ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

ΣΧΟΛΗ ΕΠΙΣΤΗΜΩΝ & ΤΕΧΝΟΛΟΓΙΑΣ ΤΗΣ ΠΛΗΡΟΦΟΡΙΑΣ
SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY

ΜΕΤΑΠΤΥΧΙΑΚΟ στην ΕΠΙΣΤΗΜΗ ΔΕΔΟΜΕΝΩΝ
MSc in DATA SCIENCE

Elements of Statistics and Probability

Exercises using R: Common operations & operations with matrices

1. Using the vectors

`x <- c(-4, 1, 0, 0, 5, -3, 2)` and `y <- c(1, 1, -5, 4, 3, -2, 0)` implement the following operations:

i. Create a new vector including the first, second and sixth element of `x`
ii. Create a new vector that includes the first four elements of `y`
iii. Create a new vector with the negative elements of `x`
iv. Create a new vector that includes all elements of `x` apart from the third one
v. Create a new vector with the elements of `x` satisfying the conditions `x<0` and `y≠1`
vi. Create a new vector with the elements of `y` satisfying the conditions `y<0` or `x≤0`
vii. Create a new vector with the first two elements of `y` replicated twice.

2. Suppose that the vectors `x` and `y` include the grades in the assignment and the written examination of a course for a specific student. Calculate the final score of the student in the following cases:
   i. The final score is the average of the two grades.
   ii. If the assignment's grade is 7 or higher the final score is the grade of the written examination, otherwise it is 4. The student also gets a bonus of one credit if the assignment's grade is more than 8.
   iii. The final score is the minimum of the two grades
   iv. If the assignment's grade is at least 5 then the final score is the average of the two grades, otherwise the final score is just the grade of the written examination.

3. Let $A = \begin{pmatrix} 1 & 2 & 6 \\ 5 & 2 & 5 \\ 6 & 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{pmatrix}$.
   i. Define the above tables in R.
   ii. What is the appropriate operation in order to get $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ from A?

iii. Calculate the following quantity:

$$5A^{-1} + 3(AA^T)^{-1} - 2BB^T + I_3 + \begin{pmatrix} 5 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

4. Assume that A is a symmetric, positive definite `pxp` matrix. Let `q<p` that can be used in order to partition A as

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

where A_1, A_2, A_3 and A_4 are matrices of dimension `qxq`, `qx(p-q)`, `(p-q)xq` and `(p-q)x(p-q)` respectively. Write commands to check whether A is symmetric and positive definite and then partition it and calculate the matrix $A_1 - A_2A_4^{-1}A_3$ and its determinant for a given value `q<p`.
Use the following matrix and `q=2` to implement your code.

$$A = \begin{bmatrix} 1 & 0 & 0.5 & -0.3 & 0.2 \\ 0 & 1 & 0.1 & 0 & 0 \\ 0.5 & 0.1 & 1 & 0.3 & 0.7 \\ -0.3 & 0 & 0.3 & 1 & 0.4 \\ 0.2 & 0 & 0.7 & 0.4 & 1 \end{bmatrix}$$