# Exercises on *n*-gram language models

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**Submit as a group of 2–3 members (unless specified otherwise in the lectures) a report for exercise 3 (max. 10 pages, PDF format). Explain briefly in the report the algorithms that you used, how they were trained, how you tuned the hyper-parameters, how you prepared your data etc. Present in the report your experimental results and demos (e.g., screenshots) showing how your code works. Do not include code in the report, but include a link to a Colab notebook (https://colab.research.google.com/) containing your code. Make sure to divide fairly the work of your group to its members and describe in your report the contribution of each member. The contribution of each member will also be checked during the oral examination of your submission. For delayed submissions, one point will be subtracted per day of delay.**

**1.** Calculate the entropy in the cases of slide 37 (e-mail messages). Hint: for $P(C) \rightarrow 0$, use L'Hôpital's rule.

*Answer:*

*a) When the training examples comprise 200 spam και 600 ham messages, then:*

$$P(C=1)=\frac{200}{800}=\frac{1}{4},\ P(C=0)=\frac{600}{800}=\frac{3}{4},$$
$$log_2\frac{1}{4}=-2,\ log_2\frac{3}{4}=log_2 3-log_2 4=1.585-2=-0.415$$

$$H(C)=\frac{1}{4}\cdot 2+\frac{3}{4}\cdot 0.415=0.811$$

*b) When the training examples comprise 400 spam και 400 ham messages, then:*

$$P(C=1)=\frac{400}{800}=\frac{1}{2},\ P(C=0)=\frac{400}{800}=\frac{1}{2},\ log_2\frac{1}{2}=-1,$$

$$H(C)=\frac{1}{2}\cdot 1+\frac{1}{2}\cdot 1=1$$

*c) When the training examples are spam, then:*

$$P(C=1)=1,\ P(C=0)=0,\ log_2 P(C=1)=0,\ log_2 P(C=0)\rightarrow-\infty$$

$$H(C)=-P(C=1)\cdot log_2 P(C=1)-P(C=0)\cdot log_2 P(C=0)=$$

$$=-1\cdot 0-\frac{log_2 P(C=0)}{\frac{1}{P(C=0)}}$$

*Using De L'Hôpital's rule for the second term, for* $P(C=0)\rightarrow 0^+$, *we get:*

$$H(C)\rightarrow-\frac{\frac{1}{P(C=0)\ln 2}}{-\frac{1}{P(C=0)^2}}=\frac{P(C=0)}{\ln 2}=0$$

*Similarly, when all the training examples are ham, again* $H(C)=0$.

**2. [Optional.]** (i) Implement (in any programing language) the dynamic programing algorithm that computes the Levenshtein distance (slides 55–60); your implementation should print tables like the one of slide 60 (without the arrows). You may want to compare the outputs of your implementation to those of http://www.let.rug.nl/~kleiweg/lev/. (ii) Extend your implementation to accept as input a word *w*, a vocabulary *V* (e.g., words that occur at least 10 times in a corpus), and a maximum distance *d*, and return the words of *V* whose Levenshtein distance to *w* is up to *d*.

→ **3.** (i) Implement (in any programming language) a bigram and a trigram language model for sentences, using Laplace smoothing (slide 8) or optionally (if you are very keen) Kneser-Ney smoothing (slides 50–51), which is much better. In practice, *n*-gram language models compute the sum of the logarithms of the *n*-gram probabilities of each sequence, instead of their product (why?) and you should do the same. Assume that each sentence starts with the pseudo-token **start** (or two pseudo-tokens **start1**, **start2** for the trigram model) and ends with the pseudo-token **end**. Train your models on a training subset of a corpus (e.g., a subset of a corpus included in NLTK – see http://www.nltk.org/). Include in the vocabulary only words that occur, e.g., at least 10 times in the *training* subset. Use the same vocabulary in the bigram and trigram models. Replace all out-of-vocabulary (OOV) words (in the training, development, test subsets) by a special token **UNK**. Alternatively, you may want to use BPEs instead of words (obtaining the BPE vocabulary from your training subset) to avoid unknown words. See Section 2.5.2 ("Byte-Pair Encoding") of the 3rd edition of Jurafsky & Martin's book (https://web.stanford.edu/~jurafsky/slp3/); for more information, check https://huggingface.co/transformers/master/tokenizer_summary.html.

(ii) Estimate the cross-entropy and perplexity of your two models on a test subset of the corpus, treating the entire test subset as a single sequence of sentences, with **start** (or **start1**, **start2**) at the beginning of each sentence, and **end** at the end of each sentence. Do not include probabilities of the form *P*(**start**|...) or *P*(**start1**|...), *P*(**start2**|...) in the computation of cross-entropy and perplexity, since we are not predicting the start pseudo-tokens; but include probabilities of the form *P*(**end**|...), since we do want to be able to predict if a word will be the last one of a sentence. You must also count **end** tokens (but not **start**, **start1**, **start2** tokens) in the total length *N* of the test corpus.

(iii) Write some code to show how your bigram and trigram language models can auto-complete an incomplete sentence. For example, given "I would like to commend the" (slide 44), generate completions like "rapporteur on his work **end**" or "president of the Commission on his intervention **end**". You can simply use the most probable next word (according to your language model) at each time-step. If you are keen, you may also want to use beam search, or methods like top-K or nucleus sampling, to improve the diversity of the texts you generate.[1] Confirm (showing some examples of generated texts) that your trigram model generates more fluent texts than your bigram model.

(iv) Develop a context-aware spelling corrector (for both types of errors, slide 14) using your bigram and/or trigram language model, a beam search decoder (slides 26–32), and the formulae of slide 20. As on slide 20, you can use the inverse of the Levenshtein distance between $w_i, t_i$ as $P(w_i|t_i)$. If you are very keen, you may want to use better estimates of $P(w_i|t_i)$ that satisfy $\sum_{w_i} P(w_i|t_i) = 1$. You may also want to use:

$$\hat{t}_1^k = \underset{t_1^k}{\operatorname{argmax}}\, \lambda_1 \log P\left(t_1^k\right) + \lambda_2 \log P(w_1^k|t_1^k)$$

[1] See, for example, https://towardsdatascience.com/decoding-strategies-that-you-need-to-know-for-response-generation-ba95ee0faadc.

to control (by tuning the hyper-parameters $\lambda_1, \lambda_2$) the importance of the language model score $\log P(t_1^k)$ vs. the importance of $\log P(w_1^k | t_1^k)$.

(v) Create an artificial test dataset to evaluate your context-aware spelling corrector. You may use, for example, the test dataset that you used to evaluate your language models, but now replace with a small probability each non-space character of each test sentence with another random (or visually or acoustically similar) non-space character (e.g., "This is an interesting course." may become "Tais is an imterestieg kourse").

(vi) Evaluate your context-aware spelling corrector in terms of Word Error Rate (WER) and Character Error Rate (CER), using the original (before character replacements) form of your test dataset of question (v) as ground truth (reference sentences), and averaging WER (or CER) over the test sentences. CER is similar to Word Error Rate (WER), but operates at the character level. See, for example:
https://huggingface.co/spaces/evaluate-metric/wer and
https://huggingface.co/spaces/evaluate-metric/cer.

You are allowed to use NLTK (http://www.nltk.org/) or other tools and libraries for sentence splitting, tokenization (including BPE tokenizers), counting *n*-grams, computing Levenshtein distances, WER and CER, but you should write your own code for everything else (e.g., estimating probabilities, computing cross-entropy and perplexity, beam search decoding). You may want to compare, however, the cross-entropy and perplexity results of your implementation to results obtained by using existing code (e.g., from NLTK or other toolkits).

Do not forget to include in your report:

- A short description of the algorithms/methods that you used, including a discussion of any data preprocessing steps that you performed.
- Cross-entropy and perplexity scores for each model (bigram, trigram) for sub-question (ii).
- Input/output examples demonstrating how your sentence completion works, including interesting cases (e.g., good and bad completions) for sub-question (iii).
- Input/output examples demonstrating how your spelling corrector works, including interesting cases it treats correctly and incorrectly, for questions (iv) and (v).
- WER and CER scores (averaged over test sentences) for question (vi).

**4.** (a) Using the following sentences as a tiny training corpus:

<start> he plays football
<start> he plays cricket
<start> she enjoys good football
<start> she plays good music
<start> he prays to god
<start> please buy me the other ball
<start> he pleases the other players by playing good football

estimate the probabilities $P(t_1^4)$ that a bigram model with Laplace smoothing would return, for each one of the following sentences $t_1^4$:

$t_1^4$: <start> he please god football
$t_1^4$: <start> he plays good football

Assume that the vocabulary $V$ contains all the words of the training corpus (excluding <start>), hence $|V| = 21$. You do not need to perform numeric calculations that can easily be computed with a calculator.

Answer:

The bigram language model estimates *P*(<start>, he, please, god, football) as follows:

*P*(he | <start>) *P*(please | he) *P*(god | please) *P*(football | god) =

(4+1)/(7+21)  (0+1)/(4+21)  (0+1)/(1+21)  (0+1)/(1+21)

where we used Laplace smoothing.

Similarly for *P*(<start>, he, plays, good, football). (Write down the calculations by yourself.)

Note: In practice, we avoid multiplying probabilities, because they often lead to very small numbers that cannot be represented well by computers. Instead, we usually compute the logarithm of the probability of a word sequence, i.e., we would compute log *P*(<start>, he, please, god, football), instead of *P*(<start>, he, please, god, football), which would lead to the following sum of four logarithms, instead of a product of four probabilities.

log[(4+1)/(7+21)] + log[(0+1)/(4+21)] + log[(0+1)/(1+21)] + log[(0+1)/(1+21)]

(b) Assume that a user wrote the sequence $w_1^4$ using the keyboard of a mobile phone:

$w_1^4$: <start> he pls gd ftball

Estimate the probabilities $P\left(t_1^4|w_1^k\right)$ of the two hypotheses (word sequences the user might have intended to write) $t_1^4$ of sub-question (a), using a noisy channel model (see slides) and the language model of sub-question (a). Assume that $P(w_i|t_i) \cong \frac{1}{LD(w_i,t_i)+1}$, where $LD(w_i, t_i)$ is Levenshtein distance from word $w_i$ to $t_i$. Show your calculations in detail, without computing the Levenshtein distances.

Answer: Using the noisy channel of the slides:

$P(t_1^4|w_1^4) = P(t_1^4)\, P(w_1^4|t_1^4) \,/\, P(w_1^4)$

For $t_1^4$= <start> he please god football:

*P*(<start>, he, please, god, football)
*P*(<start> he pls gd ftball | <start>, he, please, god, football) / $P(w_1^4)$

The probability *P*(<start>, he, please, god, football) is estimated by the language model as in sub-question (a).

Assuming $P(w_i|t_i) \cong \frac{1}{LD(w_i,t_i)+1}$, the probability *P*(<start> he pls gd ftball | <start>, he, please, god, football) becomes:

*P*(he, he) *P*(pls, please) *P*(gd, god) *P*(ftball, football) =

1/(*LD*(he, he)+1) 1/(*LD*(pls, please)+1) 1/(*LD*(gd, god)+1) 1/(*LD*(ftball, football)+1)

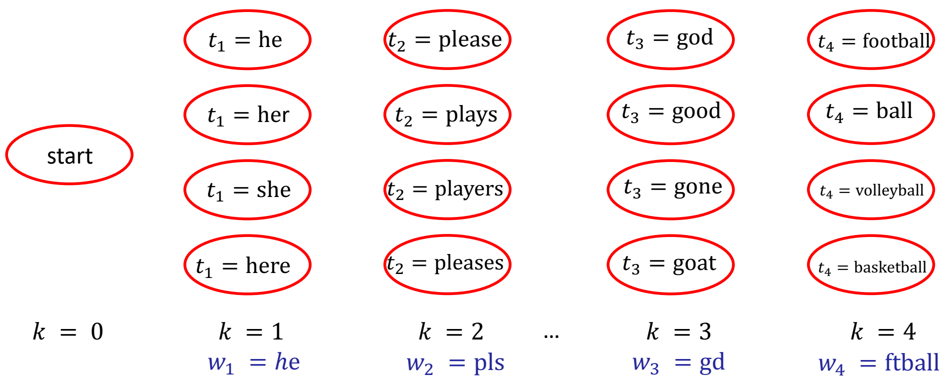
We don't need to compute $P(w_1^4)$, because it is the same for both hypotheses $t_1^4$ = <start> he please god football and $t_1^4$ = <start> he plays good football.

Similarly, we estimate $P(t_1^4|w_1^4)$ for the hypothesis $t_1^4$ = <start> he plays good football. (Write down the calculations by yourself.) Eventually, we select the hypothesis $t_1^4$ with the highest $P(t_1^4|w_1^4)$.

Note: In practice, we would again compute the logarithm of each product of probabilities, obtaining a sum of logarithms of probabilities, instead of a product of probabilities.

(c) Explain in detail how the beam search decoder of the slides would work in the following case. We use the noisy channel model of the previous sub-question, with the bigram language model of sub-question (a), trained on the tiny training corpus of that sub-question. The beam size $b = 2$.

## Beam search decoder



For $k = 1$, the candidate paths $t_1^k$ are the following. For each path, we compute $P(t_1^k)P(w_1^k|t_1^k)$. (In practice, we would compute the logarithm.) The $b = 2$ best paths are marked with asterisks.

<start, he>: P(he|start) P(he|he) = (4+1)/(7+21) 1/(0+1) = 5/28 = 0.179 **
<start, her>: P(her|start) P(he|her) = (0+1)/(7+21) 1/(1+1) = 1/28 1/2 = 0.018

<start, she>: P(she|start) P(he|she) = (2+1)/(7+21) 1/(1+1) = 3/28 1/2 = 0.054 **
<start, here>: P(here|start) P(he|here) = (0+1)/(7+21) 1/(2+1) = 1/28 1/3 = 0.012

For $k = 2$, the candidate paths $t_1^k$ are the following. Again, we compute $P(t_1^k)P(w_1^k|t_1^k)$ for each path. Complete the calculations by yourselves. Mark the $b = 2$ best paths with asterisks. Notice that we can reuse computations (parts of the products) from $k = 1$.

<start, he, please>: P(he|start) P(he|he) P(please|he) P(pls|please) = …
<start, he, plays>: P(he|start) P(he|he) P(plays|he) P(pls|plays) = …
<start, he, players>: P(he|start) P(he|he) P(players|he) P(pls|players) = …
<start, he, pleases>: P(he|start) P(he|he) P(pleases|he) P(pls|pleases) = …
<start, she, please>: P(she|start) P(he|she) P(please|she) P(pls|please) = …
<start, she, plays>: P(she|start) P(he|she) P(plays|she) P(pls|plays) = …
<start, she, players>: P(she|start) P(he|she) P(players|she) P(pls|players) = …
<start, she, pleases>: P(she|start) P(he|she) P(pleases|she) P(pls|pleases) = …

For $k = 3$, the candidate paths $t_1^k$ are the following. Again, we compute $P(t_1^k)P(w_1^k|t_1^k)$ for each path. Complete by yourselves. Mark the $b = 2$ best paths with asterisks

…

For $k = 4$, the candidate paths $t_1^k$ are the following. Again, we compute $P(t_1^k)P(w_1^k|t_1^k)$ for each path. Complete by yourselves. Which hypothesis (path) <start, $t_1, t_2, t_3, t_4$> is selected?

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