |E|$

4) $A \subseteq B \Rightarrow P(A) \leq P(B)$



$P(B) = P(A \cup (B \cap A'))$



$B = A \cup (B \cap A')$

$x \in B$ → $x \in A \Rightarrow x \in A \cup (B \cap A')$

→ $x \notin A \Rightarrow x \in A \cup (B \cap A')$

$\Rightarrow x \in A \cup (B \cap A')$

$A \cap (B \cap A') = \emptyset$

$x \in A \cup (B \cap A')$

$\Rightarrow \quad x \in A \Rightarrow x \in B$

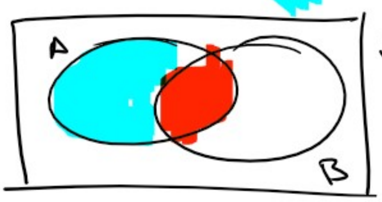
$x \in B \cap A' \Rightarrow x \in B$

$\Rightarrow x \in B$

$P(B) = P(A \cup (B \cap A')) = P(A) + P(B \cap A') \geq P(A)$

5) $P(A \cap B') = P(A) - P(A \cap B)$

$A \cap B' \triangleq A - B$



$\triangleq \quad \equiv \quad \doteq \quad \fallingdotseq$

$P(A) = P(A \cap \Omega) = P(A \cap (B \cup B')) =$

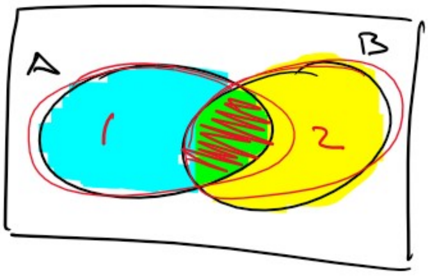
$P((A \cap B) \cup (A \cap B')) = P(A \cap B) + P(A \cap B')$

$\Rightarrow$

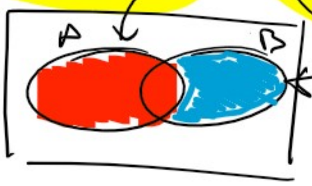
$\Rightarrow \quad P(A\cap B') = P(A) - P(A\cap B)$

6) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\#(A\cup B) = \#A + \#B - \#(A\cap B)$



$P(A\cup B) = P(A \cup (A'\cap B))$

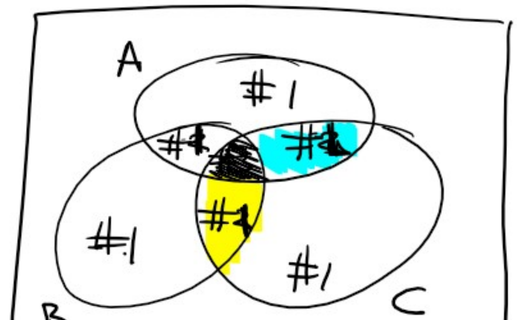


$= P(A) + P(B\cap A') = P(A) + P(B) - P(A\cap B)$

7) $P(A\cup B) \le P(A) + P(B)$ $-$ ~~$P(A\cap B)$~~

8) $P(A\cup B\cup C) = P(A) + P(A'\cap B) + P(A'\cap B'\cap C)$

9) $P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(A\cap C)$
$- P(B\cap C) + P(A\cap B\cap C)$



5 ΠΑΙΔΙΑ

P(3 ΚΟΡΙΤΣΙΑ)

ΚΚΚΑΑ   ΑΚΚΑΚ

ΚΚΚΚΚ
ΚΚΚΚΑ
ΚΚΚΑΚ
ΚΚΚΑΑ
ΚΚΑΚΚ
ΚΚΑΚΑ
ΚΚΑΑΚ
ΚΚΑΑΑ

ΚΚΚΑΑ ΑΚΚΑΚ ΚΚΑΑΚ

ΚΚΑΚΑ ΑΚΑΚΚ ΚΚΑΑΑ

ΚΚΑΑΚ ΑΑΚΚΚ ΚΑΚΚΚ

ΚΑΚΚΑ ΚΑΚΚΑ

ΚΑΚΑΚ ΚΑΚΑΚ

ΚΑΑΚΚ ΚΑΚΑΑ

ΑΚΚΚΑ ΚΑΑΚΚ

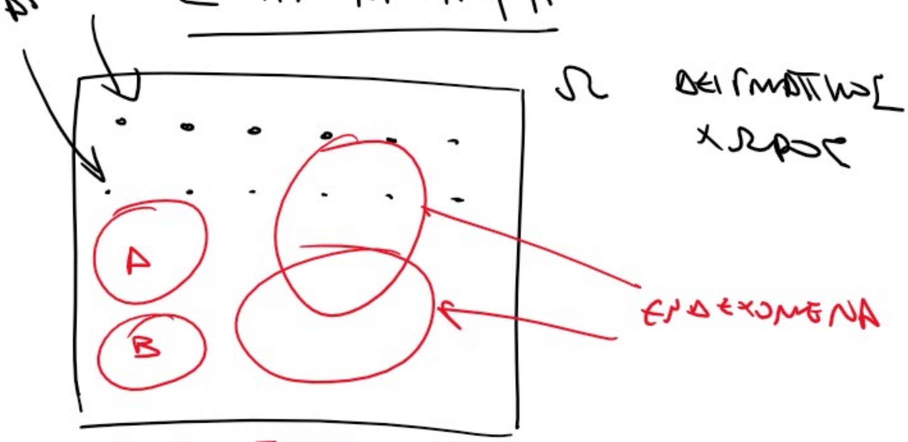
ΚΑΑΚΑ

ΚΑΑΑΚ

ΚΑΑΑΑ

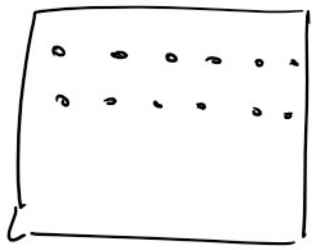
$$\binom{5}{3} = \frac{5!}{3!\,2!} = 10 \qquad \frac{10}{32}$$

ΕΠΑΝΑΛΗΨΗ



Α, Β ΞΕΝΑ

$$A = \{(1,1), (2,2), (3,3) \ldots (6,6)\}$$



Α1. $P(A) \geq 0$

Α2. $P(\Omega) = 1$

Α3. Α, Β ΞΕΝΑ

$$P(A \cup B) = P(A) + P(B)$$

ΠΑΡΑΔΕΙΓΜΑ

$$\left.\begin{array}{l} P(\{2,3\}) = 20\% \\ P(\{4,5\}) = 40\% \end{array}\right\} \Rightarrow P(\{2,3,4,5\}) = 60\%$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\left.\begin{array}{l} P(\{2,3\}) = 20\% \\ P(\{3,4\}) = 40\% \end{array}\right\} \Rightarrow P(\{2,3,4\}) = P(\{2,3\}) + P(\{3,4\}) - P(\{3\})$$

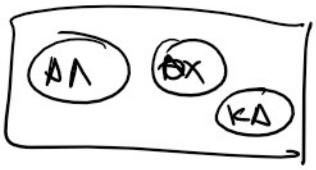
$P(\{3,4\}) = 40\%$ $\Rightarrow$ $P(\{\ldots\}) = P(\{\ldots\})$ $+ P(\{3,4\}) - P(\{3\})$

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) $P(\emptyset) = 0$

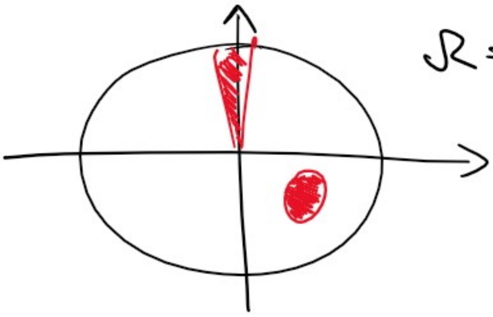
3) $P(A) \leq 1$

4) $A \subseteq B \Rightarrow P(A) \leq P(B)$



ΠΑΡΑΔΕΙΓΜΑ

0.8 0.2



$\Omega = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 40^2\} \cup \{\text{ΣΚΥΛΟΣ}\}$

ΠΑΡΑΔΕΙΓΜΑ 1.13

→ 30% ΤΟΥΛΑΧΙΣΤΟΝ 1 ΔΕΝ ΛΕΙΤΟΥΡΓΕΙ

→ 10% ΑΚΡΙΒΩΣ 2 ΔΙΚΤΥΑ ΔΕΝ ΛΕΙΤΟΥΡΓΟΥΝ

5% ΔΕΝ ΛΕΙΤΟΥΡΓΕΙ ΚΑΝΕΝΑ

P(ΛΕΙΤΟΥΡΓΕΙ ΑΚΡΙΒΩΣ 1) = ;

$\Omega = \{000, 001, 010, 011, 100, 101, 110, 111\}$

$B_0 = \{000\}$ $B_1 = \{100, 010, 001\}$ ΔΙΑΜΕΡΙΣΗ

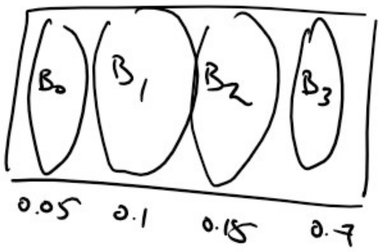
$B_2 = \{110, 101, 011\}$, $B_3 = \{111\}$

1) $P(B_0 \cup B_1 \cup B_2) = 0.3$

2) $P(B_1) = 0.1$

2) $P(B_1) = 0.1$

3) $P(B_0) = 0.05$



$P(B_2) = ;$

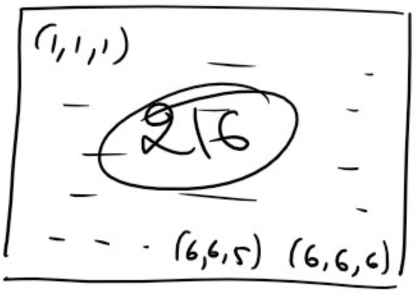
$$P(B_0 \cup B_1 \cup B_2) = P(B_0) + P(B_1) + P(B_2)$$

$0.3 \qquad\qquad 0.05 \qquad 0.1 \qquad 0.15$

$\Rightarrow \quad P(B_2) = 0.15$

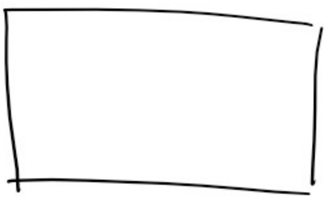
$$P(B_3) = 1 - P(B_3') = 1 - P(B_0 \cup B_1 \cup B_2) = 0.70$$

$P(Z_1 + Z_2 + Z_3) = 13$



$$\frac{21}{216}$$

(6,6,1) (2,5,6) (4,4,5)
(1,6,6) (2,6,5) (4,5,4)
(6,1,6) (3,6,4) (5,4,4)
(6,2,5) (3,4,6) (5,5,3)
(6,5,2) (4,6,3) (5,3,5)
(5,2,6) (4,3,6) (3,5,5)
(5,6,2) (6,3,4)
(6,4,3)



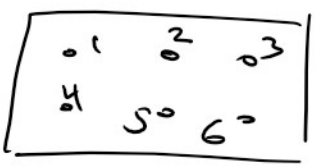
Ω A1. $P(A) \geq 0$

A2. $P(\Omega) = 1$

A3. $P(A \cup B) = P(A) + P(B)$

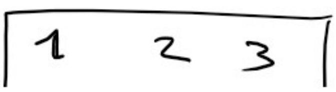
ΠΑΡΑΔΕΙΓΜΑ 1.14

$P(\{i\}) = \frac{1}{6} \qquad i = 1, 2, \ldots, 6.$



$$P(\{2,4,6\}) = P(\{2\}) + P(\{4\}) + P(\{6\}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

ΠΑΡΑΔΕΙΓΜΑ 1.15



$P(1) = P(3) = P(5) = 0$

1	2	3
4	5	6

$$P(1) = P(3) = P(5) = 0$$
$$P(2) = P(4) = P(6) = \frac{1}{3}$$

$$P(\{1, 2, 3\}) = 0 + \frac{1}{3} + 0 = \frac{1}{3}$$

rand()

ΛΗΜΜΑ 1.2

$$\Omega = \{\omega_1, \omega_2, \dots, \omega_n\} \qquad n = |\Omega|$$

$$P(A) = \frac{|A|}{|\Omega|} \quad \forall A \subseteq \Omega \qquad \binom{52}{5}$$

A1 $P(A) \geq 0$ A2. $P(\Omega) = \frac{|\Omega|}{|\Omega|} = 1$

A3. A, B $\quad P(A \cup B) = P(A) + P(B) \qquad \#\Omega$

$$P(A \cup B) = \frac{|A \cup B|}{|\Omega|} = \frac{|A| + |B|}{|\Omega|} = \frac{|A|}{|\Omega|} + \frac{|B|}{|\Omega|}$$
$$= P(A) + P(B)$$

ΠΑΡΑΔΕΙΓΜΑ

52 ΦΥΛΛΑ

ΠΑΙΡΝΩ 5

$$P(\overbrace{\text{ΚΑΡΕ ΑΣΟΥ}}^{A})$$

$$\Omega = \{\text{ΣΥΝΔΥΑΣΜΟΙ 5 ΦΥΛΛΩΝ ΑΠΟ 52}\}$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{|A|}{\binom{52}{5}} = \frac{48}{\binom{52}{5}}$$

$A_1 A_2 A_3 A_4 K_1$
-"- K_2
-"- K_n

$|\Omega|$

$\binom{52}{5}$ $\binom{52}{5}$ -//- k_2

-//- k_3

-//- k_4

$$\binom{52}{5} = \frac{52!}{5!\,47!}$$

ΠΑΡΑΔΕΙΓΜΑ

$p = 0.8$

10 ΔΙΠΛΩΜΑΤΑ

P(7 ΝΕΟΙ)

$$1024 = 2^{10}$$

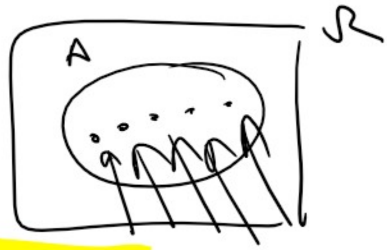
ΛΗΜΜΑ 1.3

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_n\} \qquad |\Omega| = n$$

$\omega_1 \to p_1$, $\omega_2 \to p_2$, …, $\omega_n \to p_n$

$$p_i \geq 0 \quad i = 1, \ldots, n \qquad \sum_{i=1}^{n} p_i = 1$$

$$P(A) = \sum_{\omega_i \in A} p(\omega_i)$$



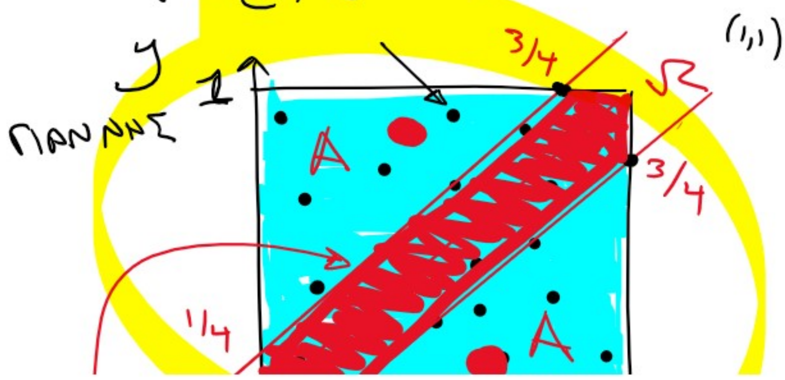
ΠΑΡΑΔΕΙΓΜΑ

12 + [0, 1]

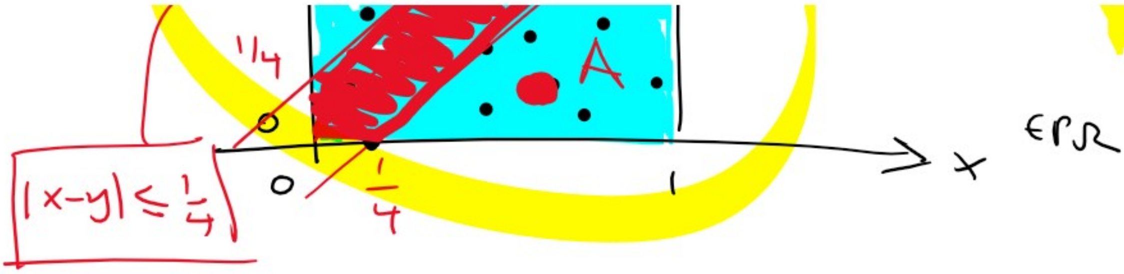
12 + [0, 1]

P(ΣΤΗΣΙΜΟ > 15 ΛΕΠΤΑ)

$X \in [0,1]$ $\quad Y \in [0,1]$ $\quad P(\underbrace{|X - Y| > \tfrac{1}{4}}_{A})$



$$P(A) = 2 \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$$



$= \frac{9}{16}$