AUEB COMPUTER GRAPHICS GROUP

COMPUTER GRAPHICS COURSE

Geometry Representation

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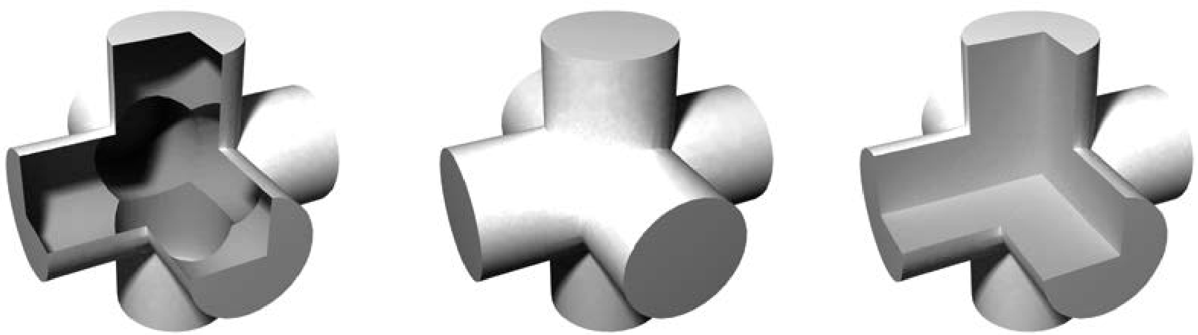
How do we Represent our World?

- In general, the mathematical world we display in graphics consists of entities describing:
  - The geometry of the surface of objects
  - The volume of the space inside and outside the surfaces
  - The energy (light) that is transmitted
  - The materials (substance qualities) the energy interacts with
  - The spatial relationships of entities
  - The dynamics of all the above (motion)

Representing Geometry - Surfaces

- In most cases, we are interested in displaying surfaces, i.e. the “shell” of 3D objects that is the interface between one medium and another
- They are 2D embeddings in a 3D space



In rendering, we are usually interested in the surface of an object

- This is because most lighting “events” occur on or near this interface

Representing Geometry - Volume

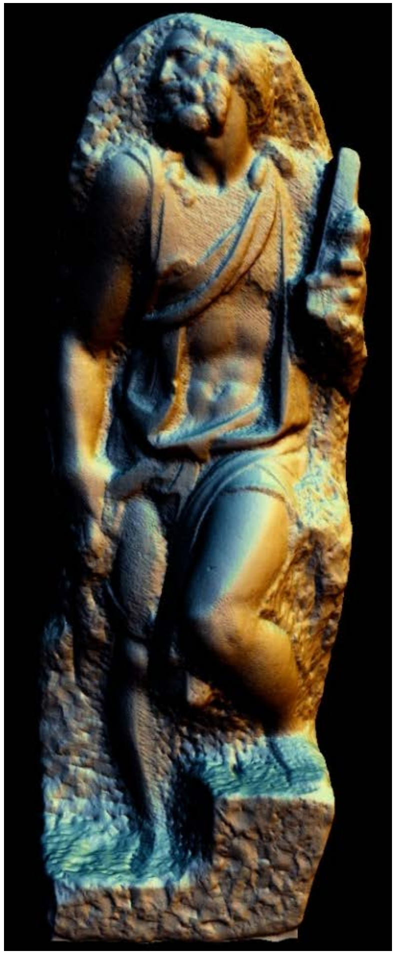
- The volume of a 3D entity becomes important when light interacts with it as it travels though the medium
- Most objects are opaque, absorbing the transmitted light fast
- However, the rendering of important light scattering phaenomena must take into account transmission through dense media such as:
  - Clouds, smoke, wax, milk etc.
  - These are expressed as volumetric data, i.e. data defined everywhere inside a *boundary surface*

Representing Geometry - Curves

- 2D/3D curves are by definition infinitesimally thin and insubstantial, and therefore non-renderable per se.
- They are 1D embeddings in 2D or 3D space
- They are used in graphics to define boundaries, trajectories and higher-dimension entities (e.g. parametric surfaces)
- We sometimes “plot” the curves, i.e. approximate them by pixels (of non-zero area) on an image plane

Representing Geometry - Points

- Points (isolated vertices) in 3D and 2D space are sometimes drawn to represent:
  - Scattered data of variable density
  - "very small" (sub-pixel) objects, such as particles
- We typically use these in massive quantities to approximate either surface data or volume data (a processes called *point-based* rendering)
- Points are not necessarily rendered as single pixels



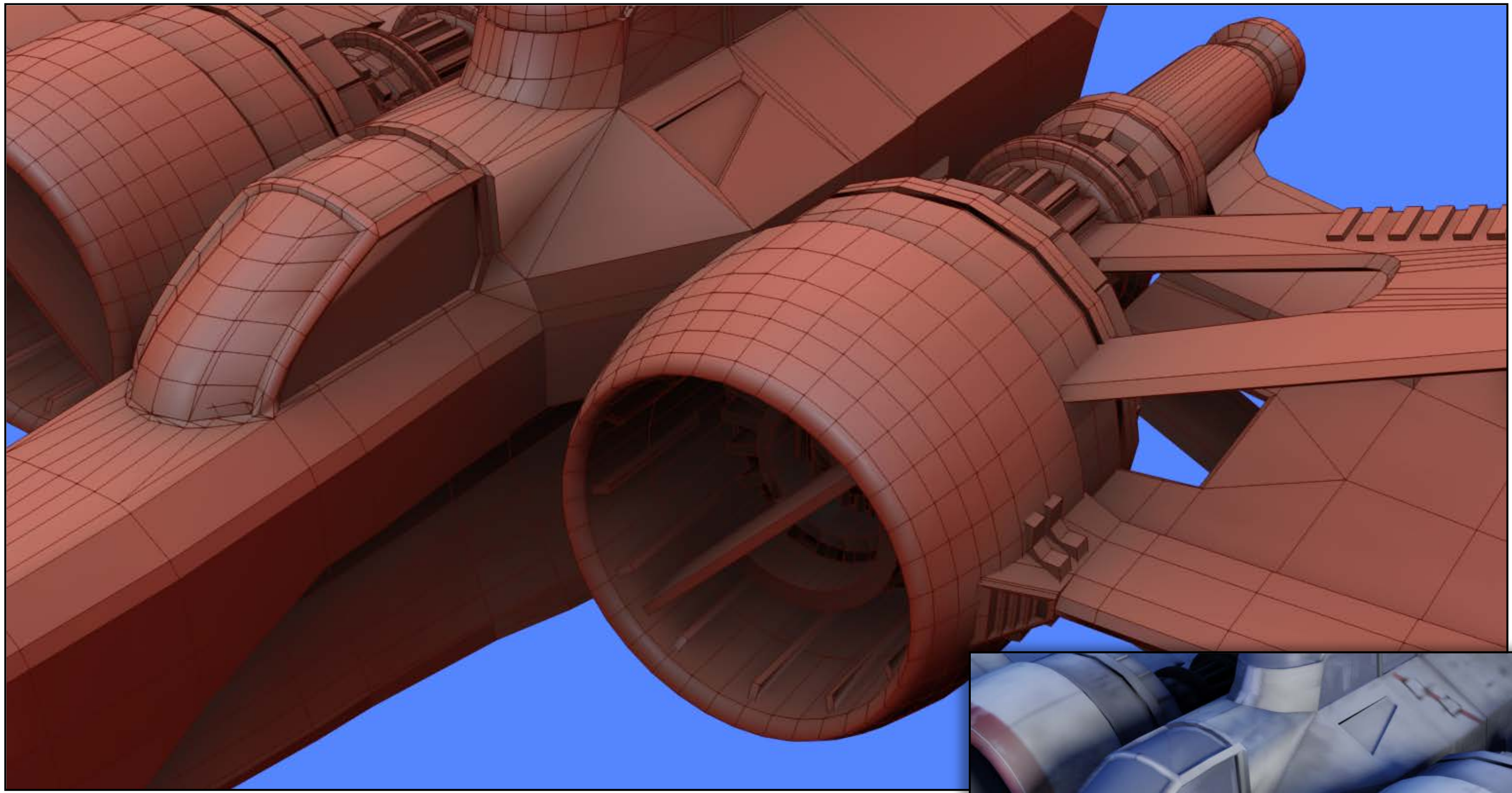
http://www-sop.inria.fr/reves/Basilic/2005/BDS05/

Surfaces

- Surfaces are composed or modelled via an aggregation of surface elements
- These elements can be:
  - Curved parametric patches
  - Flat polygons → usually triangles

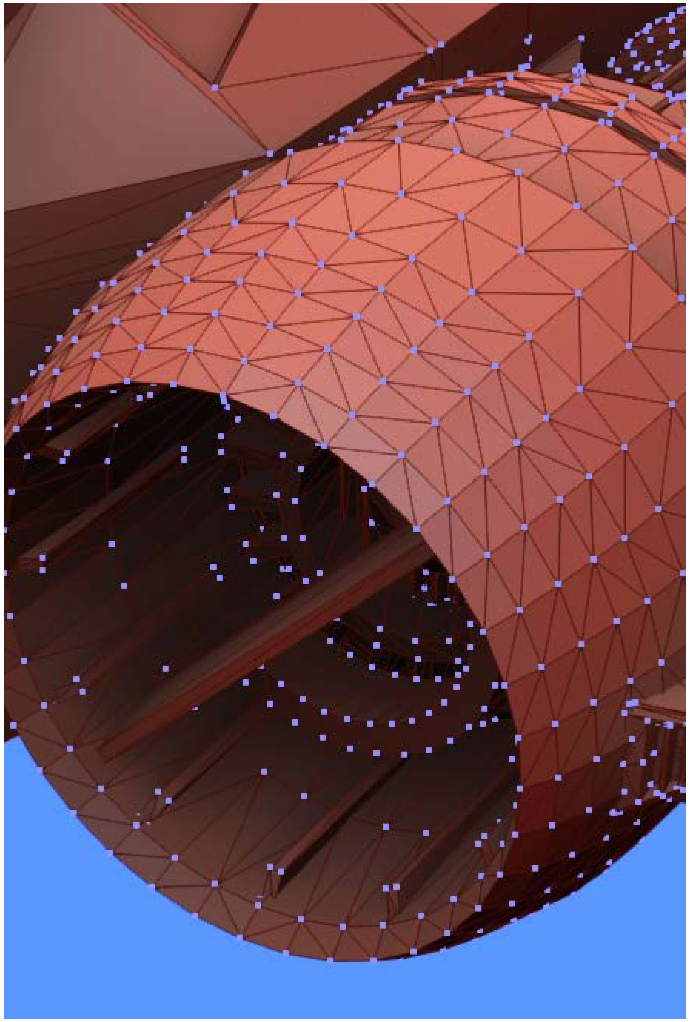
- An organization of a surface into connected polygonal surface elements is called a mesh

Mesh Example



Triangle Meshes

- The most common type of polygonal mesh representation
- Very convenient to use in real-time rendering!

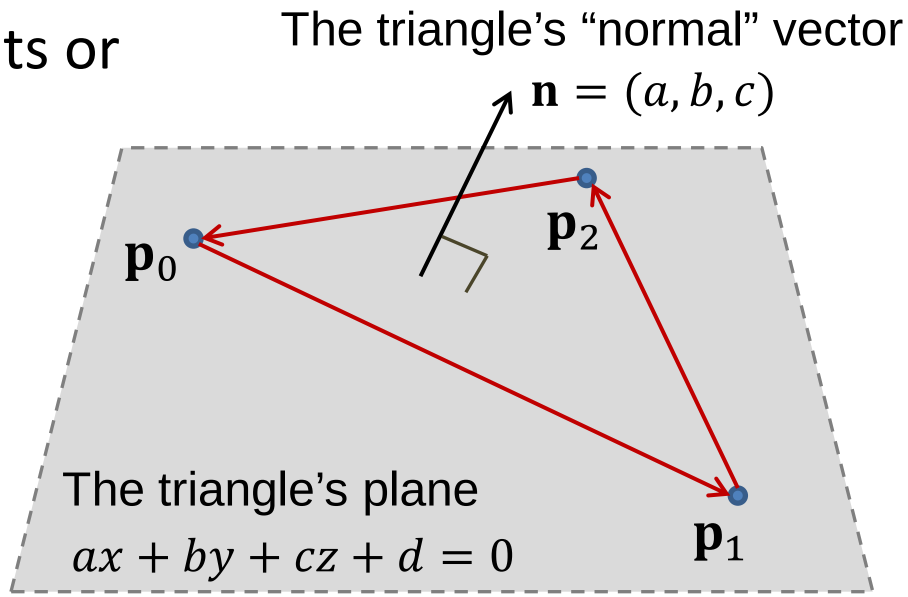


The Ingredients of Meshes

- What are their data?
  - Flat elements approximating both curved and flat surfaces
- Where are they?
  - Vertices of triangles (points on the plane or in space)
- What is their shape?
  - Connectivity among vertices defines the structure of the mesh
- How do they look?
  - Material and shading attributes per vertex – interpolated / predicted inside each triangle

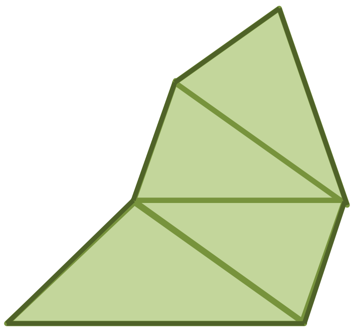
The Triangle

- A set of three (ordered) vertices (points in space)
- Connectivity:
  - Implied by order of points or
  - Given explicitly



Triangles - Useful Properties

- Minimal: The most elementary surface shape (3 points define an oriented plane)
- Always convex!
  - This is not true for generalized *n*-gons
  - A very useful property in calculations: its boundary coincides with its convex hull
- Any other polygon can be decomposed into a set of triangles!



Linear Combinations on the Plane

- Any point on the plane of the triangle can be uniquely written as a linear combination of the three triangle vertices. In fact:
- For any number *k* of vectors and scalar coefficients:

$$\mathbf{q} = \sum_{i=1}^{k} a_i \mathbf{v}_i$$ is their linear combination

Representing the Triangle Interior

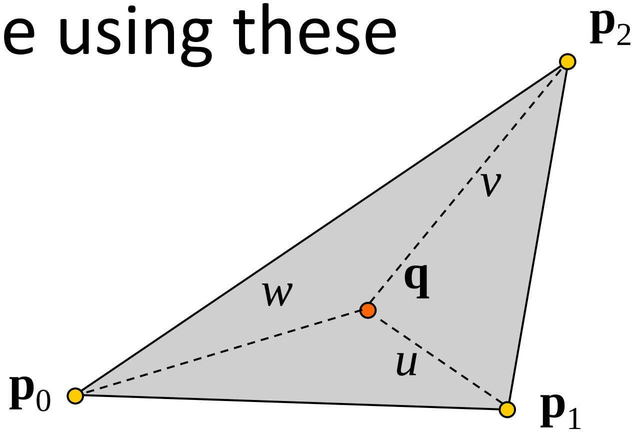
- Given the three triangle vertices (corners), any point inside the triangle (both in 2D and 3D) can be expressed as an affine combination of them
- This is an important property that translates to:
  - We only need the three corners of a triangle to fully describe its interior
- Relates to the topic of “affine transformations”

Barycentric Coordinates

- To interpolate parameters across a triangle we need to find the set of (unique) parameters u, v, w that define $\mathbf{q}$ as the linear combination of all 3 vertices

$$\mathbf{q} = w\mathbf{p}_0 + u\mathbf{p}_1 + v\mathbf{p}_2$$

- All vertex attributes can be linearly interpolated at arbitrary locations on the plane using these barycentric coordinates

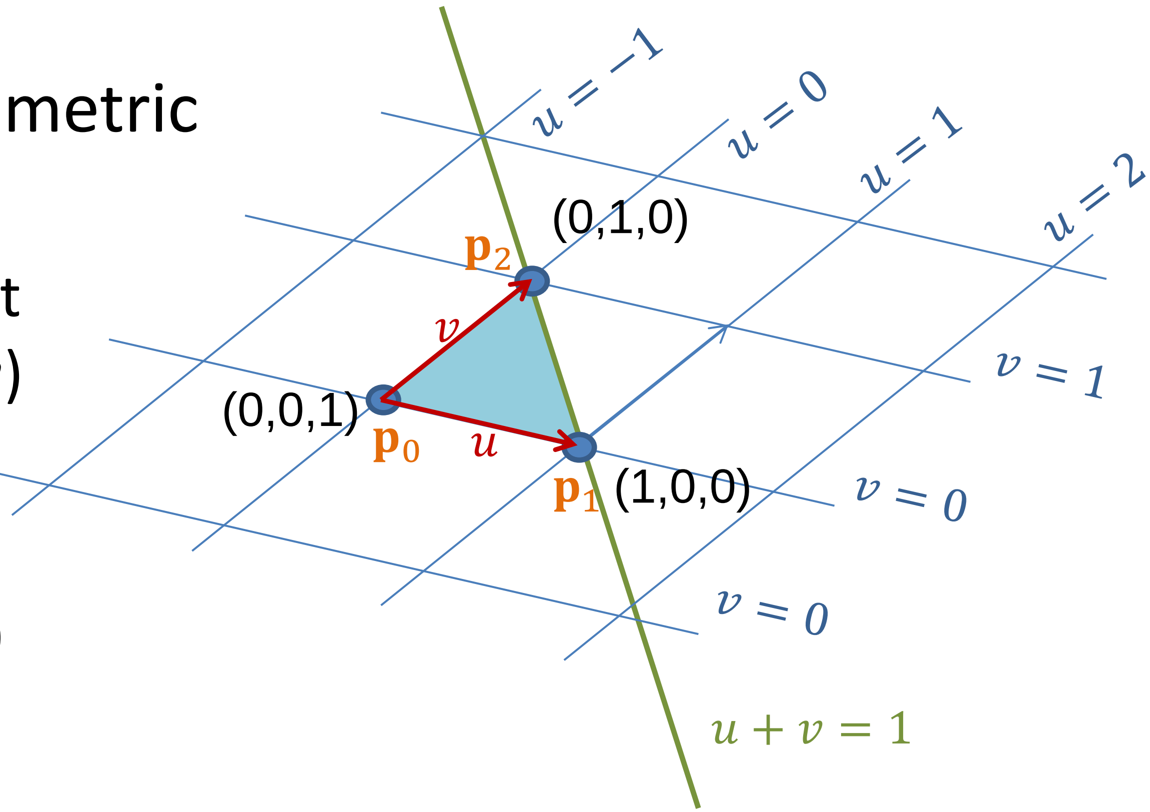


Barycentric Coordinates - Usage

- Barycentric coordinates are extremely useful in triangle rendering, as they are used for:
  - Interpolating triangle properties for arbitrary points inside the triangle
  - Performing point containment tests for various primitive intersection tests

Interpreting Barycentric Coordinates (1)

- They form a parametric space of:
  - two independent parameters (u, v)
  - One dependent parameter $(w = 1 - u - v)$



- All points with:

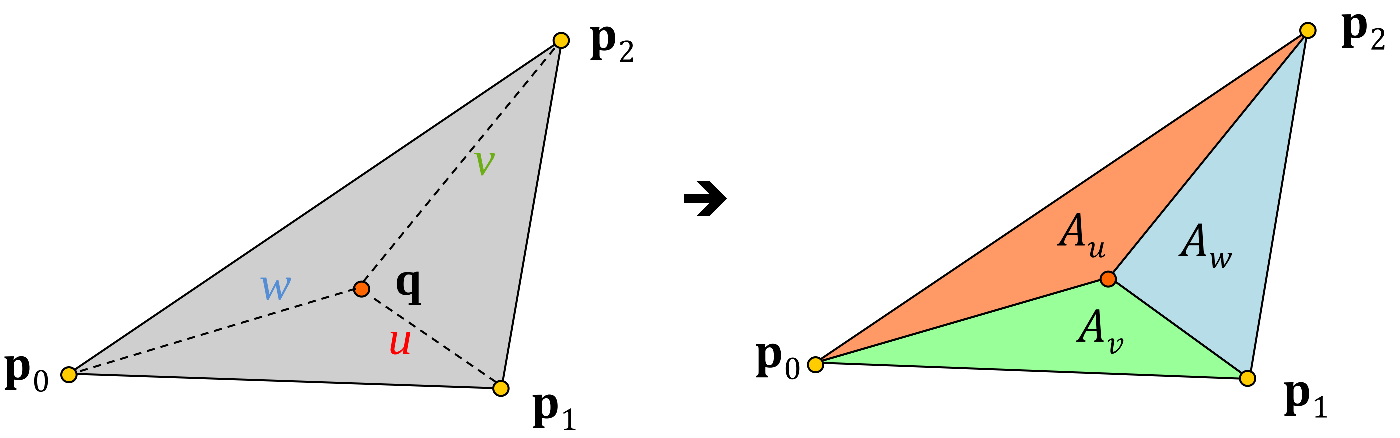
  $u > 0 \; AND \; v > 0 \; AND \; w > 0$

  Lie inside the triangle

Interpreting Barycentric Coordinates (2)

- Geometric properties:
  - Barycentric coordinates equal the ratio of triangle areas formed by the opposite side and the query point against the total triangle area
  - Can be exploited to computed them:

$$w = 1 - u - v, \qquad u = \frac{A_u}{A_{total}}, \qquad v = \frac{A_v}{A_{total}}$$



Finding the Barycentric Coordinates (1)

- Let $\mathbf{p}_i = (x_i, y_i, z_i), i = 0..2$ the triangle vertices and $\mathbf{q} = (x_q, y_q, z_q)$ the query point. Then:

$$\begin{aligned} wx_0 + ux_1 + vx_2 &= x_q \\ wy_0 + uy_1 + vy_2 &= y_q \\ wz_0 + uz_1 + vz_2 &= z_q \end{aligned} \xrightarrow{w=1-u-v} \begin{aligned} x_0 + u(x_1 - x_0) + v(x_2 - x_0) &= x_q \\ y_0 + u(y_1 - y_0) + v(y_2 - y_0) &= y_q \\ z_0 + u(z_1 - z_0) + v(z_2 - z_0) &= z_q \end{aligned}$$

Dropping the coordinate line with the “least precision” *, say here z :

$$\begin{aligned} u(x_1 - x_0) + v(x_2 - x_0) &= x_q - x_0 \\ u(y_1 - y_0) + v(y_2 - y_0) &= y_q - y_0 \end{aligned} \Leftrightarrow \begin{bmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} x_q - x_0 \\ y_q - y_0 \end{bmatrix}$$

* To be discussed during course

Finding the Barycentric Coordinates (2)

- Using Cramer's rule, we can solve the 2X2 linear system analytically to obtain (u, v) and therefore w

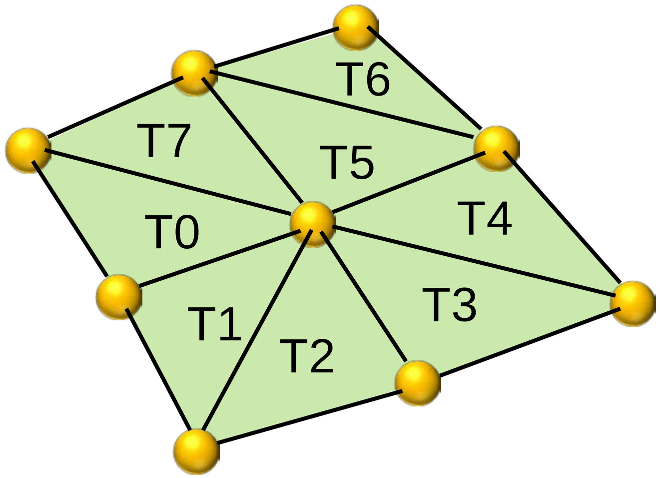
$$u = \frac{\begin{vmatrix} (x_q - x_0) & (x_2 - x_0) \\ (y_q - y_0) & (y_2 - y_0) \end{vmatrix}}{\begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix}} = \frac{(x_q - x_0)(y_2 - y_0) - (y_q - y_0)(x_2 - x_0)}{(x_1 - x_0)(y_2 - y_0) - (y_1 - y_0)(x_2 - x_0)}$$

$$v = \frac{\begin{vmatrix} (x_1 - x_0) & (x_q - x_0) \\ (y_1 - y_0) & (y_q - y_0) \end{vmatrix}}{\begin{vmatrix} (x_1 - x_0) & (x_2 - x_0) \\ (y_1 - y_0) & (y_2 - y_0) \end{vmatrix}} = \frac{(x_1 - x_0)(y_q - y_0) - (y_1 - y_0)(x_q - x_0)}{(x_1 - x_0)(y_2 - y_0) - (y_1 - y_0)(x_2 - x_0)}$$

$$w = 1 - u - v$$

Mesh Data Representation and Storage

- (Triangle) meshes are collections of triangles
- Can be:
  - A set of disjoint, independent triangles (a triangle “soup”)
  - A set of vertices “shared” among triangles, given a set of “connections”:



Triangle Representation (1)

- Triangle data are typically stored in:
  - Self-contained triangle arrays
  - Indexed attribute arrays
- Vertex attributes:
  - All data related to a single vertex of a triangle
  - These include at least the position of each vertex
  - May include additional data such as color, per vertex normal vector, texture coordinates, user-defined variables etc
- Per triangle attributes:
  - Data associated to the entire triangle (e.g. material, geometric normal etc)

Triangle Data: An Example (1)

```
class Triangle
{
   float vertex[3][3];          // 3 vertices X 3 coords
   float vertex_normal[3][3]; // 3 vertices X 3 coords
   float vertex_color[3][3];  // 3 vertices X 3 components
   float plane_normal [3];    // xyz
   class Material * p_mat;    // pointer to material data
};
```

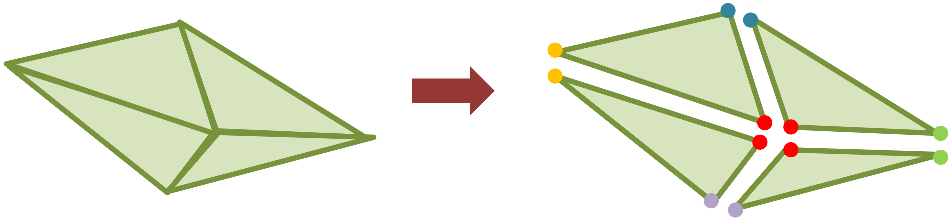
Per vertex attributes

Shared material: many triangles use the same material, so the triangle only points to this single material instance

Can I do better?

Triangle Data: An Example (2)

- Can I do better?
  - Observe that many vertices are repeated in the triangle data:



- We can save significant storage and bandwidth (this will become important later – see real-time graphics) if we:
  - Keep per vertex attribute data separately
  - Index the vertex attributes

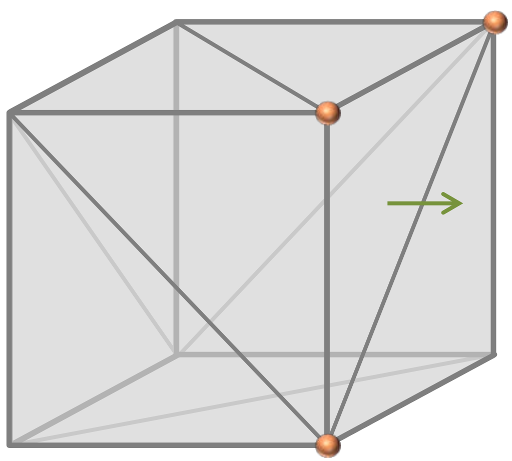
Triangle Data: An Example (3)

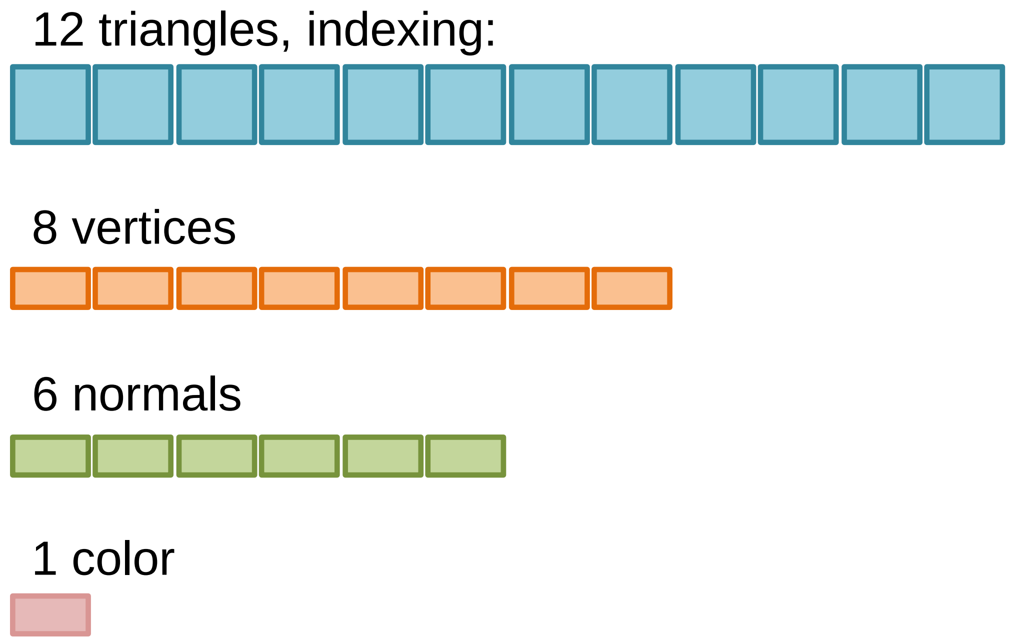
Per vertex attributes

```
class Triangle
{
    unsigned int v_index[3];    // 3 vertices X 1 index
    unsigned int n_index[3];    // 3 vertices X 1 index
    unsigned int c_index[3];    // 3 vertices X 1 index
    float plane_normal [3];     // xyz
    class Material * p_mat;     // pointer to material data
};
```

Triangle Data: Attribute Buffers

- Now we can have separate attribute buffers of different size:





Unindexed vs Indexed Data

- Memory savings from this indexing (cube example):
  - Cost for per vertex data in unindexed cube triangles:

    27 floats (4 bytes each) X 12 triangles = **1296 bytes**
  - Cost for per vertex data in indexed cube triangles:

    9X12 integer indices (4 bytes each) = 432 bytes

    8 vertices X 3 floats = 96 bytes

    6 normals X 3 floats = 72 bytes

    1 color X 3 floats = 12 bytes

    total: **612 bytes**

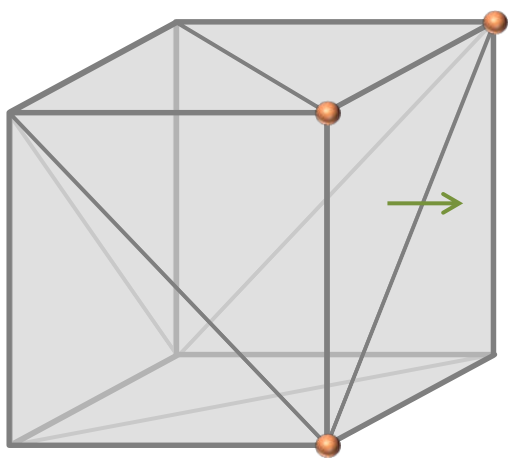
Attribute Structures (1)

- In this example, we waste a lot of memory in indexing
- Alternatively, we could use a per vertex structure and index a single array of *vertex objects*:

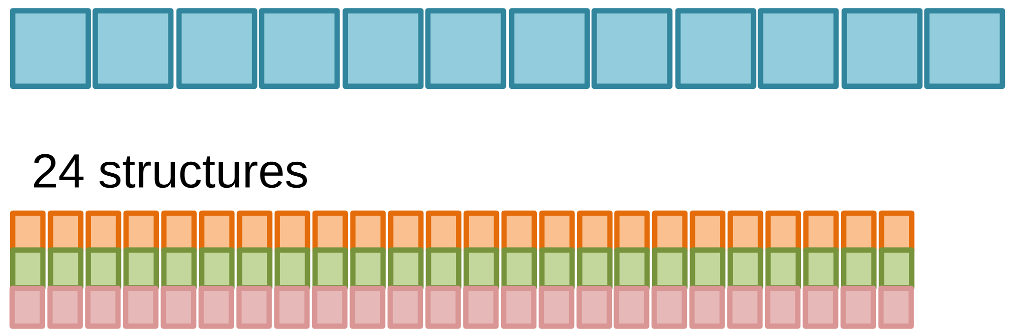
```
class Triangle
{
   unsigned int index[3];      // 3 vertices X 1 structure index
   float plane_normal [3];     // xyz
   class Material * p_mat;     // pointer to material data
};
```

Attribute Structures (2)

- Now we have a single attribute buffer:



12 triangles, indexing:



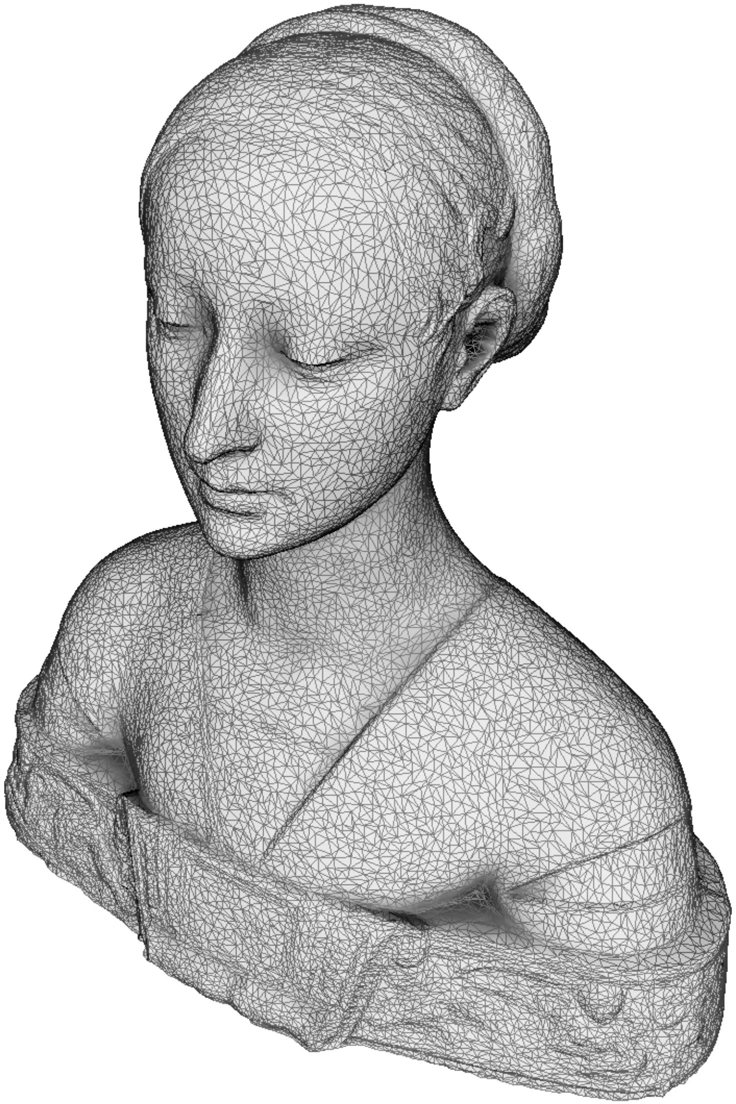
Total bytes: 144 (indices) + 864 (data) = 1008 bytes

This is larger than the separate data buffer size!!! Then why use it…???

Attribute Structures (3)

- Why use a more "wasteful" indexing?
  - The particular cube example is an extreme case that reuses many data
  - Most typical objects have attribute buffers of comparable length
  - But most importantly:
- Using a single per vertex index, abstracts the data that a vertex carries!
  - The triangle (i.e. connectivity) structure does not have to care about how many attributes each vertex has

A More Typical Indexed Mesh



- Using unindexed triangles (positions+normals):

  3,596,688 bytes
- Using separate attribute buffers:

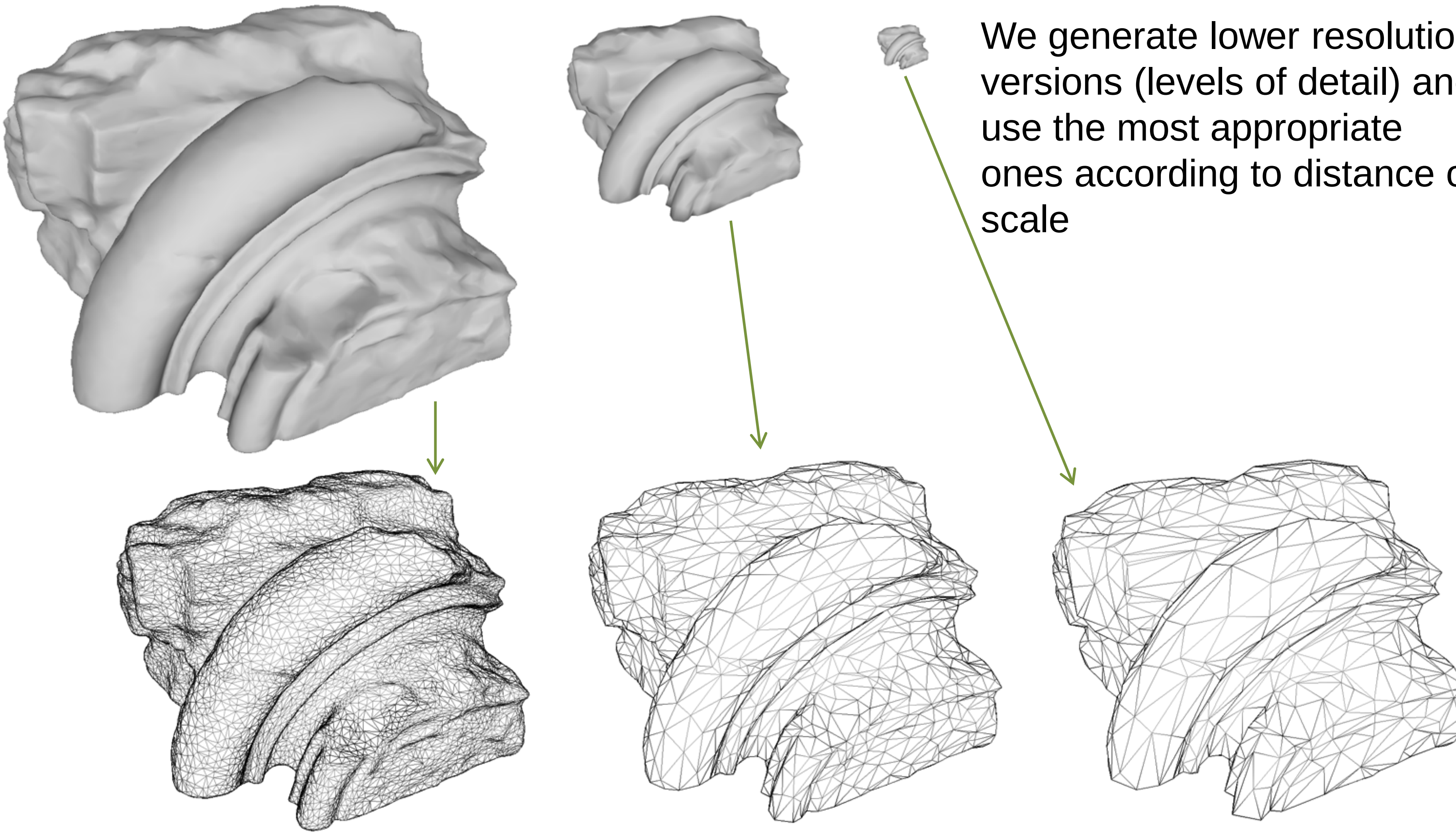
  1,867,560 bytes
- Using a single vertex object buffer:

  1,268,112 bytes

Level of Detail (1)

- For distant/small versions of objects, detail is lost, i.e. cannot be sufficiently sampled and displayed
- There is no point in attempting to display rich detail if we are not going to see it! → waste of bandwidth and processing power

Level of Detail (2)



We generate lower resolution versions (levels of detail) and use the most appropriate ones according to distance or scale

20,000 Triangles

3,000 Triangles

1,500 Triangles

LOD – Selection (1)

- The selection criteria of a LOD must answer the question:
  - What is the simplest LOD for which no visual artifacts appear?
  - Sometimes, in order to be more aggressively efficient, we allow artifacts to become noticeable, or
  - Completely disable the rendering of an object
- Criteria usually map the expected on-screen scale of an object (in pixels) to a LOD

LOD – Selection (2)

- Sometimes, we allow LODs to coexist → blending between different levels

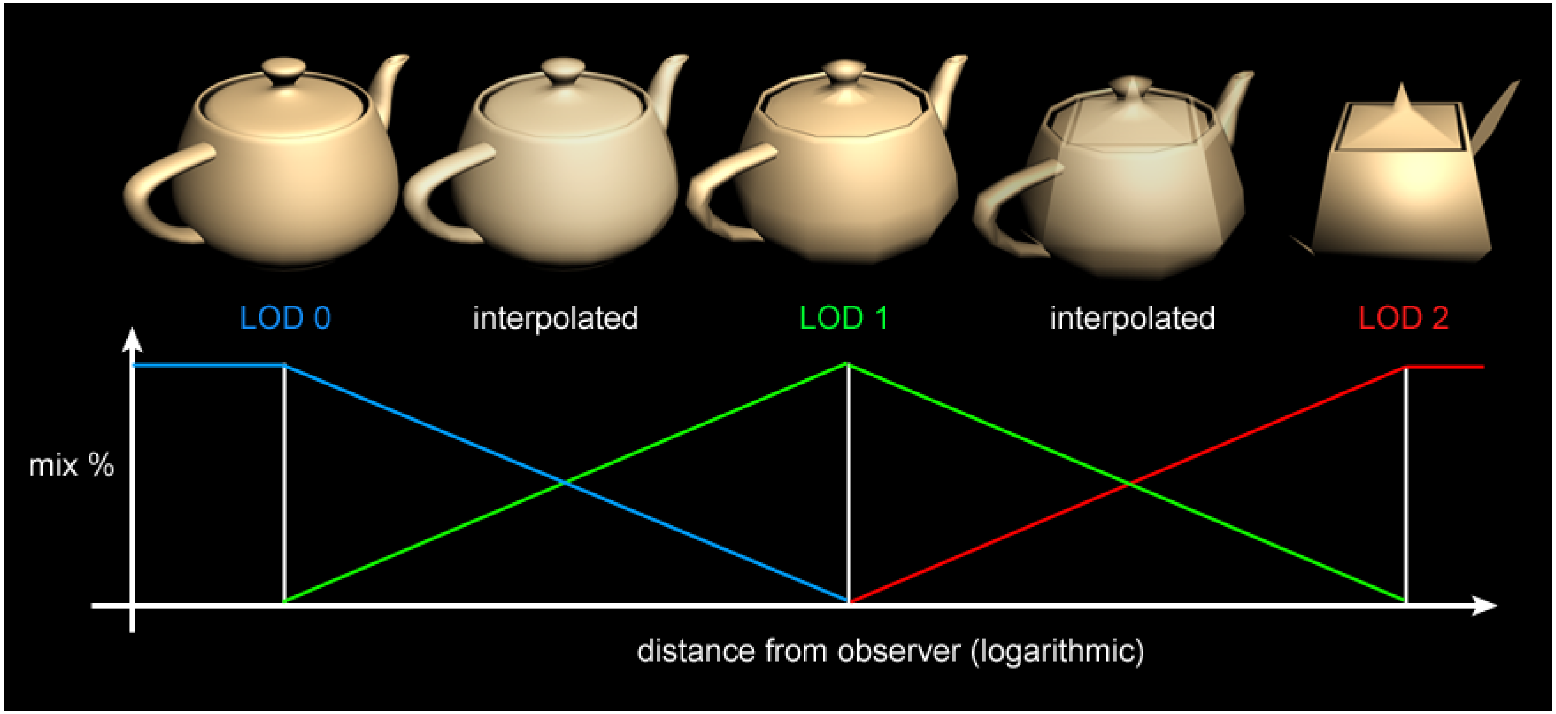


Image-based Rendering: Proxies (1)

- For distant or simply too many repeated objects, it is not efficient to approximate certain LODs with polygonal models
- We can use polygonal proxies (billboards, fins etc.) to “host” an image representation of the object, often with transparency

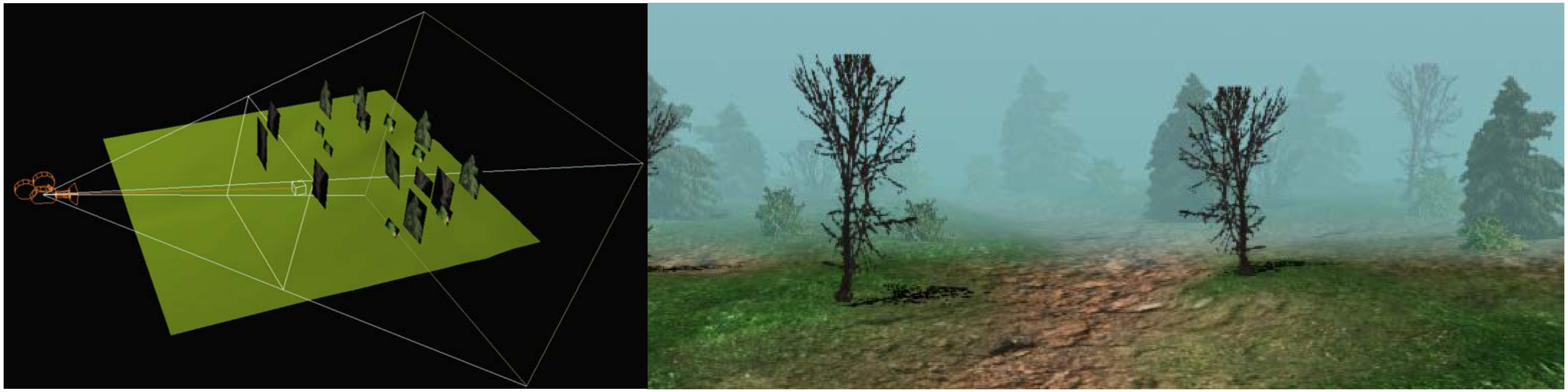
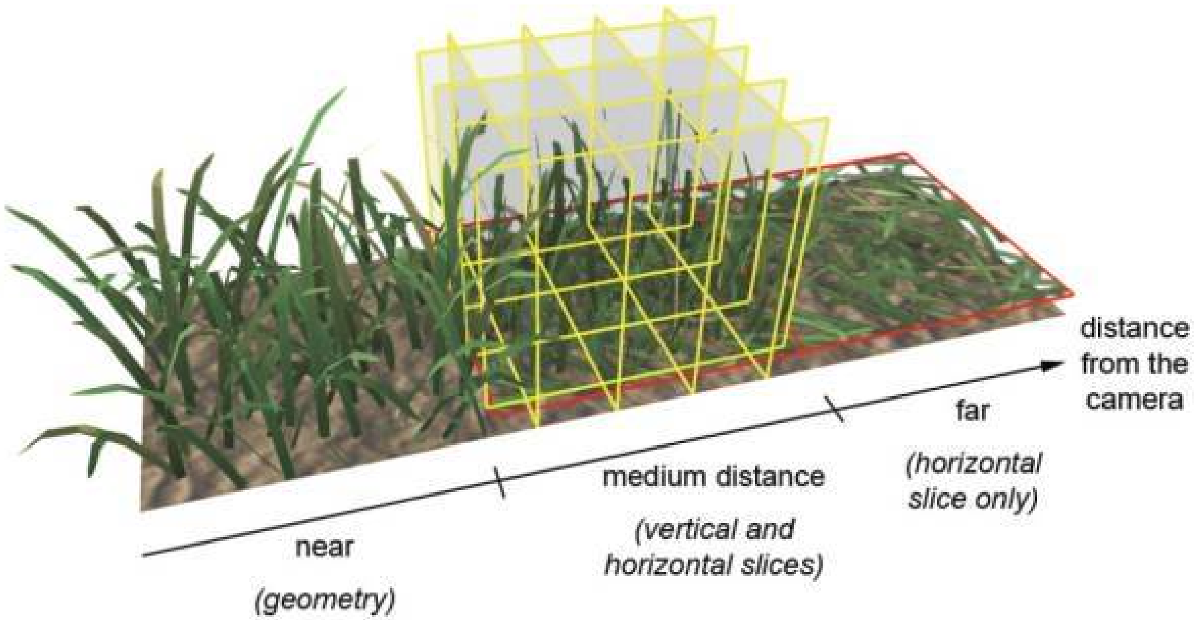


Image-based Rendering: Proxies (2)



Contributors

- Georgios Papaioannou

- Sources:
  - T. Theoharis, G. Papaioannou, N. Platis, N. M. Patrikalakis, Graphics & Visualization: Principles and Algorithms, CRC Press