Math for Data Science

Stavros Toumpis, Assistant Professor, AUEB, toumpis@aueb.gr

Overview

The course is a brief overview of the basic tools from Linear Algebra and Multivariable Calculus that will be needed in subsequent courses of the program.

Key Outcomes

By completing the course the students will have been reminded of the basic tools of Linear Algebra and Multivariable Calculus needed in subsequent courses in the program, notably:

- Fundamental properties of matrices, their norms, and their applications
- Differentiating/integrating multiple variable functions, and the role of the gradient and the hessian matrix.
- Basic properties of optimization problems involving matrices and functions of multiple variables.

Requirements and Prerequisites

This course is meant to be a refresher. Students are assumed to have developed some mathematical maturity through a few mathematically oriented classes that involve the standard undergraduate treatment of linear algebra and single-variable calculus.

Bibliography

1. Gilbert Strang, *Linear Algebra and Its Applications*, Thomson/Brooks Cole (Available in a Greek Translation)
2. Thomas M. Apostol, *Calculus*, Wiley, 2nd Edition, 1991, ISBN 960-07-0067-2.
3. Michael Spivak, *Calculus*, Publish or Perish, 2008, ISBN 978-0914098911.
4. Ross L. Finney, Maurice D. Weir, and Frank R. Giordano, *Thomas's Calculus*, Pearson, 12th Edition, 2009.
5. David C. Lay, Linear Algebra and Its Applications, 4^{th} Edition.
6. Yousef Saad, ‘Iterative Methods for Sparse Linear Systems’

Grading

There will be a written examination at the end of the course.

Course Syllabus

The course comprises twelve one-hour lectures which will be spread out in the first two weeks of the first semester.

Lecture 1: Matrices and Basic Operations, Special structures

Matrices and Basic Operations, interpretation of matrices as linear mappings, and some examples

Lecture 2: Square Matrices, Determinants

Properties of determinants, singular and non-singular matrices, examples, finding an inverse matrix

Lecture 3: Eigenvalues and Eigenvectors

Characteristic Polynomial, Definition of Left/right Eigenvalues and Eigenvectors, Caley-Hamilton theorem, Singular Value Decomposition, interpretation of eigenvalues/vectors

Lecture 4: Normed Spaces, Vector Spaces, Matrix Norms

Definition of complete normed and vector spaces and some examples. Matrix norms and properties, applications to series of matrices and their convergence

Lecture 5: The Range and the Null space of a Matrix

Definition and basic properties, orthogonality, Gram-Schmidt algorithm

Lecture 6: Positive-Definite Matrices and the Taylor Expansion of a two-variable function

Definition of positive-definiteness and the role of the eigenvalues. Physical meaning and importance in real-life problems

Lecture 7: Linear Systems

Definition, applications, solving linear systems, linear inequalities, linear programming

Lecture 8: Real-valued functions of two or more variables.

Definition, examples, simple demos, applications

Lecture 9: Analysis elements

Distance, Limits, continuity, differentiability, the gradient and the Hessian

Lecture 10: Optimization problems

Simple examples, motivation, the role of the Hessian, maxima and minima and related extrema conditions

Lecture 11: Integration

Double integrals, Fubini's theorem, properties, applications

Lecture 12: Elements of Convex Optimization

Functions of n variables. Convex sets, convex functions, convex problems, and their basic properties. Examples of convex problems, convexity versus non-convexity