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ΣΧΟΛΗ ΕΠΙΣΤΗΜΩΝ & ΤΕΧΝΟΛΟΓΙΑΣ ΤΗΣ ΠΛΗΡΟΦΟΡΙΑΣ — SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY

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Ειδικά Θέματα Αλγορίθμων

Ασκήσεις Φροντιστηρίου #6

Algorithms for flows and matchings

1. Show that splitting an edge in a flow network yields an equivalent network. More formally, suppose that flow network G contains edge (u, v), and define a new flow network G' by creating a new vertex x and replacing (u, v) by new edges (u, x) and (x, v) with $c(u, x) = c(x, v) = c(u, v)$. Show that a maximum flow in G' has the same value as a maximum flow in G.

2. Extend the flow properties and definitions to the multiple-source, multiple-sink problem. Show that any flow in a multiple-source, multiple-sink flow network corresponds to a flow of identical value in the single-source, single-sink network obtained by adding a supersource and a supersink, and vice versa.

3. Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is each vertex v has a limit $l(v)$ on how much flow can pass through v. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G. How many vertices and edges does G' have?

4. Suppose that each source s_i in a flow network with multiple sources and sinks produces exactly pi units of flow, so that $\sum_{v \in V} f(s_i, v) = pi$. Suppose also that each sink t_j consumes exactly q_j units, so that $\sum_{v \in V} f(v, t_j) = q_j$, where $\sum_i p_i = \sum_j q_j$. Show how to convert the problem of finding a flow f that obeys these additional constraints into the problem of finding a maximum flow in a single-source, single-sink flow network.

5. Given a graph G with unary weights. Show that the least amount edges that need to be deleted, in order to disconnect two vertices s and t, is equal with the amount of maximum flow from s to t given that each edge has capacity equal to 1. **Hint:** What is the connection of the above quantities with the amount of edge-disjoint paths from vertex s to t in graph G.