Flows in Networks

(informal) problem statement:

Suppose we want to transport some quantity of a good within a given network, from some source to a destination

The good can be

- Oil to be transported through a network of oil pipes
- Information through a computer network
- Etc

Constraints: each edge in the network has a *capacity*, i.e., the maximum quantity it can carry

- oil pipes have a volume capacity
- A link in a computer network has limits on its bandwidth

Goal: find a way to route the good through the network so as to maximize the total quantity shipped

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More formally:

Consider a graph G = (V, E), with a source node s $\in$ V, and a sink node t $\in$ V

Capacity constraints: for every edge e $\in$ E, there is a capacity c_e

A feasible flow is an assignment of a flow f_e to every edge so that

1. $f_e \leq c_e$
2. For every node other than source and sink:
   incoming flow = outgoing flow (preservation of flow)

Goal: find a feasible flow so as to maximize the total amount of flow coming out of s (or equivalently going into t)

Flow going out of s: $\sum_{(s,u)\in E} f_{su}$

By preservation of flow this equals: $\sum_{(u,t)\in E} f_{ut}$

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Example:

- Figure (a): network with capacities
- Figure (b): a feasible flow
- In fact, the flow in (b) is optimal (7 units)