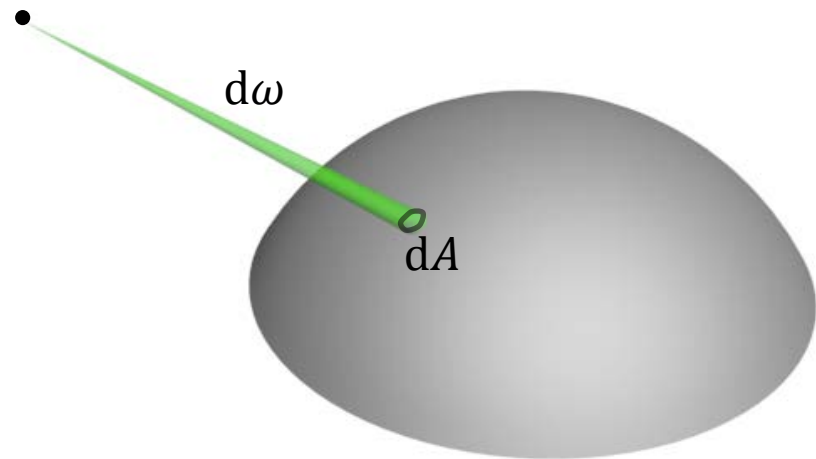
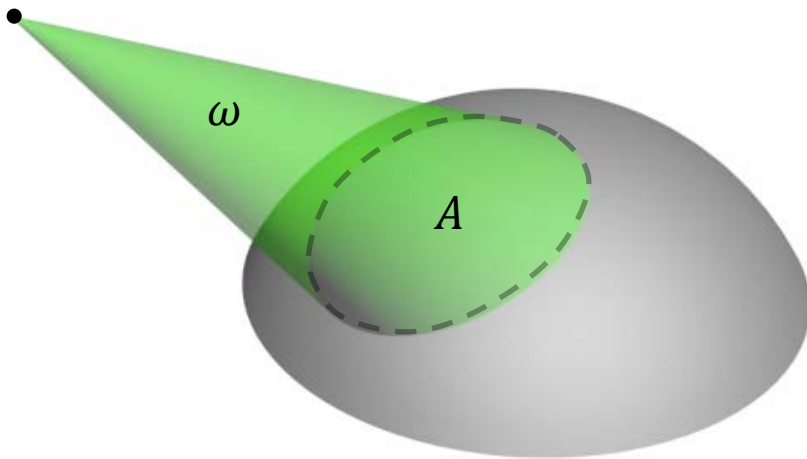


Appearance: Local Shading



- Central to the energy transport are two measures:
 - The surface area (and the differential surface area)
 - The solid angle (and of course the differential solid angle)

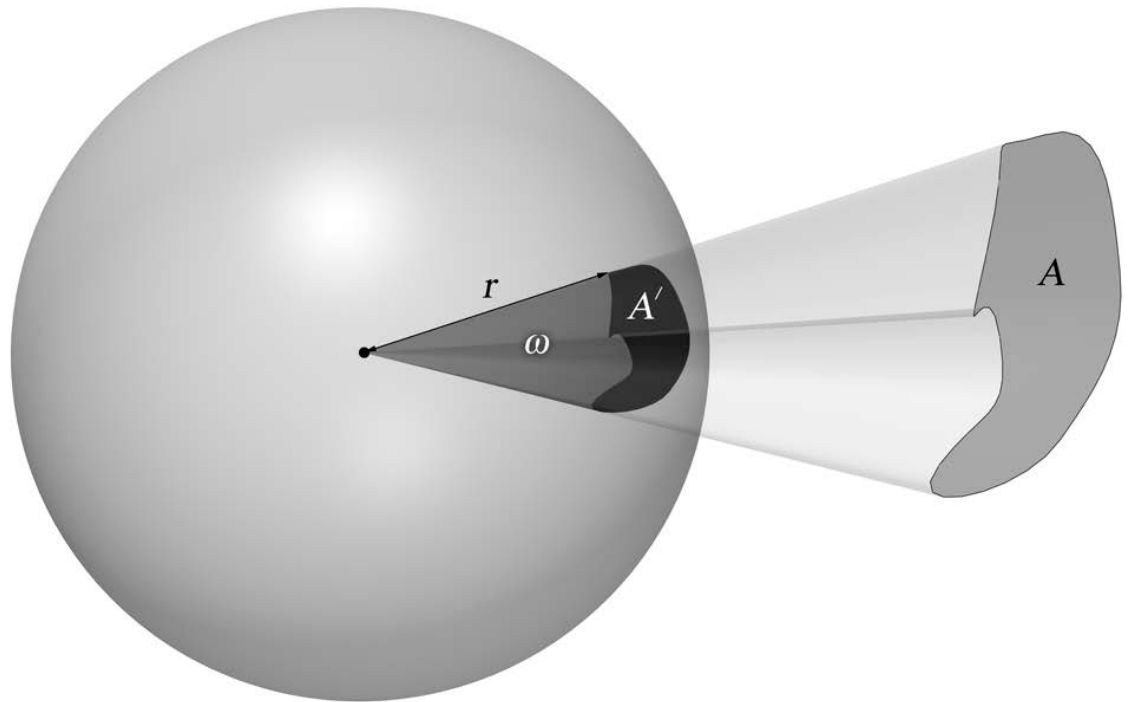


Solid Angles

- The solid angle ω subtended by a surface patch A is defined as the area of the projection of A on the surface of a sphere of radius r , divided by r^2
- Unit: steradian

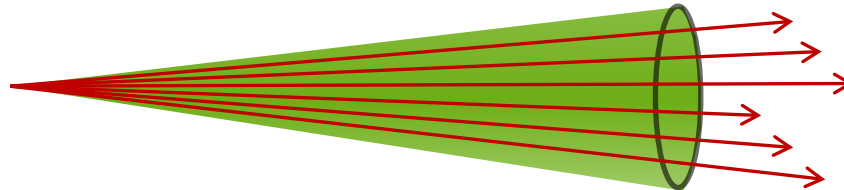
$$\omega = \frac{A'}{r^2}$$

$$\omega_{sphere} = 4\pi$$



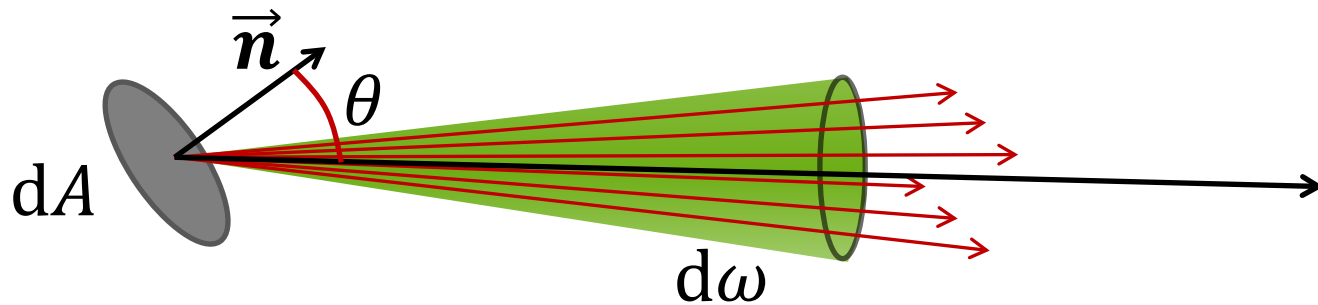
Basic Radiometric Quantities (1)

- Radiant **energy** Q : the energy carried by photons emitted from a light source (Joules)
- Radiant **power** (**flux**) $\Phi = dQ/dt$: Rate of energy (W)
- Radiant **intensity** $I = d\Phi/d\omega$: perceived light through a given solid angle in space (W/steradian)



Basic Radiometric Quantities (2)

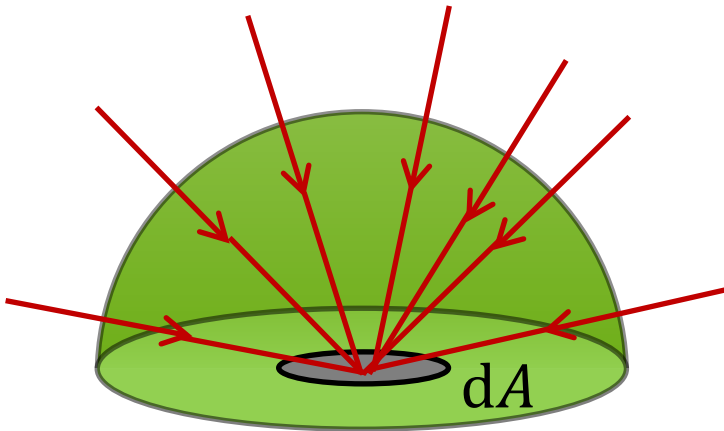
- **Radiance L** : The flow of radiant power emitted through a solid angle that crosses a tilted differential surface dA ($W/(\text{steradians} \cdot \text{m}^2)$)



$$L = \frac{\partial^2 \Phi}{\partial A \partial \omega \cos \theta}$$

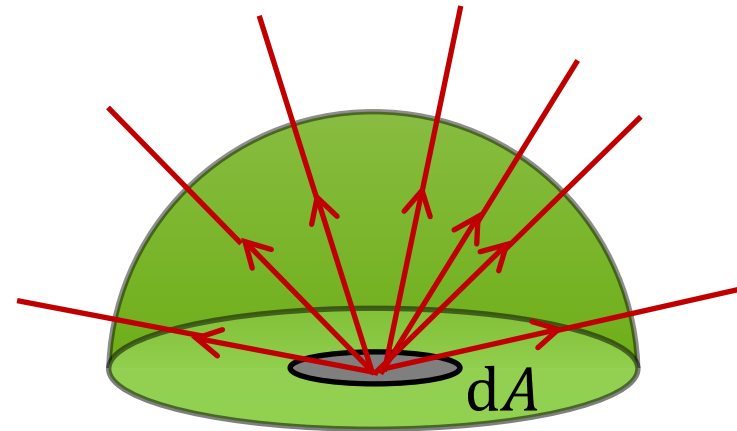
Basic Radiometric Quantities (3)

- **Irradiance E** : Incident flux from all directions on a differential patch dA :
- **Radiosity B** : Total flux exiting a differential patch dA :



irradiance

$$E = \frac{d\Phi_i}{dA}$$



radiosity

$$B = \frac{d\Phi_o}{dA}$$

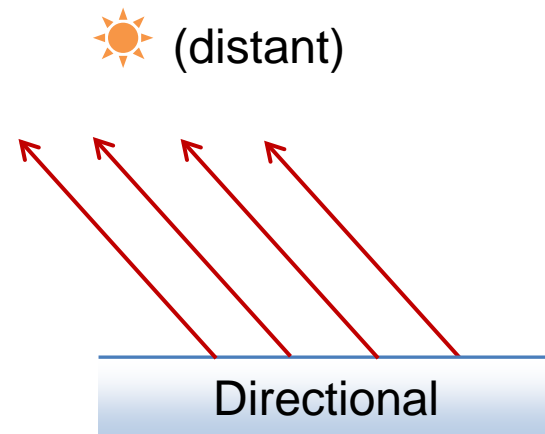
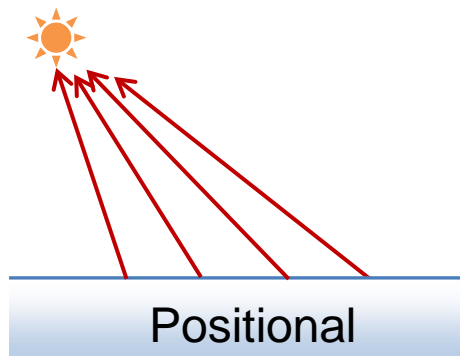
Local Shading Models

- Local shading **only regards the light interactions at a single point**
- All geometry inter-reflections are omitted
- The same holds for shadowing (light blocking)
- Local shading models can be computed at isolated locations, without requiring knowledge about the entire scene → **ideal for shader computations**

- For a local shading model to work we need emitting bodies
- In graphics, light emitters are frequently modelled separately as **light sources**
- For simplicity, proper luminaries with mass and surface can be approximated by **punctual** (point) light sources

Punctual Lights

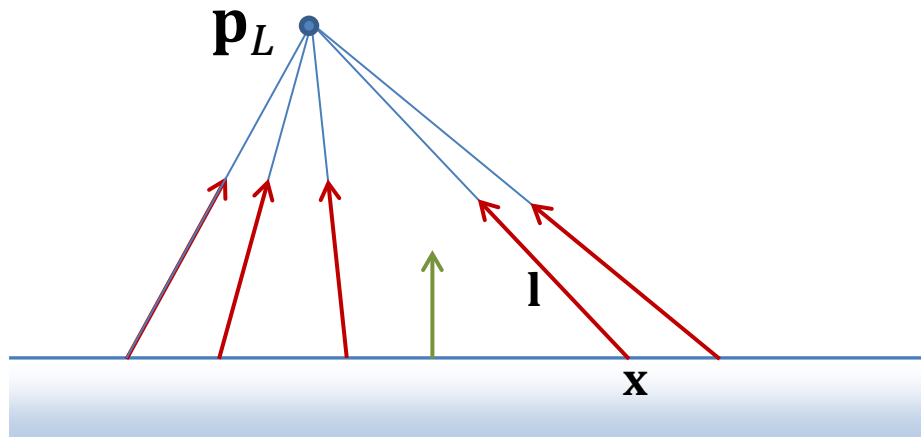
- Infinitesimally small (point) sources
 - Infinite power density
 - Can explicitly define exitant radiance and intensity
- Can be:
 - Directional (distant compared to world scale)
 - Positional



Punctual Lights - Positional

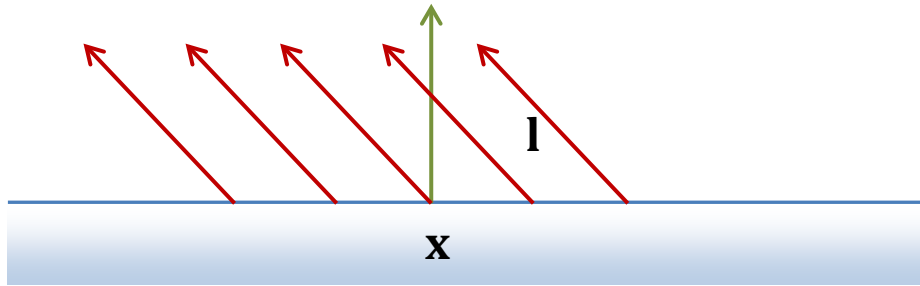
- We are given a light source position \mathbf{p}_L
- The direction towards the light is calculated per shaded point:

$$\mathbf{l} = \frac{\mathbf{p}_L - \mathbf{x}}{|\mathbf{p}_L - \mathbf{x}|}$$



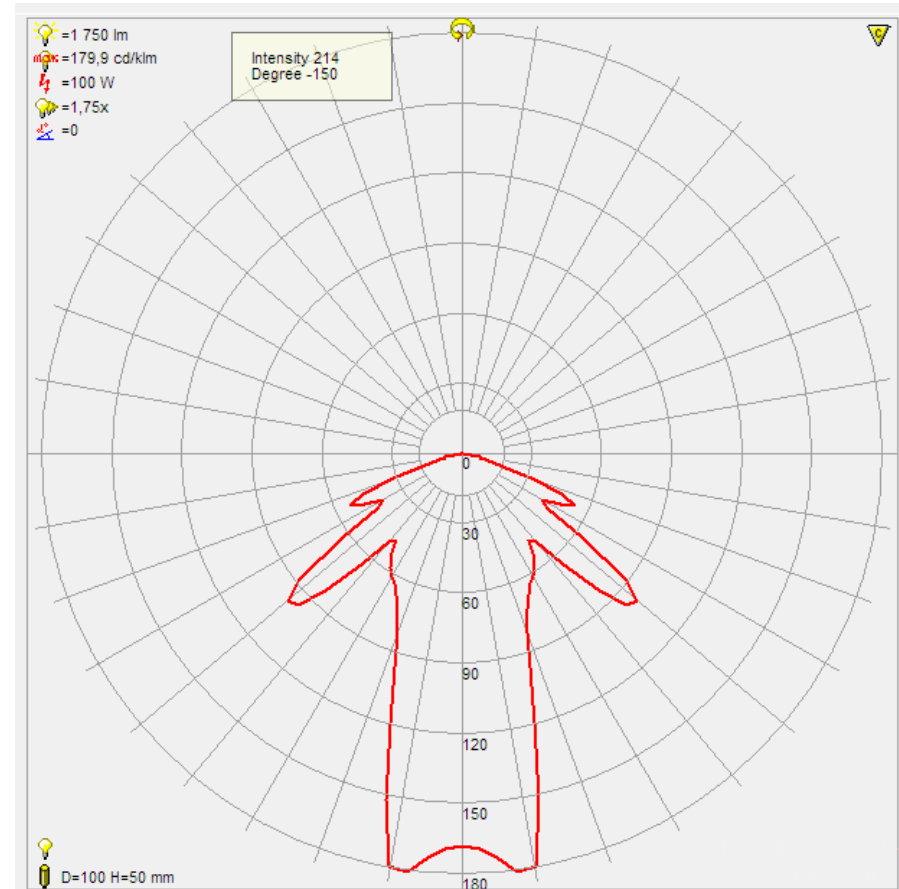
Punctual Lights - Directional

- We are given a light source direction \mathbf{l} explicitly



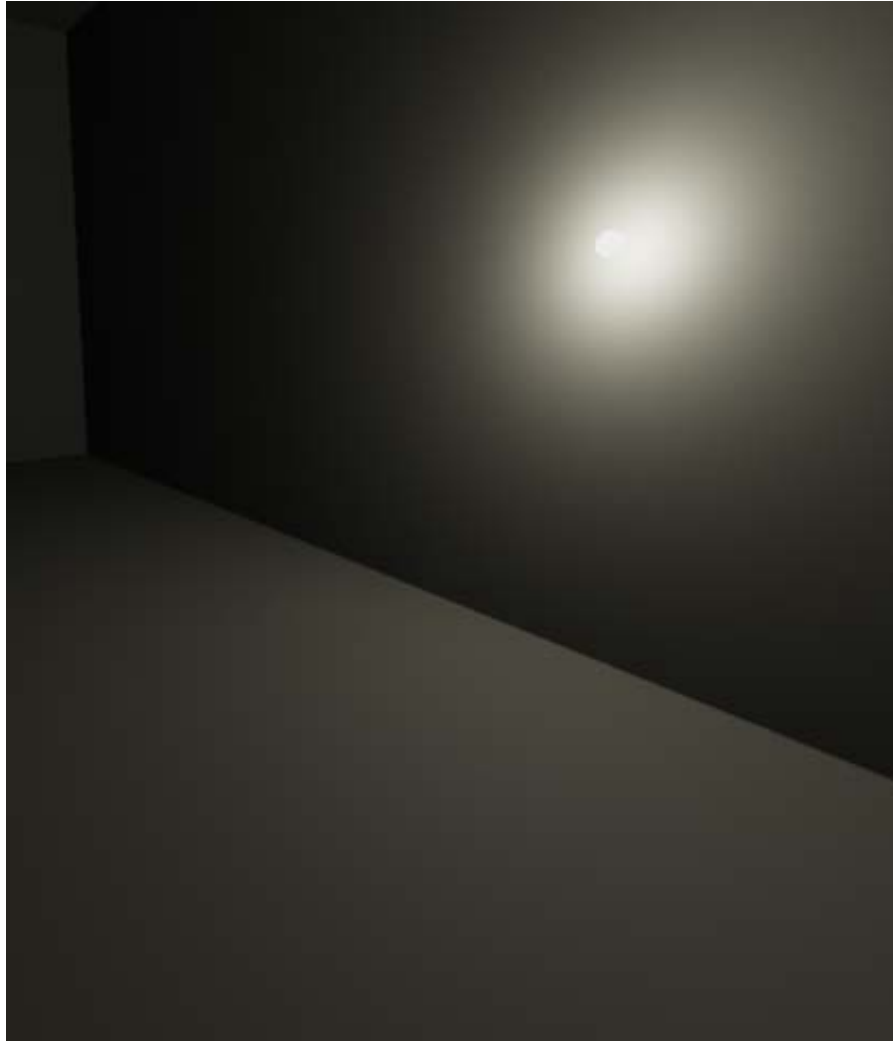
Lighting Distribution (1)

- Light sources practically emit a different amount of energy per direction
- We can model this as a distribution $f_e(\omega)$
- For convenience, we usually create punctual lights of constant emission



Lighting Distribution (2)

Uniform



Custom



Lighting Distribution (3)

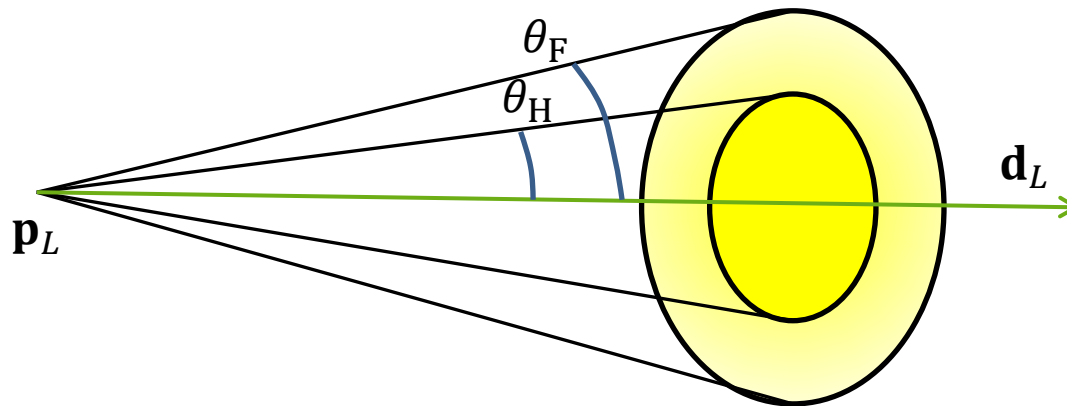
Examples



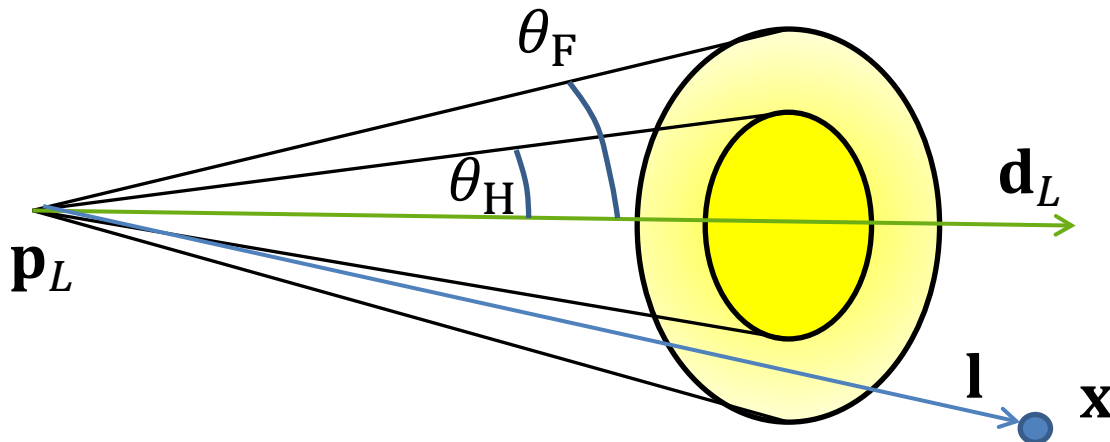
- There exists a standardization for defining the emissive properties of light sources
- IES Light description defines the emission of realistic (or measured) light sources for a given set of directions $\omega = (\theta, \varphi)$
 - Supports symmetrical luminaries, too

Light Sources – Spotlights (1)

- Very frequently, we use spotlights in computer graphics, as a procedural way to define a light source with tight emission cone
- Spotlights have 2 brightness zones:
 - Hotspot (full, maximum emission)
 - Fall-off zone (gradual dimming to zero emission)



Light Sources – Spotlights (2)



$$l = \frac{x - p_L}{|x - p_L|}$$

$$f_L(l) = \begin{cases} 1, & l \cdot d_L > \cos \theta_H \\ 1 - \frac{\cos \theta_H - l \cdot d_L}{\cos \theta_H - \cos \theta_F}, & \cos \theta_H \geq l \cdot d_L > \cos \theta_L \\ 0, & \text{otherwise} \end{cases}$$

Area Lights

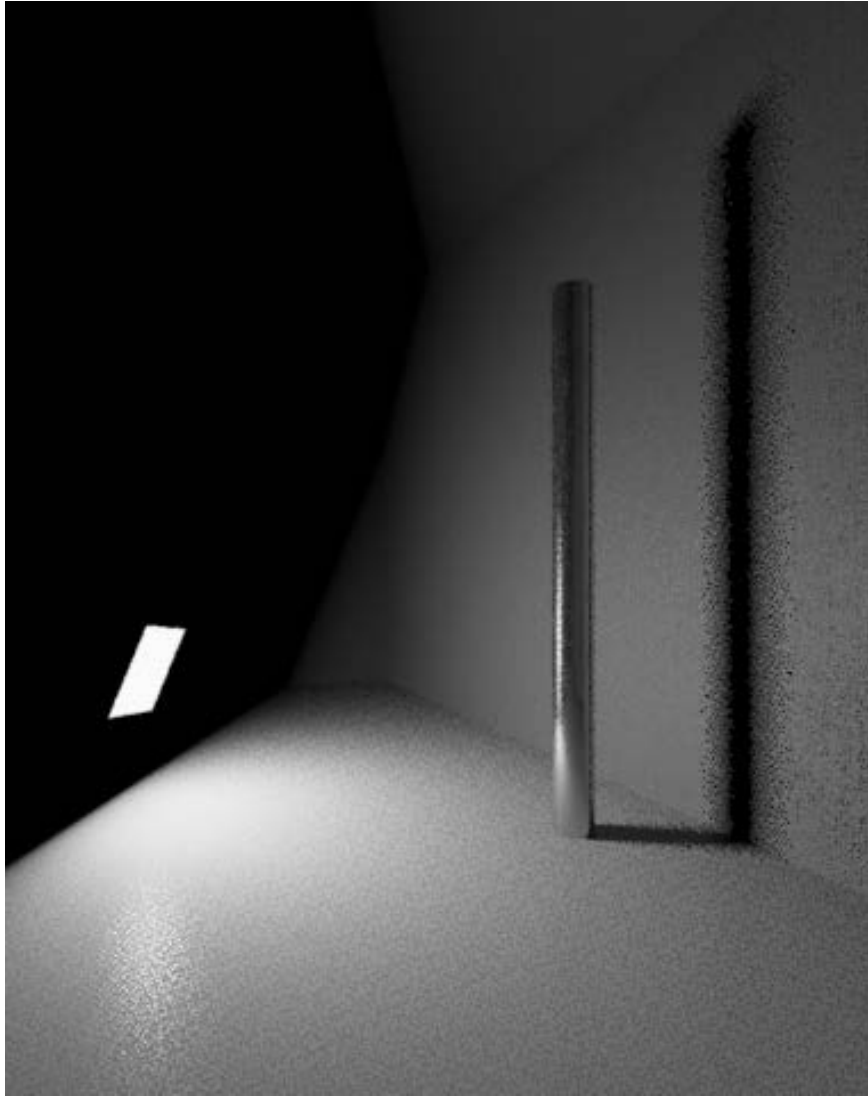
- In reality, there are no punctual light sources!
- Physical light sources are light emitting bodies
 - They have physical properties like surface area and volume



Light Contribution of Area Lights (1)

- Can be done analytically for certain light source geometry types (e.g. spheres, disks etc.)
 - Difficult to handle shadows (see shadowing presentation)
- However, usually area lights are point sampled
 - A number of point samples are chosen on them (see also Monte Carlo light sampling)
 - Each one is treated as a punctual light source
 - Each punctual light sample has its properties derived from the area light (radiance, flux etc.)
 - The sample configuration changes per shaded point to avoid patterns

Light Contribution of Area Lights (2)



Units for Lighting - Watts

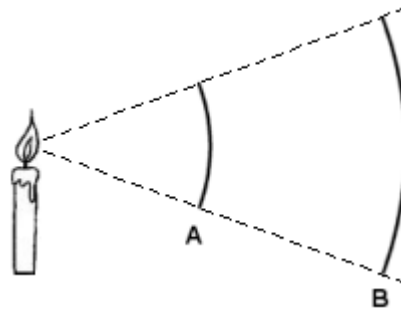
- Radiant flux is the total power that emanates from a light emitter (in Watts)
 - Caution: this is the actual **produced power**, not the consumed power (e.g. electrical)
 - Measured at the emitter surface
 - **Over the entire spectrum**
 - Our eyes are not equally sensitive to all wavelengths! A lot of energy is wasted (outside the visible spectrum)

Units for Lighting – Lumen

- Is the unit of **luminous flux**, i.e. the apparent (visible) flux (lm)
- It is related to the radiant power via the **luminous efficacy**, i.e the ability of source to produce usable lighting per Watt of produced energy
- Maximum possible efficacy: 683 lm/W (at $\lambda=555\text{nm}$)
- Example:
 - A 100W light bulb with an average efficacy of 30lm/W emits 3000lm

Units for Lighting – Candela

- Is a measure of light intensity, i.e. flux per solid angle
- We can obtain luminous flux by integrating the measured intensity over all emitting directions of the light source

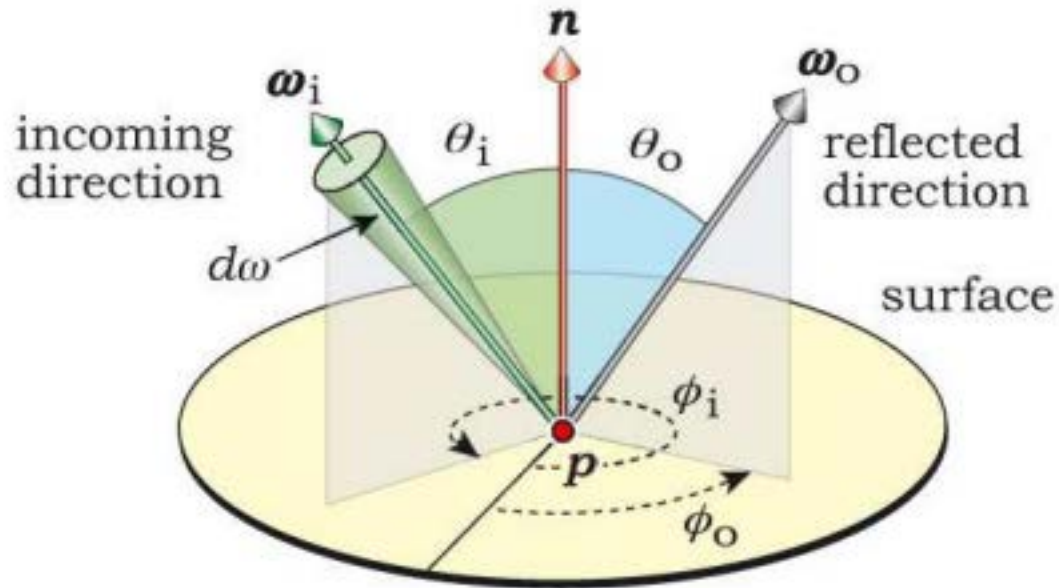


BRDF – The Reflectance Equation (1)

- What is the equilibrium of energy at a differential patch dA ?
- Energy leaving the surface in a direction ω_o is the result of:
 - Energy reflected from all incident directions ω_i
 - Energy scattered from all incident directions ω_i as a local effect (diffuse reflection)
 - Energy from all incident directions ω_i being absorbed

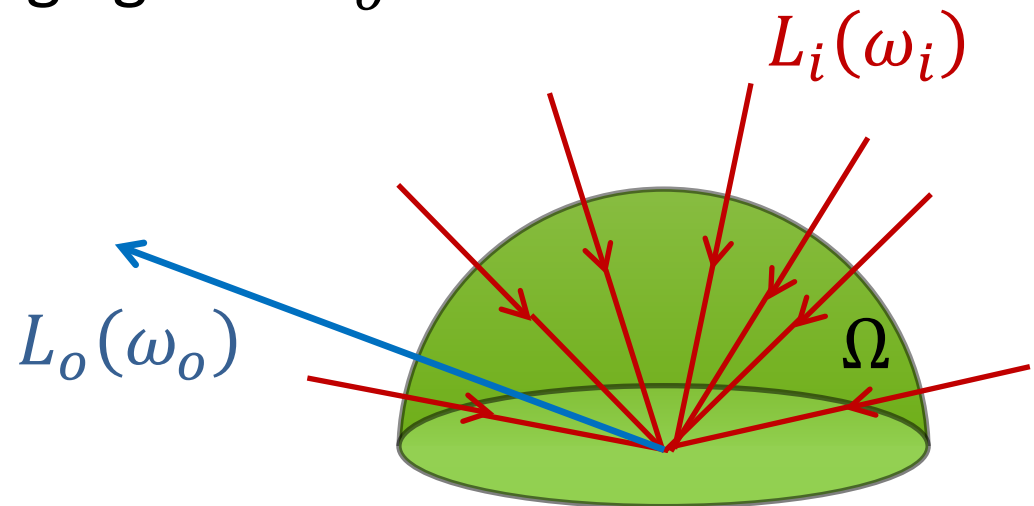
BRDF – The Reflectance Equation (2)

- The setup:



BRDF – The Reflectance Equation (3)

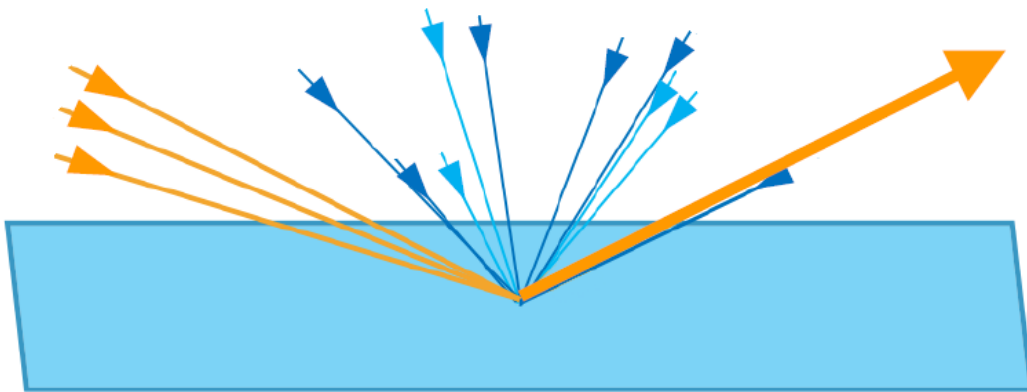
- Therefore, given a function $f(\omega_i, \omega_o)$ that indicates how much light from incident direction ω_i contributes to outgoing light in ω_o direction:



$$L_o(\omega_o) = \int_{\Omega} f(\omega_i, \omega_o) L_i(\omega_i) \cos \theta_i d\omega_i$$

The BRDF (1)

- $f(\omega_i, \omega_o)$ is the **Bidirectional Reflectance Distribution Function**
- Provides the relative contribution of each incoming direction to the outgoing lighting in a given direction



$$f(\omega_i, \omega_o) = \frac{dL_o(\omega_o)}{L_i(\omega_i) \cos \theta_i d\omega_i}$$

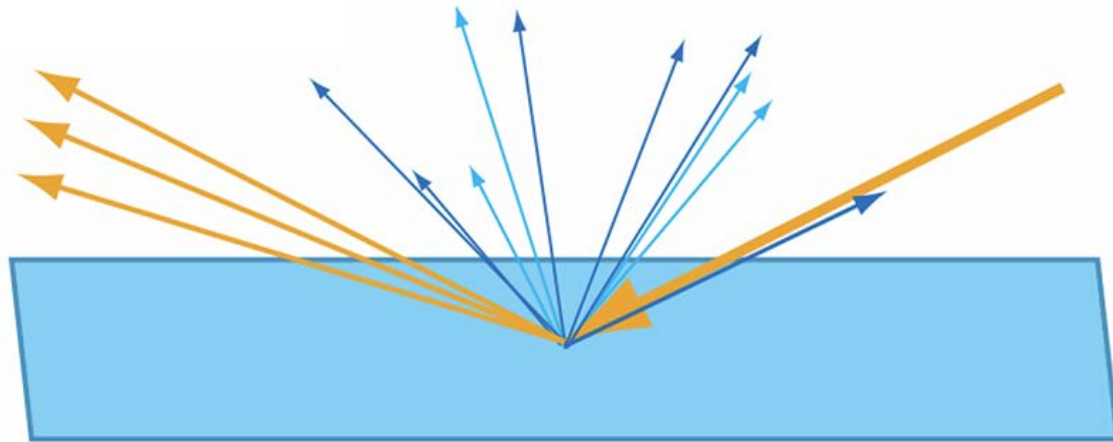
The BRDF (2)

- The BRDF characterizes the **surface material**
- The BRDF is a function of:
 - In/out latitude and longitude
 - Wavelength (so it is different for each R,G,B channel)
- A BRDF can be measured for real materials and
- **Approximated by models** in most calculations

- Properties:
 - Should be positively defined
 - Linear operator
 - The integral of the BRDF over the entire hemisphere should be ≤ 1 (it is a distribution of non-absorbed radiance)
 - Helmholtz reciprocity: For most materials $f(\omega_i, \omega_o) = f(\omega_o, \omega_i)$. Important property for many algorithms

The BRDF (4)

- Therefore, the BRDF also describes how incident light from a given direction is distributed w.r.t outgoing directions



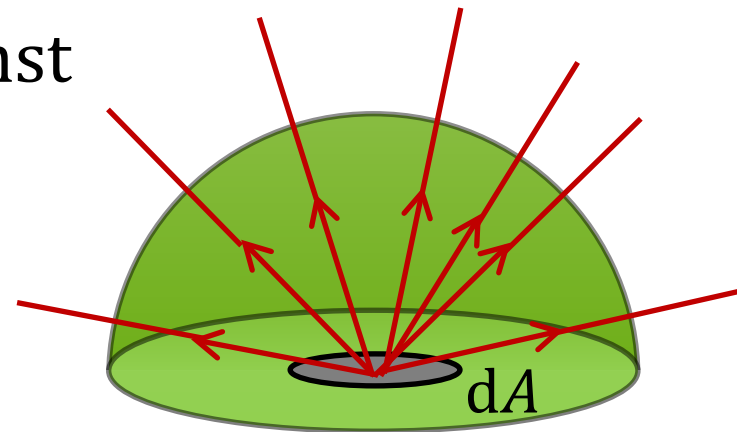
Specular and Diffuse BRDFs

- For the modeling of the BRDF, we typically regard the local scattering and reflection separately:
 - Diffuse BRDF
 - Specular BRDF
- We combine the contribution of both to the reflected color

Diffuse BRDF

- The most common modeling of the local scattering of light (diffuse reflection) is the uniform light scattering
- This BRDF is called a Lambertian BRDF

$$f_d(\omega_i, \omega_o) = f_d = \text{const}$$



Value?

- For ideally diffuse surfaces (pure white), the reflectance integral should be 1 using unit incoming energy:

$$1 = \int_{\Omega} f_d \cos \theta_i d\omega_i \Rightarrow 1 = f_d \int_{\Omega} \cos \theta_i d\omega_i = f_d \pi \Rightarrow$$

$$f_d = \frac{1}{\pi}$$

And accounting for absorption, we have loss of energy:
Replace 1 with the albedo ρ (or k_d) of the surface:

$$f_d = \frac{\rho}{\pi}$$

Specular BRDF – A Simple Model

- The commonest model for specular BRDFs is the Phong model
- It was later modified by Blinn (Blinn-Phong model)
- It is an empirical model, not a physically-based one
- Tries to model the specular highlight by using:
 - A specular color (the reflectance color K_s)
 - A specular exponent factor (“tightness” of the highlight)

The Phong Model for Specular BRDF (1)

- With the Phong model, outgoing radiance is directly given by a custom reflectance equation:

Diffuse reflection Specular reflection

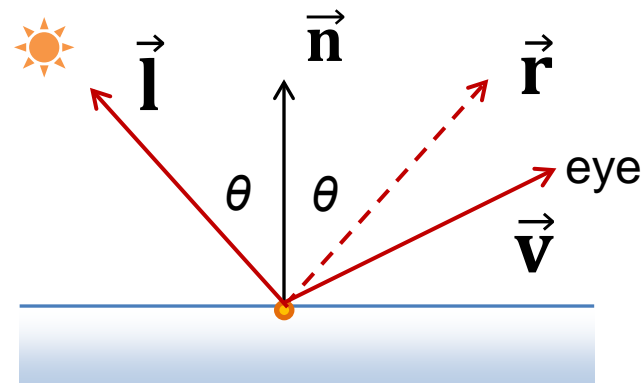
$$L_o(\vec{\mathbf{v}}) = \begin{cases} (k_d \vec{\mathbf{n}} \cdot \vec{\mathbf{l}} + k_s (\vec{\mathbf{r}} \cdot \vec{\mathbf{v}})^n) L_i(\vec{\mathbf{l}}), & \vec{\mathbf{n}} \cdot \vec{\mathbf{l}} > 0 \\ 0, & \text{otherwise} \end{cases}$$

The Phong Model for Specular BRDF (2)

(should be at most) $\frac{\rho}{\pi}$ $\cos \theta_i$ Custom specular color Specular exponent (tightness)

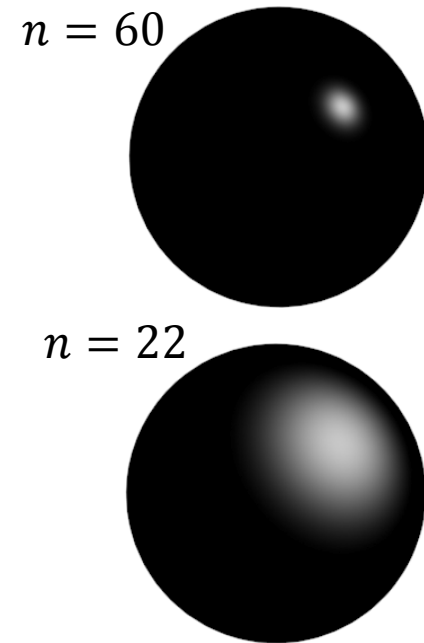
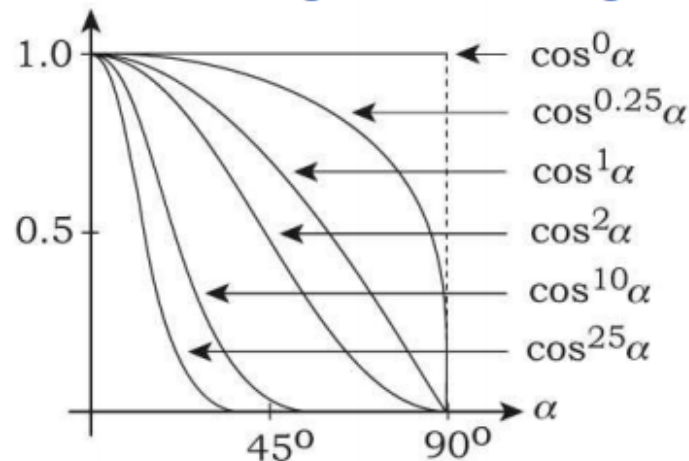
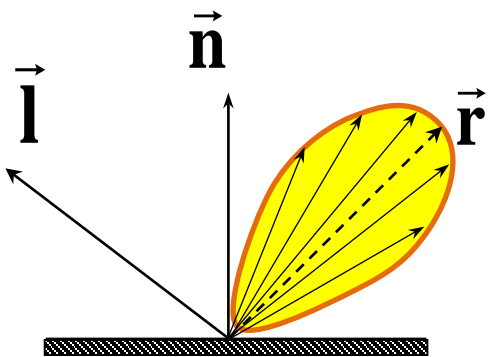
$$k_d \bar{\mathbf{n}} \cdot \bar{\mathbf{l}} + k_s (\bar{\mathbf{r}} \cdot \bar{\mathbf{v}})^n$$

- l**: Vector towards light source (ω_i)
- n**: Surface normal vector
- v**: Vector towards the eye (ω_o)
- r**: Direction of ideal reflection



The Phong Model for Specular BRDF (3)

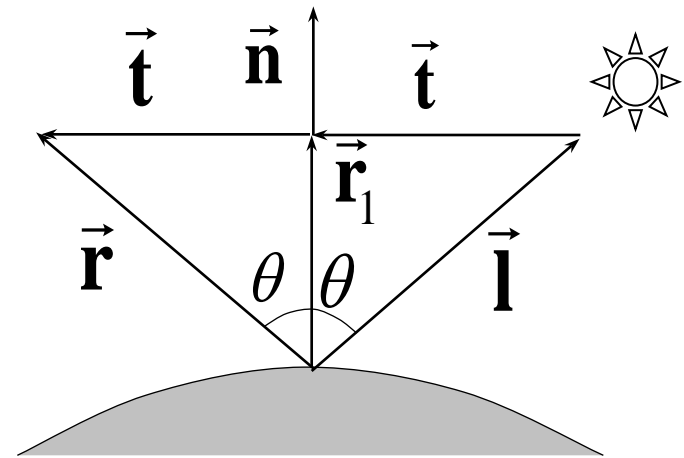
- Interpretation:
 - Outgoing directions near the ideal reflection direction receive more energy
 - The falloff of this distribution is controlled by the tightness of the highlight (exponent)



Phong Model: The Reflection Vector

- If \vec{r}_1 is the projection of \vec{r} on \vec{n} , then $\vec{r}_1 = \vec{n} \cos\theta = \vec{n}(\vec{n} \cdot \vec{l})$
- Additionally, $\vec{t} = \vec{r}_1 - \vec{l}$, and $\vec{r} = \vec{l} + 2\vec{t}$:

$$\vec{r} = 2\vec{n}(\vec{n} \cdot \vec{l}) - \vec{l}$$

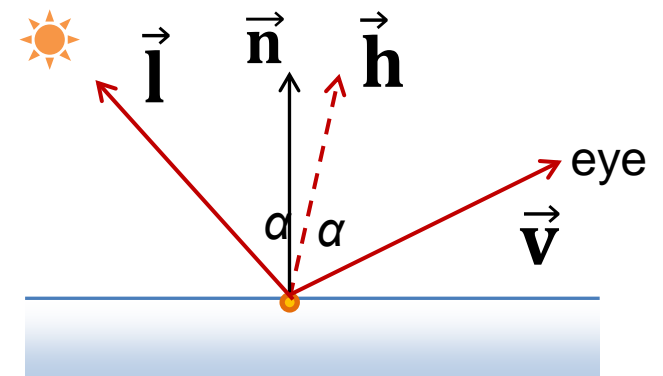


The Blinn Model (1)

- Similar to Phong
- Replaces the specular part with the following:

$$k_s (\vec{n} \cdot \vec{h})^n$$

- Where \vec{h} is the “halfway” vector between the incident and outgoing direction:



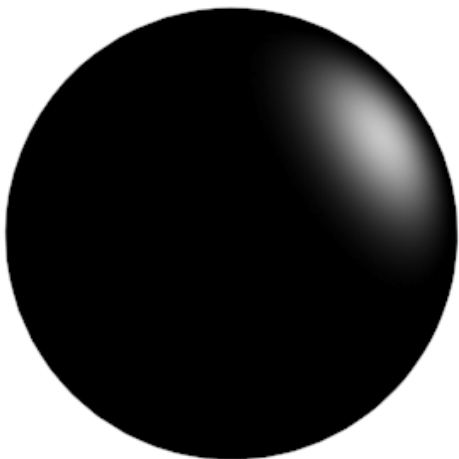
The Blinn Model (2)

- Why is the Blinn model better?
 - More consistent with the notion of “micro-facet” geometry at a microscopic level (see next)
 - Validated to be more accurate (compared to photos)
 - Faster to compute:

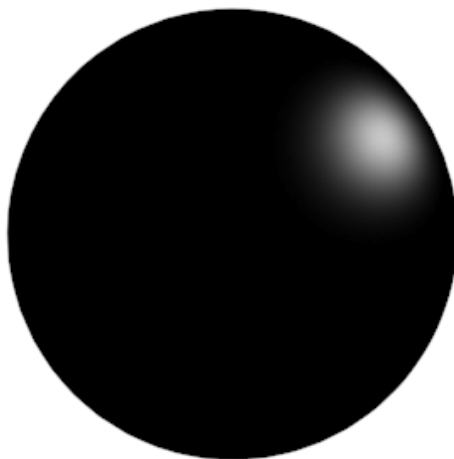
$$\vec{\mathbf{h}} = \frac{\vec{\mathbf{v}} + \vec{\mathbf{l}}}{|\vec{\mathbf{v}} + \vec{\mathbf{l}}|}$$

The Blinn Model (3)

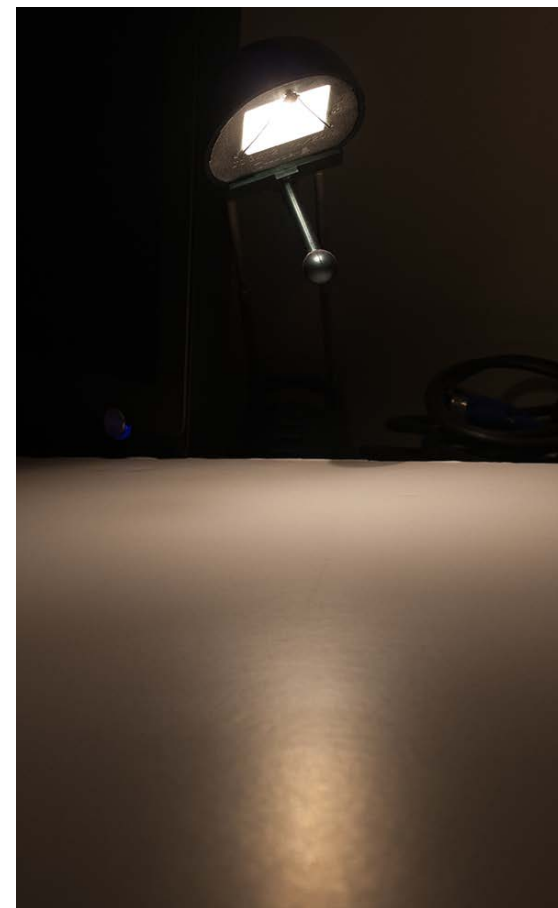
Phong



Blinn-Phong



Real glossy surface



Is the Phong-Blinn Model Realistic?

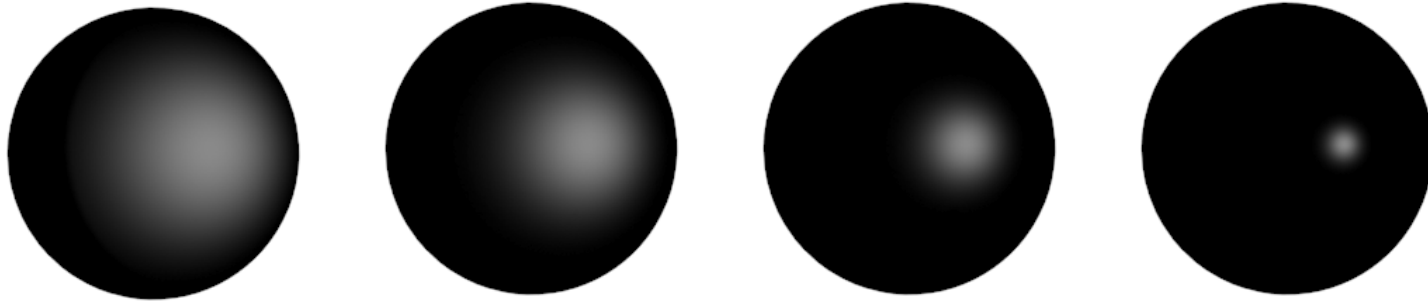
- It is **not a physically-based** shading model
- It could have been a “plausible” model, if it were not for the fact that **it is not normalized**
- It takes some manipulation to **convert to a BRDF**

The Importance of Being Normalized (1)

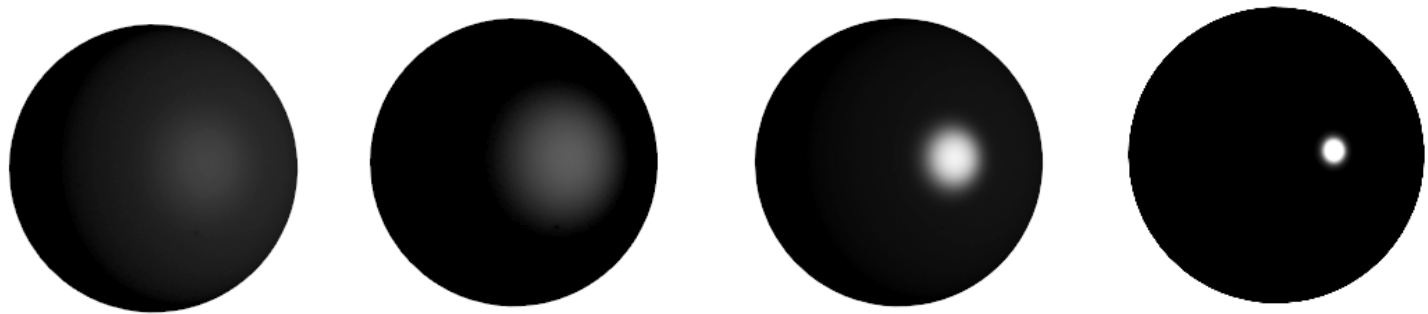
- Why is it important for the model to be normalized?
- Remember, the BRDF represents a distribution!
- For a fixed reflectivity, the total flux leaving the surface (i.e. the surface radiosity) must be constant wr.t. input energy
 - Energy preserving
- So, we must normalize the BRDF so that the reflectance integral is ≤ 1

The Importance of Being Normalized (2)

- Clearly this is not the case here:

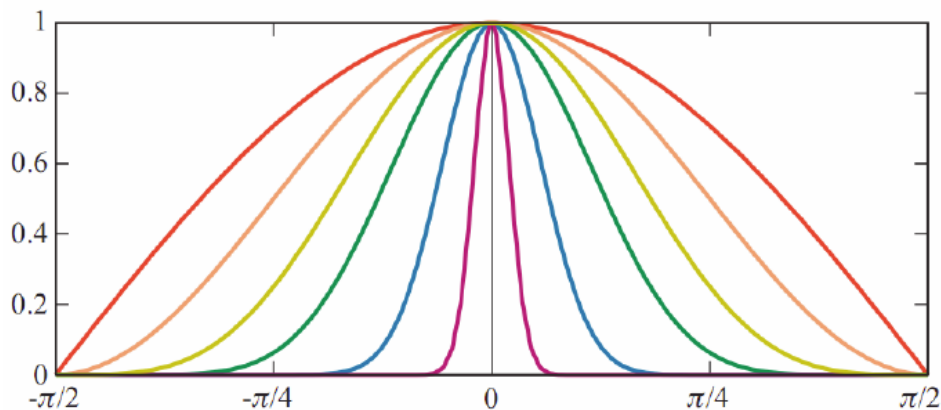


- Should be:

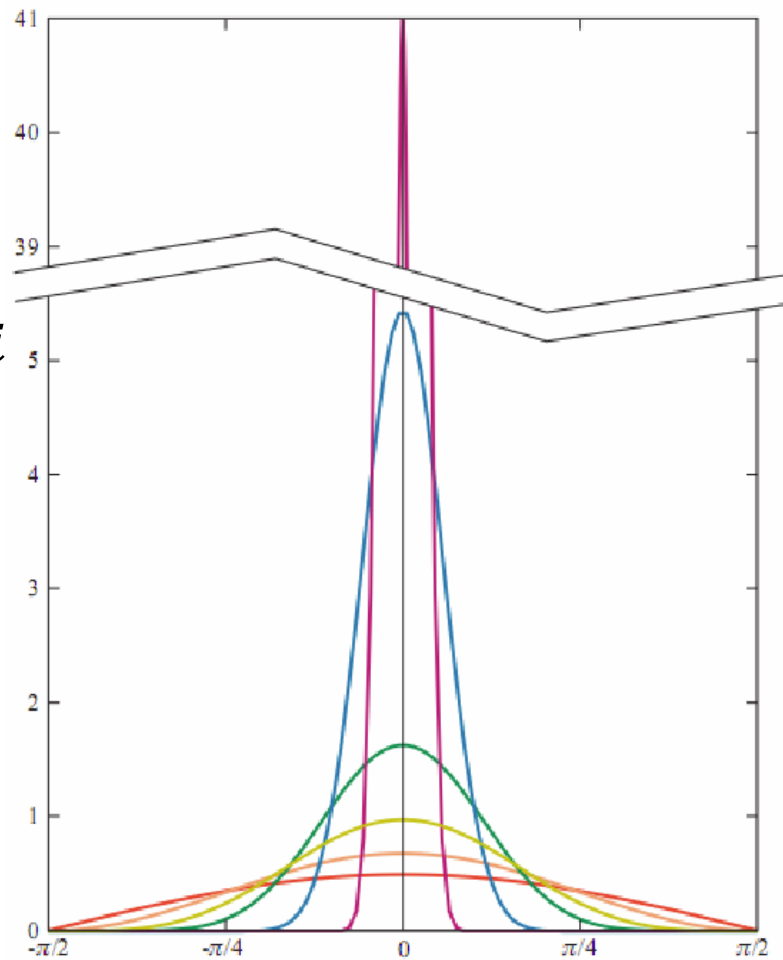


Normalized Blinn-Phong Model

$$L_{\text{specular}}(\vec{\mathbf{v}}) = \frac{n + 2}{4\pi(n + 2^{-n/2})} k_s (\vec{\mathbf{n}} \cdot \vec{\mathbf{h}})^n L_i$$



Unnormalized



Normalized
(area under the function
plot is constant)

Converting the Blinn-Phong Model to BRDF

- When the Blinn-Phong model is used within the reflectance equation, the entire integral has to be normalized (for maximum reflectivity)
- Also, $k_s + k_d$ must equal 1 or less (reflected vs transmitted and scattered back)

Blinn-Phong BRDF (1)

- By requiring the specular BRDF integral to be 1 for $k_s = 1$ and maximum flow direction $\vec{v} = \vec{n}$,

$$1 = \int_{\Omega} (\vec{n}\vec{h})^n \cos \theta \, d\omega \xrightarrow{\vec{h} = \frac{\vec{n} + \vec{l}}{|\vec{n} + \vec{l}|}} 1 = \int_{\Omega} \cos^n \frac{\theta}{2} \cos \theta \, d\omega$$

- the normalization factor f_n becomes:

$$f_n = \frac{(n + 2)(n + 4)}{8\pi(n + 2^{-\frac{n}{2}})}$$

Blinn-Phong BRDF (2)

- Therefore, the complete BRDF becomes:

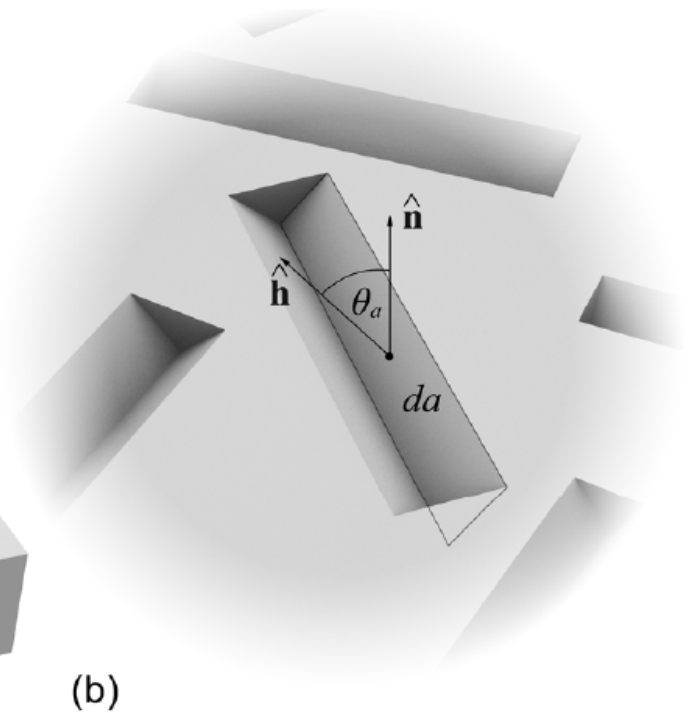
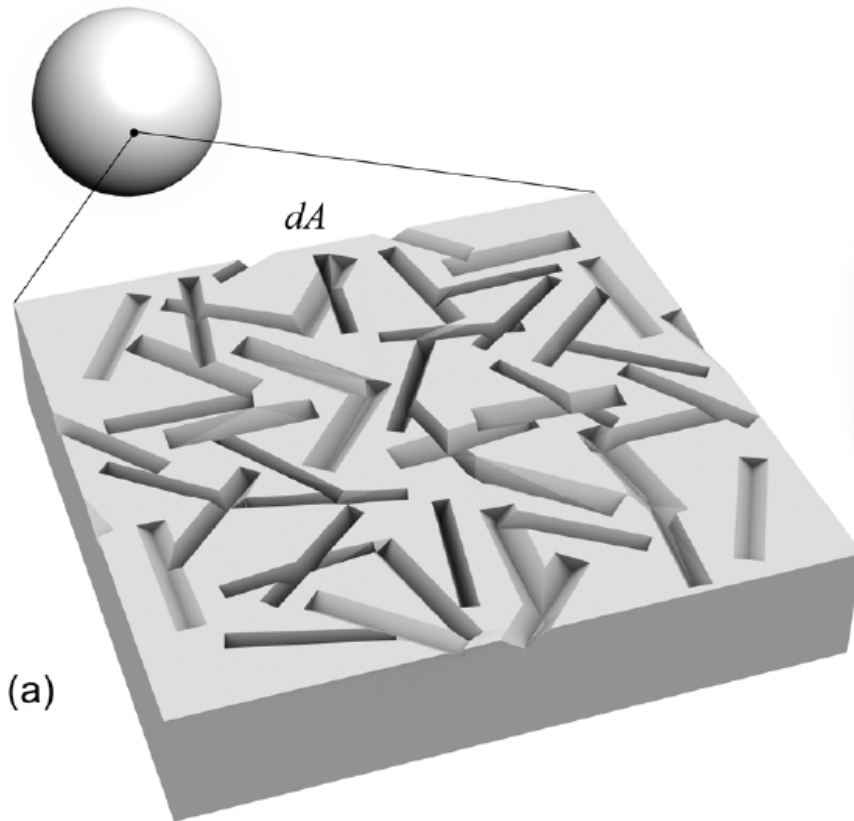
$$f_{Blinn}(\omega_o, \omega_i) = \frac{k_d}{\pi} + k_s \frac{(n+2)(n+4)}{8\pi(n+2\frac{n}{2})} \cos^n a$$

a being the angle between the halfway vector and the normal

- The Blinn-Phong BRDF:
 - Requires the user to guess the coefficients k_s, k_d
 - These coefficients vary with angle of incidence
 - Cannot correctly model the behaviour of metals (different reflectivity at normal and grazing angles)
 - Is based on a counter-intuitive notion of an exponent to set the “glossiness” of a surface

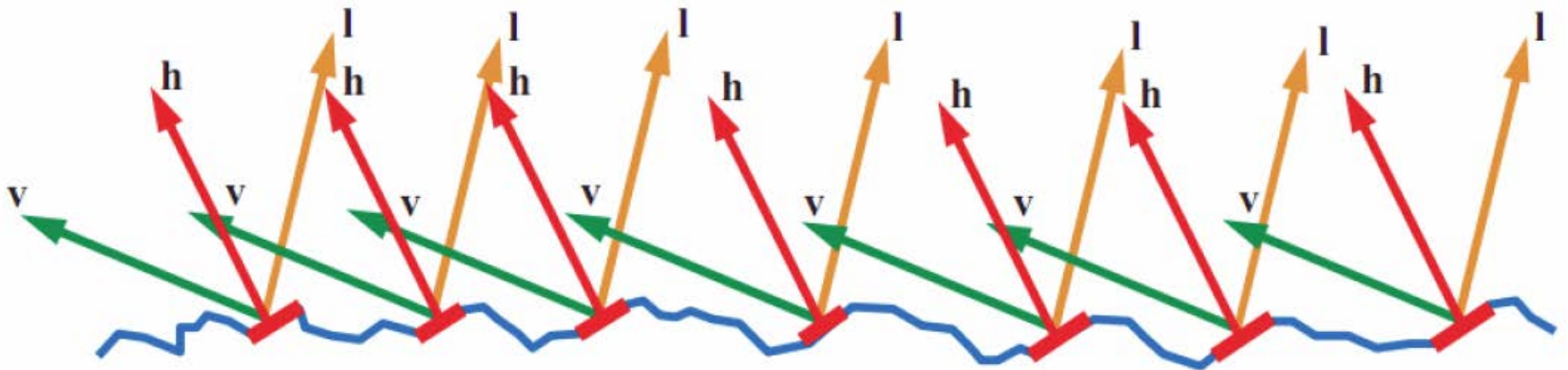
The Torrance – Sparrow Microfacet Model (1)

- Models arbitrarily “rough” surfaces
- Aggregation of V-shaped grooves



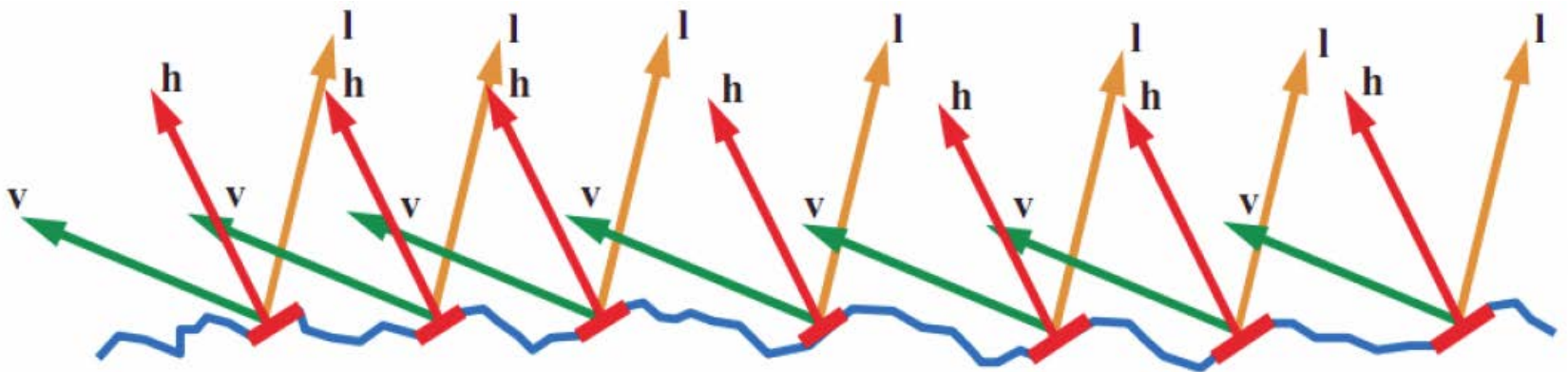
The Torrance – Sparrow Microfacet Model (2)

- Assumes surface is composed of many microfacets – individual optically flat surfaces too small to be seen
- Each microfacet reflects an incoming ray of light in only one outgoing direction (ideal reflector)
- Only those microfacets which happen to have their surface normal \mathbf{m} oriented exactly halfway between \mathbf{l} and \mathbf{v} (i.e. \mathbf{h}) will reflect visible light



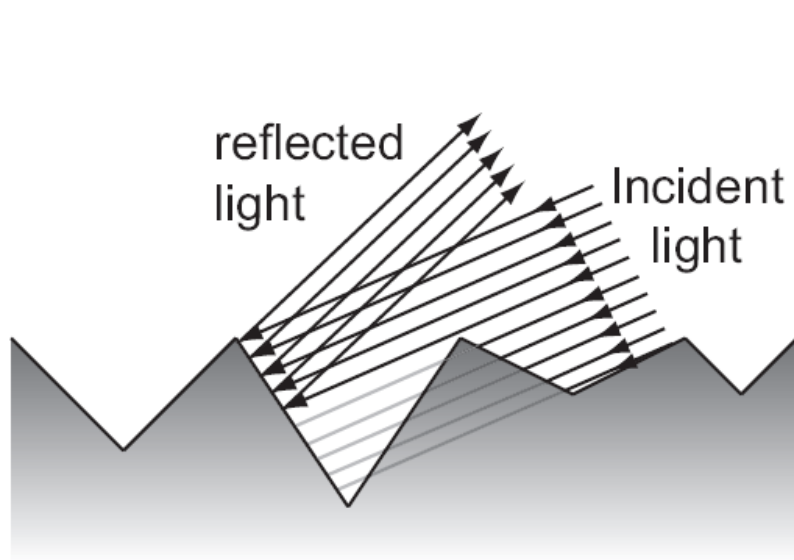
The Torrance – Sparrow Microfacet Model (3)

- Perfect mirror sides:
 - On/off contribution of micro-facets
 - Specular component **proportional to fraction of facets facing in the \mathbf{h} direction**

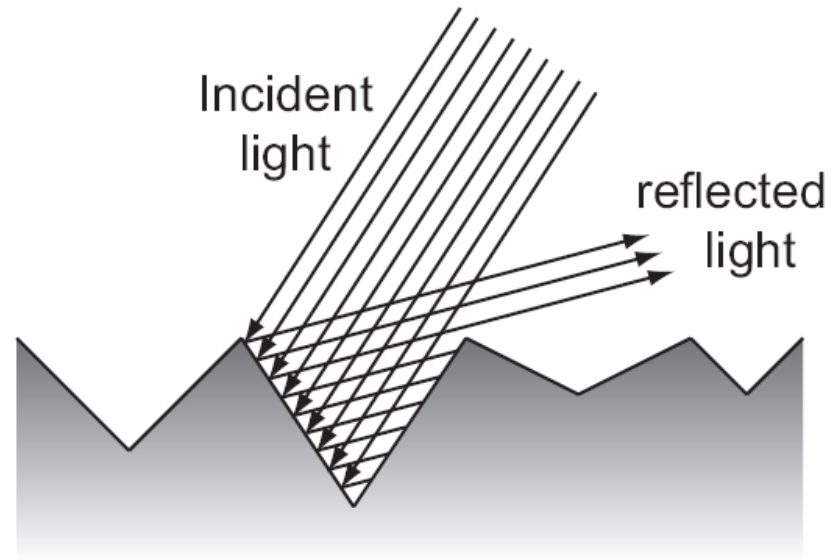


Shadowing and Masking

- Not all microfacets with $\mathbf{m} = \mathbf{h}$ will contribute
- Some will be blocked by other microfacets from either 1 (*shadowing*) or \mathbf{v}



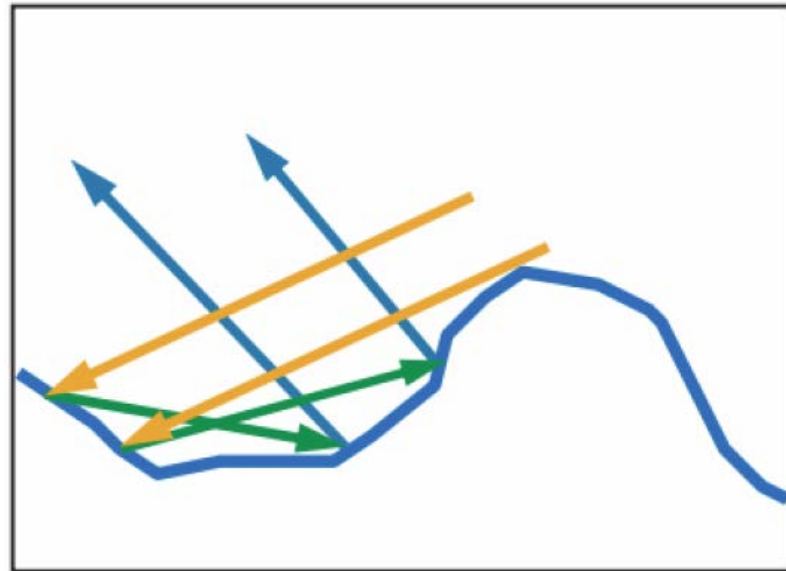
masking (shadow)



interception

Inter-reflections

- In reality, blocked light continues to bounce; some will eventually contribute to the BRDF
- Microfacet BRDFs ignore this – blocked light is lost (see Oren-Nayar model for inter-reflection contribution)



The Cook – Torrance Model (1)

- Uses the Torrance-Sparrow surface model
- Accounts for self-shadowing / masking light attenuation
- Accounts for directional reflectivity changes (Fresnel term)

The Cook – Torrance Model (2)

$$f_s = \frac{1}{\pi} \frac{DGF}{(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

- **D**istribution, **G**eometry, **F**resnel
- Saturates color to light source color with full brightness at grazing angles
- In the original model, maximum reflectivity if not attenuated (similar to Lambert diffuse scattering)

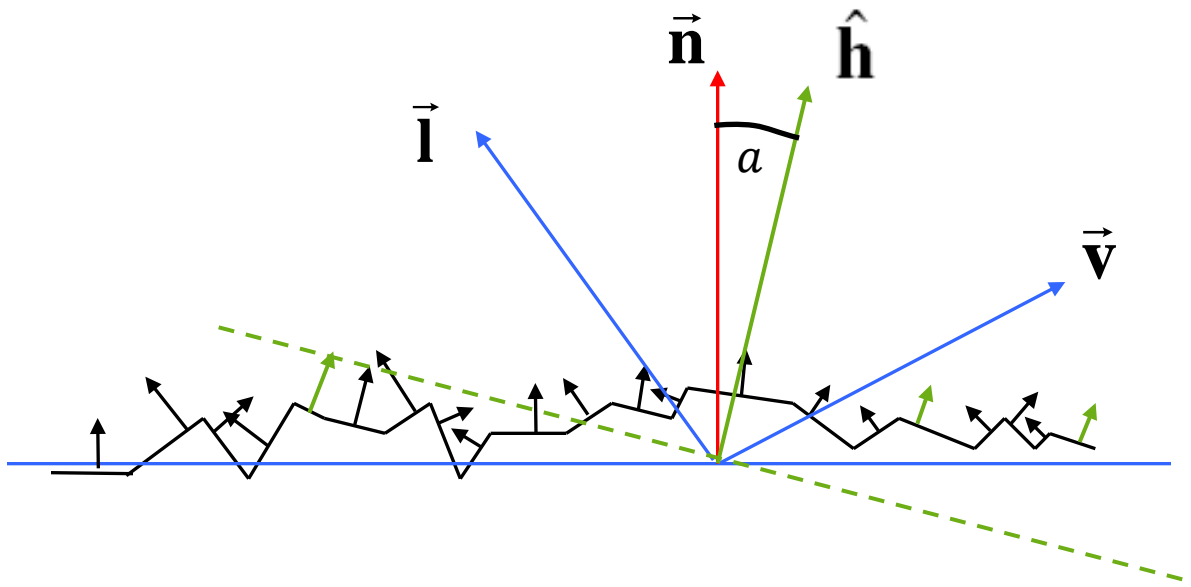
Cook – Torrance – Distribution Term (1)

- Represents the micro-facet density oriented along a direction **\mathbf{h}**
- It is a **normalized term**: expresses the fraction (probability) of facets turned towards **\mathbf{h}**
- Any distribution function can be used!
- Some reasonable ones though:
 - Gauss
 - Beckmann
 - Normalized Blinn-Phong

$$f_s = \frac{1}{\pi} \frac{DGF}{(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

Cook – Torrance – Distribution Term (2)

- An example:



$$D(a) = 3/21$$

Beckmann Distribution:

- Typically used in BRDFs

$$D(a) = \frac{e^{-\frac{\tan^2 a}{m^2}}}{\pi m^2 \cos^4 a}, \quad a = \arccos(\mathbf{n} \cdot \mathbf{h})$$

- Physically-based: m represents the RMS slope of the micro-facets
 - $m \rightarrow 0$: polished materials (caution with near zero values)

Beckmann Distribution:

- Relatively expensive (not preferred for RT graphics)
- A faster alternative (no $\text{acos}()$, no $\text{tan}()$, just dot products):

$$D(a) = \frac{e^{-\frac{\tan^2 a}{m^2}}}{\pi m^2 \cos^4 a}$$

$$\tan^2 a = \frac{\sin^2 a}{\cos^2 a} = \frac{1 - \cos^2 a}{\cos^2 a}, \quad \cos a = \mathbf{n} \cdot \mathbf{h}$$

Beckmann distribution example

No other factor apart from the $\mathbf{n} \cdot \mathbf{l}$ and the Beckmann distribution of micro-facets:



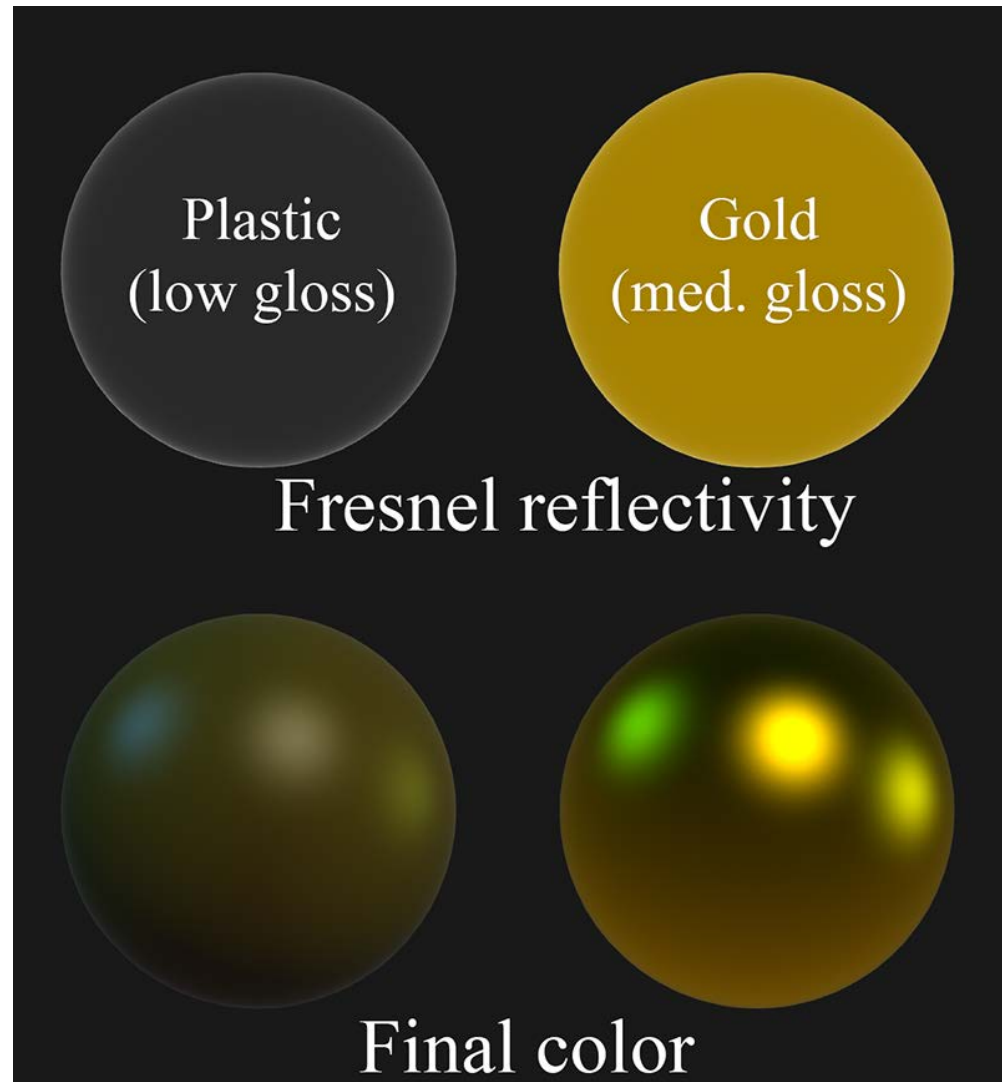
Cook – Torrance – Fresnel Term (1)

- Fraction of light reflected from optically flat surface given light direction \mathbf{l} and surface normal \mathbf{h}
- Value range: 0 to 1, spectral (RGB)
- Transmitted light = 1 - reflected

$$f_s = \frac{1}{\pi} \frac{DGF}{(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$

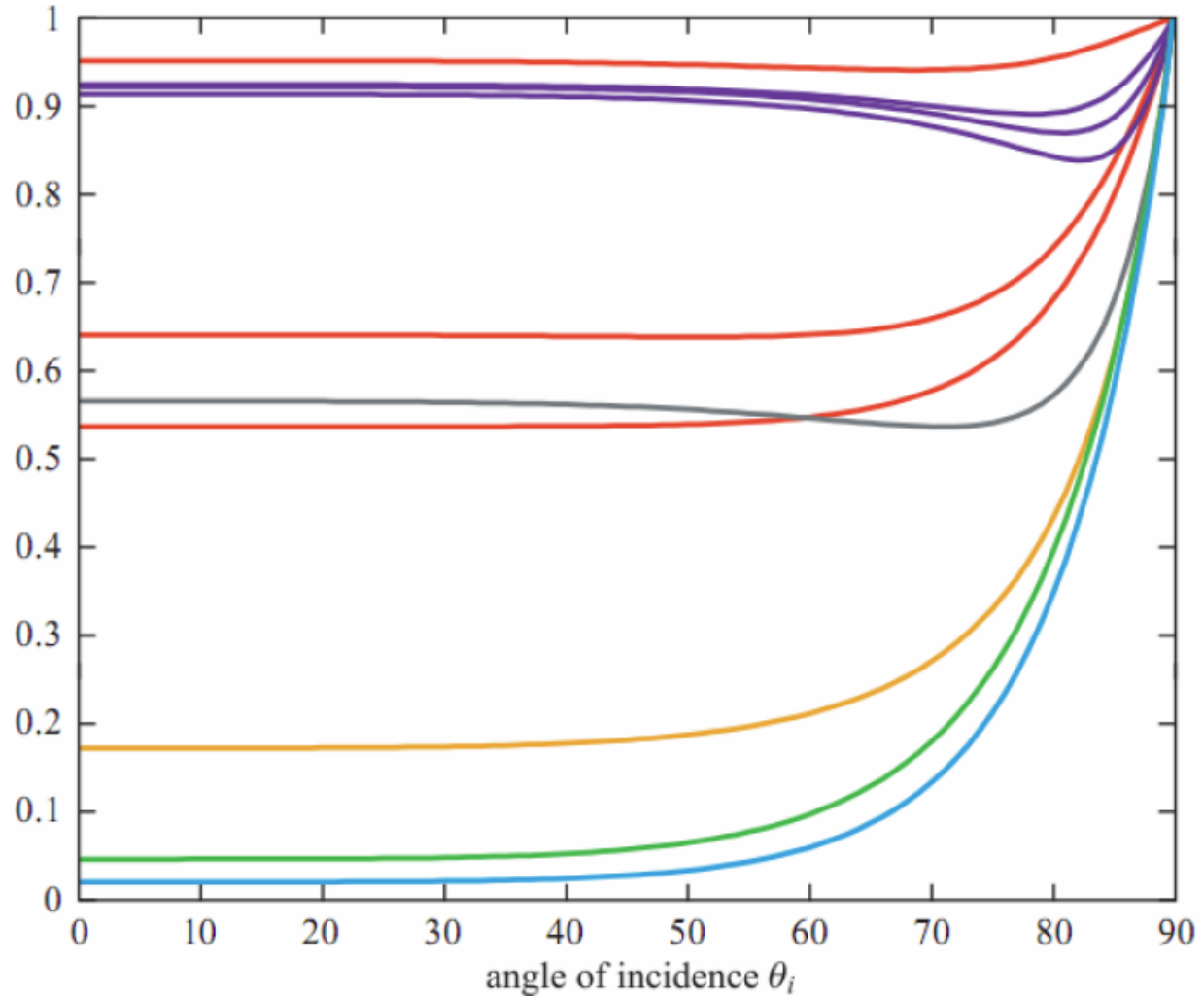
Cook – Torrance – Fresnel Term (2)

- Depends on refractive index and light angle
 - As angle increases, at first the reflectance barely changes, then for very glancing angles goes to 1 at all wavelengths



Cook – Torrance – Fresnel Term (3)

Fresnel Reflectance



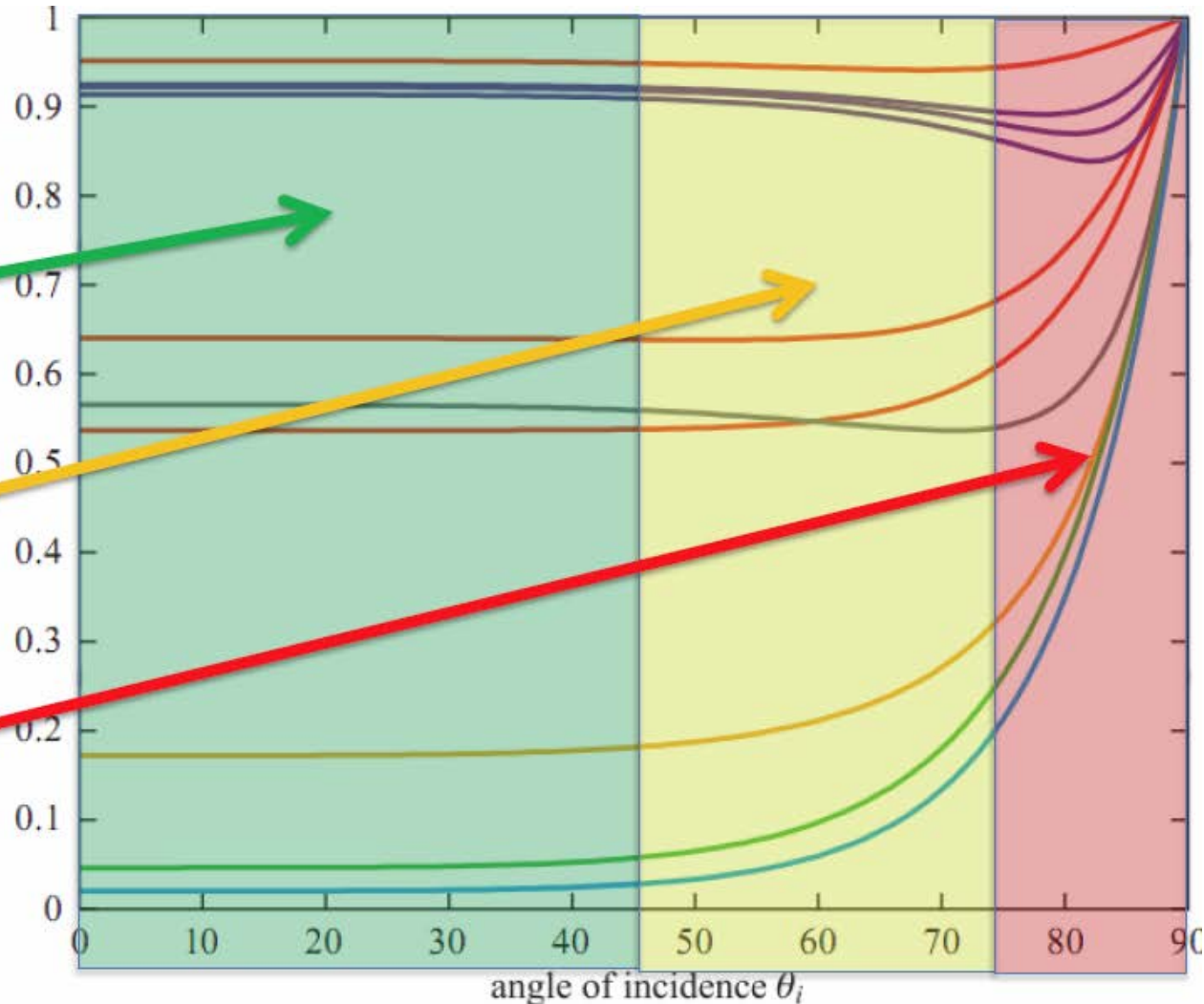
Cook – Torrance – Fresnel Term (4)

Fresnel Reflectance

barely changes

changes somewhat

goes rapidly to 1



Cook – Torrance – Fresnel Term (5)

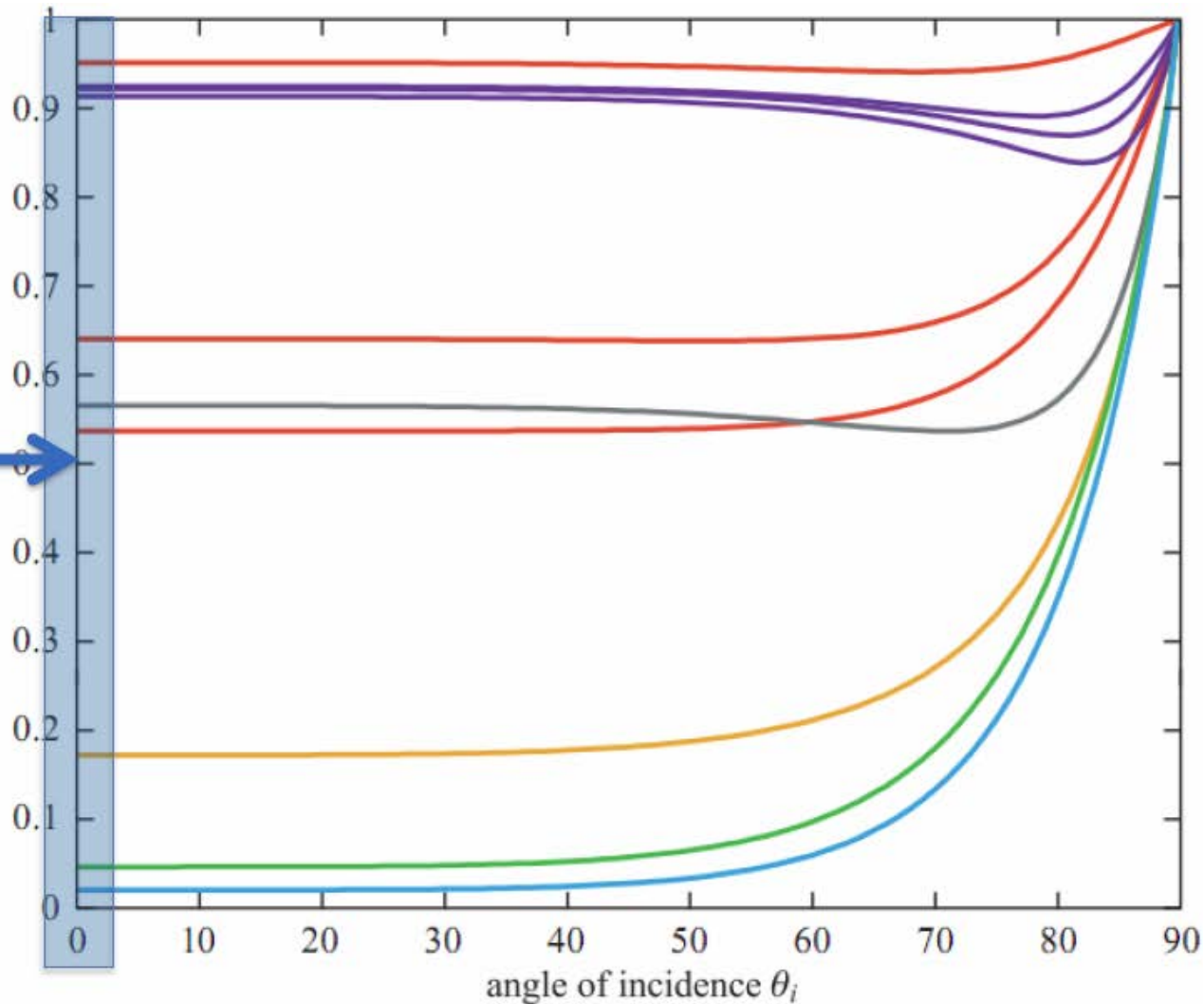
Fresnel Reflectance

The normal-incidence
Fresnel reflectance:

$$F(0^\circ)$$


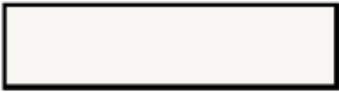



Is the surface's
characteristic
specular color :

$$c_{\text{spec}}$$








Normal-Incidence Fresnel for Metals

- No subsurface term; this is only source of color

Metal	$F(0^\circ)$ (Linear)	$F(0^\circ)$ (sRGB)	Color
Gold	1.00,0.71,0.29	1.00,0.86,0.57	
Silver	0.95,0.93,0.88	0.98,0.97,0.95	
Copper	0.95,0.64,0.54	0.98,0.82,0.76	
Iron	0.56,0.57,0.58	0.77,0.78,0.78	
Aluminum	0.91,0.92,0.92	0.96,0.96,0.97	

Normal-Incidence Fresnel for Non-Metals

- Subsurface term (diffuse) usually also present in addition to this Fresnel reflectance

Insulator	$F(0^\circ)$ (Linear)	$F(0^\circ)$ (sRGB)	Color
Water	0.02,0.02,0.02	0.15,0.15,0.15	
Plastic / Glass (Low)	0.03,0.03,0.03	0.21,0.21,0.21	
Plastic High	0.05,0.05,0.05	0.24,0.24,0.24	
Glass (High) / Ruby	0.08,0.08,0.08	0.31,0.31,0.31	
Diamond	0.17,0.17,0.17	0.45,0.45,0.45	

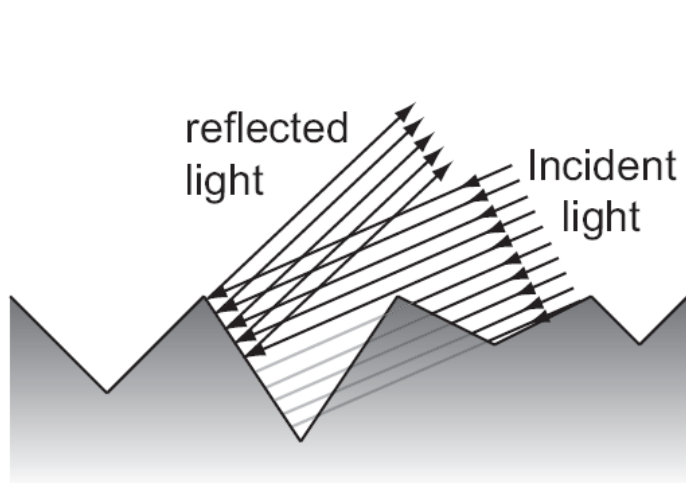
- Fresnel equations produce the reflectivity of polarized and unpolarized light
- A simple formula can approximate reasonably the reflectivity for unpolarized light (Schlick approximation formula):

$$F_{Schlick} = c_0 + (1 - c_0)(1 - \mathbf{l} \cdot \mathbf{h})^5$$

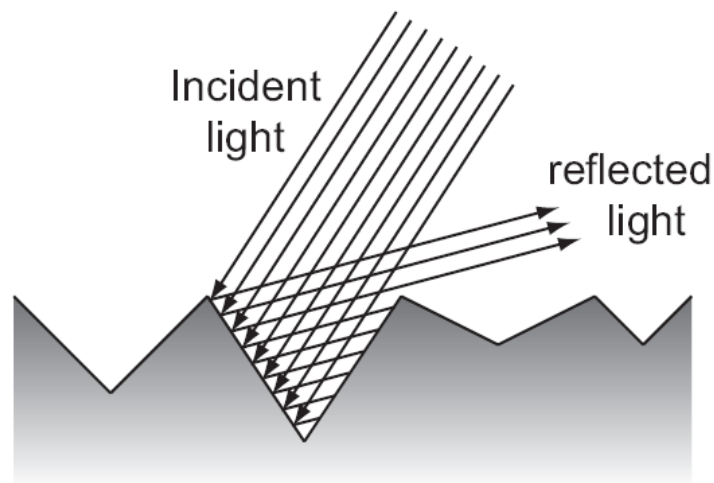
Cook – Torrance – Geometric Term (1)

- Accounts for the loss of light due to either light interception or shadowing

$$f_s = \frac{1}{\pi} \frac{DGF}{(\mathbf{n} \cdot \mathbf{l})(\mathbf{n} \cdot \mathbf{v})}$$



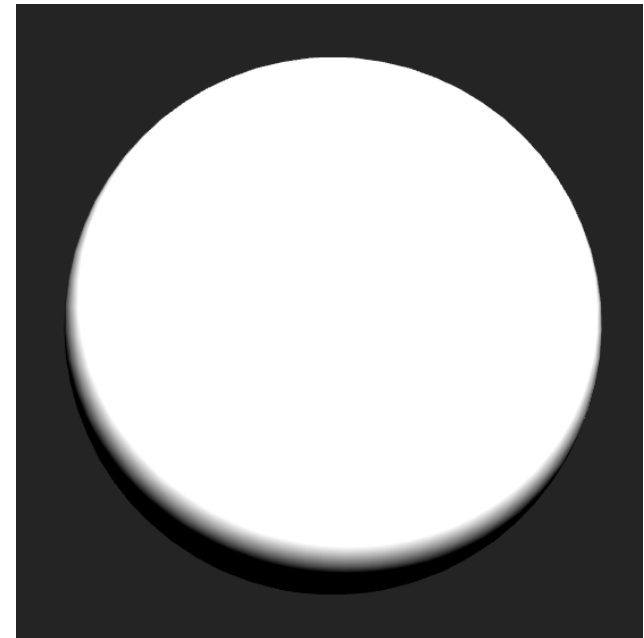
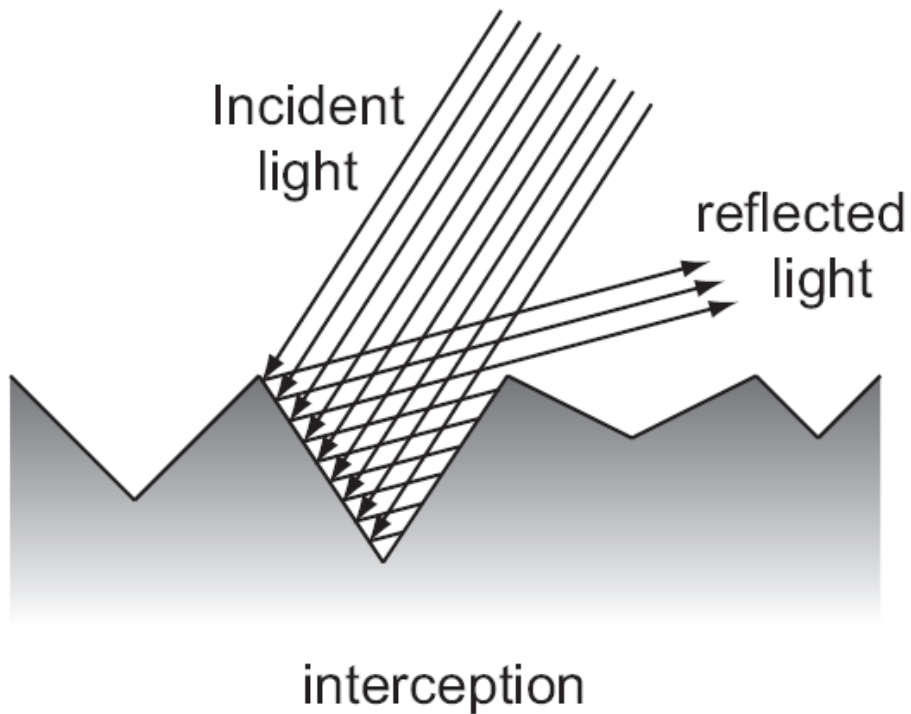
masking (shadow)



interception

Cook – Torrance – Geometric Term (2)

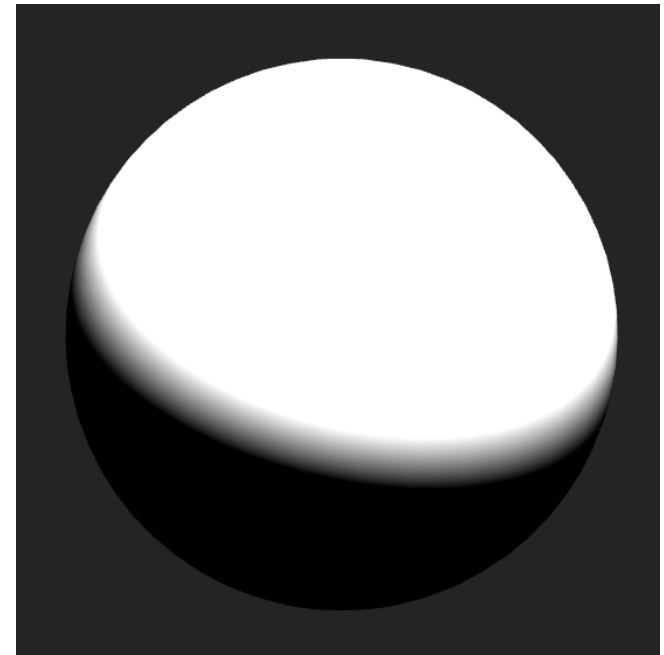
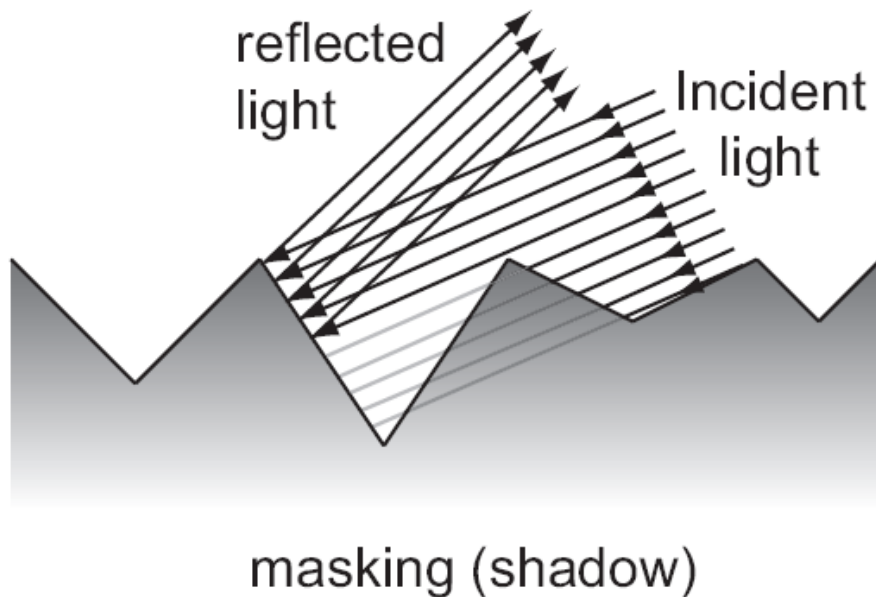
$$G_{\text{intercept}} = \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}})}{\hat{\mathbf{v}} \cdot \hat{\mathbf{h}}}$$



Cook – Torrance – Geometric Term (3)

- If we swap the roles of light and view direction:

$$G_{\text{shadow}} = \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{\hat{\mathbf{l}} \cdot \hat{\mathbf{h}}} = \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{\hat{\mathbf{v}} \cdot \hat{\mathbf{h}}}$$

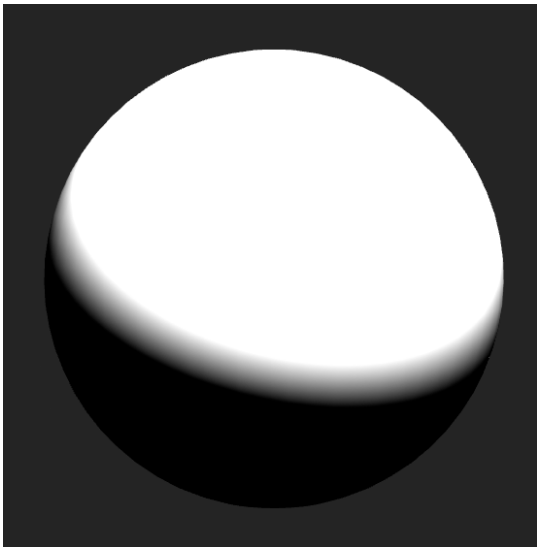


Cook – Torrance – Geometric Term (3)

- Combining both and keeping the most dominant (smallest) factor:

$$G = \min \left\{ 1, \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{v}})}{\hat{\mathbf{v}} \cdot \hat{\mathbf{h}}}, \frac{2(\hat{\mathbf{n}} \cdot \hat{\mathbf{h}})(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})}{\hat{\mathbf{v}} \cdot \hat{\mathbf{h}}} \right\}$$

Combined G



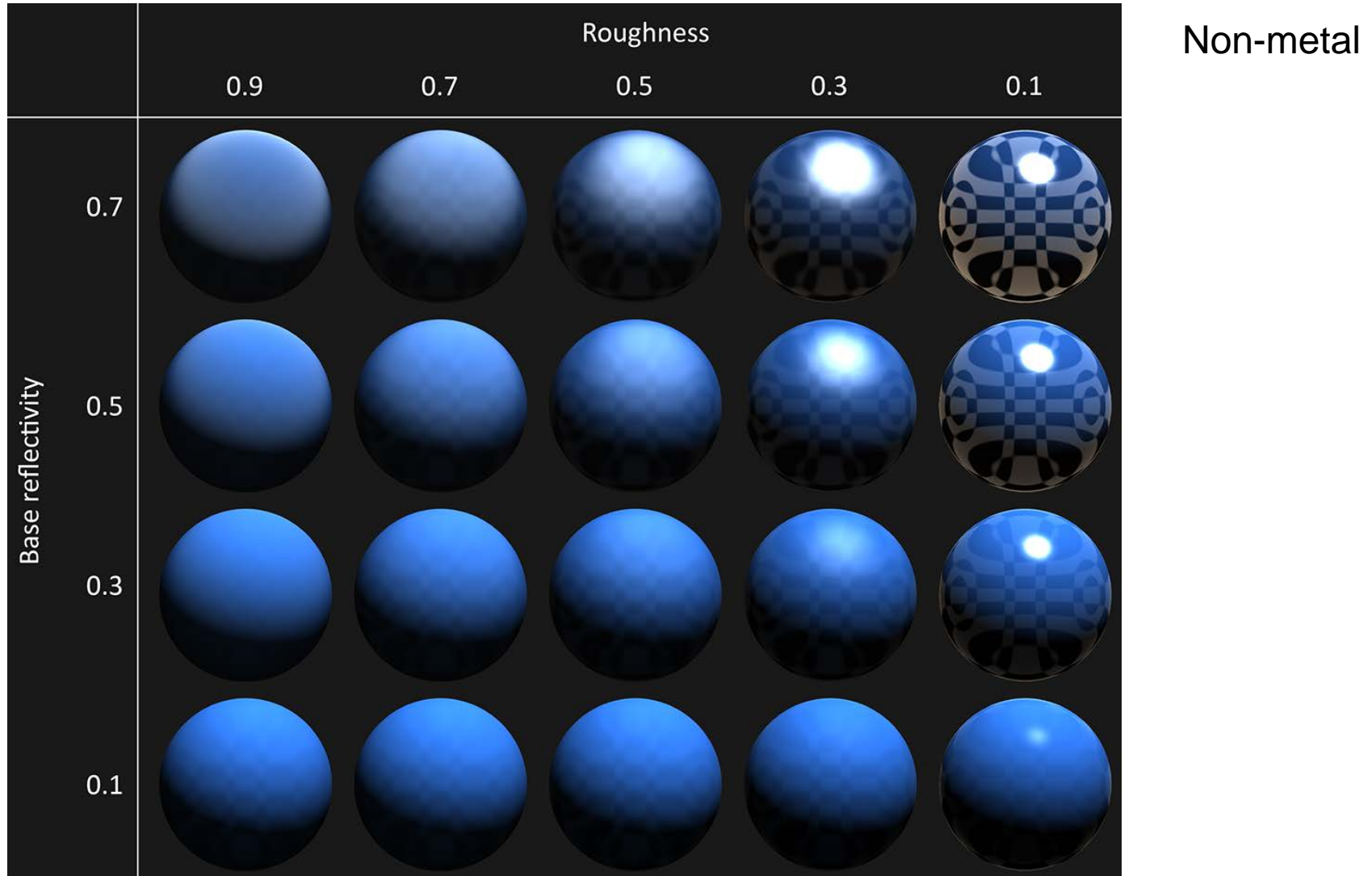
Without G



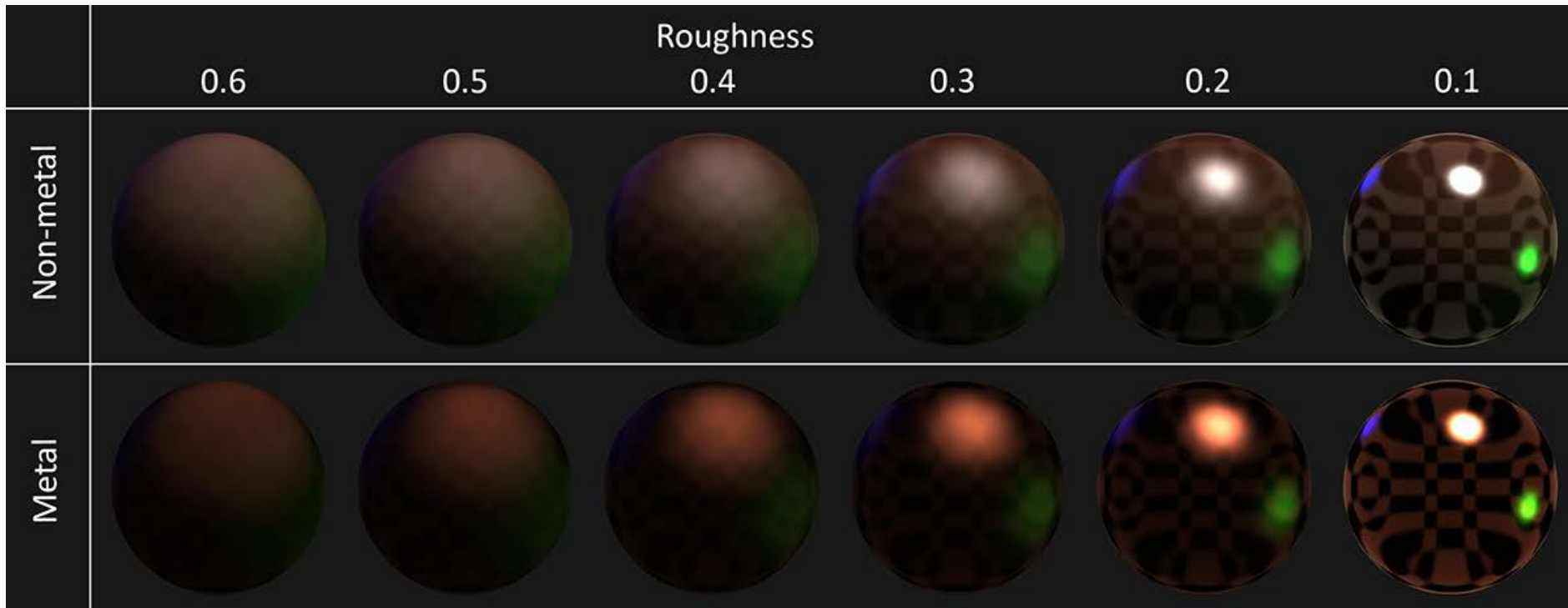
With G



Cook Torrance – Comparison (1)



Cook Torrance – Comparison (2)



Working with Shading Models

- Implementing a shading model (correctly) requires:
 - Understanding **what quantities we handle** within our rendering system (radiance, intensity, flux, irradiance etc)
 - A firm grasp of the **conversions** between the above
 - Understanding what the visible (fragment) **geometry** represents
 - Correct **definition of light sources** and their properties
 - Properly normalized (**energy-conserving**) models

- For area lights, we typically sample radiance from points on their surface
- For point lights, the above process has no meaning
- We rather rely on the intensity of the source for that:
 - Given the total flux of the source Φ :
 - $\Phi = \int_{sphere} I(\omega) d\omega \xrightarrow{uniform} \Phi = 4\pi I \Rightarrow I = \Phi/4\pi$
 - For point sources sufficiently far from a shaded point, $L_i \approx I/r^2$, r the distance to the light source

Plausible Shading – BRDFs

- Use energy conserving BRDFs
 - Make sure to balance the reflected vs transmitted (diffuse + sub. scattered + refracted) energy
 - Use the Fresnel terms for this
 - Take care of metallic surfaces (remember they do not transmit / scatter light from the surface substrate)
- See separate example shader (demo) for putting all these together

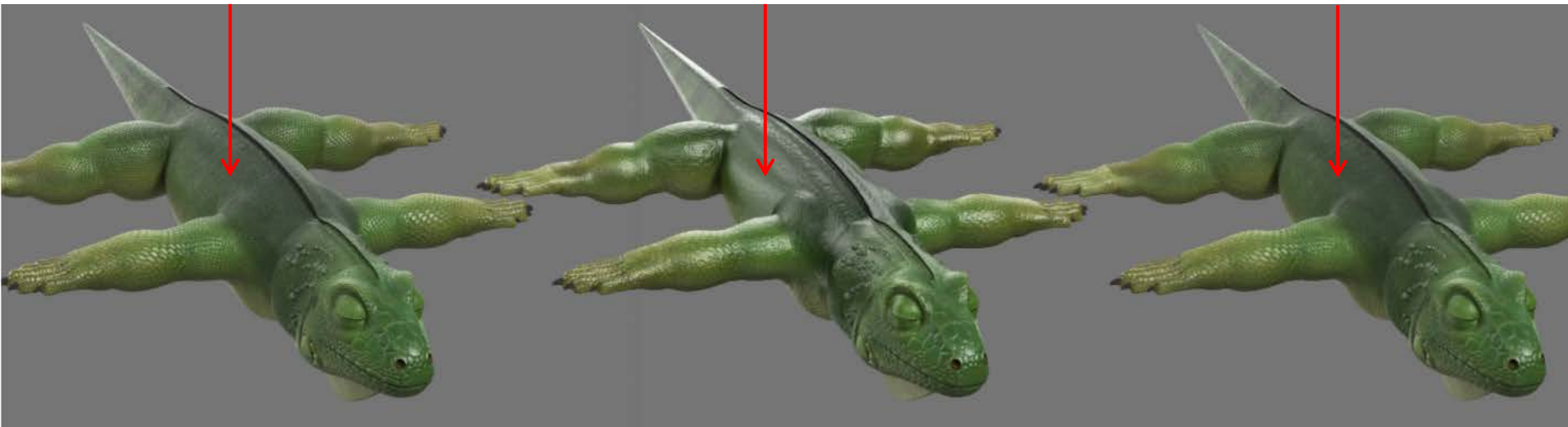
Plausible Shading – The Scale Effect

- Remember, micro-facet geometry behaves differently at different scales
 - You may need to introduce macro-scale irregularities into the BRDF roughness for distant objects

Ground truth

Incorrect shading: skin bumps
now at micro-facet level!

Corrected



Plausible Shading - Texturing

- Even the most perfect surfaces exhibit subtle details that vary spatially
- We provide texturing for important attributes of the surface to simulate reality (weathering, chaotic structure etc.)



- Georgios Papaioannou
- Sources:
 - [PBSM] SIGGRAPH 2010 Course: Physically Based Shading Models in Film and Game Production
 - T. Theoharis, G. Papaioannou, N. Platis, N. M. Patrikalakis, Graphics & Visualization: Principles and Algorithms, CRC Press