Athens University of Economics and Business
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Industrial Economics — TA Session 2

Problem Sets III & IV: Stackelberg in Prices & Collusion

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Today's Roadmap

PS III — Dynamic Oligopoly

- **A**: Stackelberg in prices, homogeneous
- **B**: Stackelberg in prices, differentiated

PS IV — Collusion

- **C**: Tacit collusion (3 firms + mergers)
- **D**: Asymmetric costs
- **E**: Cournot collusion

After the break

- Quick quiz (6 MC questions)
- ⋆ **F**: Stackelberg in quantities
- ⋆ **G**: Collusion, 4 firms

Recurring theme

Sequential games reward whoever can commit; **repeated games** reward whoever can punish.

Exercise A — Problem Statement

Exercise A (PS III, Problem 2)

Sequential price-setting duopoly. Demand: $D(p) = 1 - p$. Lowest-price firm supplies the whole market; ties are shared.

(i) Same marginal cost $C(q_i) = c\,q_i$ for both firms, $0 < c < 1$. Firm 1 chooses p_1 first, firm 2 observes and chooses p_2.

(ii) Now $c_1 < c_2$. Find equilibrium prices when $c_2 > p_1^m$ and when $c_2 < p_1^m$, where p_1^m is firm 1's monopoly price.

A (i) — Set Up: What Does Backward Induction Tell Us?

Two stages, perfect information after stage 1:

- **Stage 2:** firm 2 observes p_1 and picks p_2.
- **Stage 1:** firm 1 picks p_1 anticipating firm 2's reaction.

★ Your Turn

Before doing any algebra: in the simultaneous Bertrand game with homogeneous products and symmetric costs, what was the unique Nash equilibrium?

Does giving one firm the *move-first* advantage change anything?

A (i) — Stage 2: Firm 2's Best Response

Given p_1, firm 2's profit is

$$\pi_2(p_2 \mid p_1) = \begin{cases} (p_2 - c)(1 - p_2) & \text{if } p_2 < p_1 \\ \frac{1}{2}(p_2 - c)(1 - p_2) & \text{if } p_2 = p_1 \\ 0 & \text{if } p_2 > p_1 \end{cases}$$

- If $p_1 > c$: undercut by ε and capture the market $\Rightarrow p_2 = p_1 - \varepsilon$.
- If $p_1 = c$: firm 2 is indifferent between matching or staying out — profit is zero either way.
- If $p_1 < c$: firm 2 prefers no sale $(p_2 \geq c)$ to a loss.

A (i) — Stage 1: Firm 1's Move and Equilibrium

Firm 1 anticipates firm 2's reaction:

- $p_1 > c \;\Rightarrow$ firm 2 undercuts $\Rightarrow$ firm 1 sells nothing $\Rightarrow \pi_1 = 0$.
- $p_1 = c \;\Rightarrow$ both firms earn $\pi = 0$ either way.
- $p_1 < c \;\Rightarrow$ losses.

Equilibrium

$$p_1^* = p_2^* = c, \quad \pi_1^* = \pi_2^* = 0.$$

The **Bertrand paradox persists** under sequential price setting. Moving first confers no advantage when the good is homogeneous and costs are symmetric.

A (ii) — Asymmetric Costs: Two Regimes

Now $c_1 < c_2$. Firm 1's monopoly price (if rivals are out of the picture) is $p_1^m = (1 + c_1)/2$.

Large asymmetry: $c_2 > p_1^m$

Firm 2 cannot profitably undercut p_1^m (would price below its own MC). Firm 1 prices as an unconstrained monopolist.

$$p_1^* = p_1^m = \frac{1+c_1}{2}, \quad p_2^* \geq c_2$$

Small asymmetry: $c_2 < p_1^m$

Firm 1 *limit-prices*: just below c_2 to keep firm 2 out.

$$p_1^* \approx c_2 - \varepsilon, \quad p_2^* = c_2$$

Firm 1 captures the whole market; firm 2 is out.

★ Your Turn

Why doesn't firm 1 just charge its own monopoly price p_1^m in the *small* asymmetry case?

A (ii) — Firm 1's Equilibrium Profit

Small asymmetry ($c_2 < p_1^m$):

$$\pi_1^* = (c_2 - c_1)(1 - c_2)$$

This is the **limit-pricing profit**: the cost advantage $(c_2 - c_1)$ scaled by the volume at the rival's marginal cost.

Large asymmetry ($c_2 \geq p_1^m$):

$$\pi_1^* = \pi_1^M = \frac{(1 - c_1)^2}{4}$$

Firm 1 ignores firm 2 entirely — standard monopoly result.

△ Watch Out

The Stackelberg structure does *not* add anything here: the same outcome is obtained in simultaneous Bertrand with asymmetric costs. Once again, the leader's commitment is wasted on a homogeneous-product market.

Exercise B (PS III, Problem 3)

Two Bertrand competitors with differentiated goods. Common MC $= 0$. Each firm i faces

$$q_i(p_i, p_j) = 10 - p_i + g\, p_j, \qquad 0 < g < 1.$$

(i) Simultaneous choice of prices: find the equilibrium price and profit of each firm.

(ii) Sequential choice: firm 1 chooses p_1 first. Find the equilibrium and compare with (i). Who wants to move first?

B (i) — Simultaneous Bertrand: Best Responses

Firm i's profit: $\pi_i = p_i(10 - p_i + g\,p_j)$. FOC: $10 - 2p_i + g\,p_j = 0$.

Best response function

$$p_i^{BR}(p_j) = \frac{10 + g\,p_j}{2}$$

Slope $= g/2 > 0 \Rightarrow$ **prices are strategic complements**.

Symmetric Nash: $p_1^* = p_2^* = p^*$, so $p^* = (10 + g\,p^*)/2$, giving

$$p^* = \frac{10}{2-g}, \quad q^* = 10 - p^* + g\,p^* = \frac{10}{2-g}, \quad \pi^* = p^*\,q^* = \frac{100}{(2-g)^2}.$$

B (ii) — Sequential: Stage 2 (Follower)

Firm 2 observes p_1. Its problem is identical to the simultaneous case, treating p_1 as given:

$$\max_{p_2}\ p_2\,(10 - p_2 + g\,p_1)\ \Rightarrow\ p_2^{BR}(p_1) = \frac{10 + g\,p_1}{2} = 5 + \tfrac{g}{2}\,p_1.$$

Slope is still $g/2 > 0$: if the leader sets a high price, the follower also sets a high price (but *not* as high — intercept is 5).

B (ii) — Stage 1: Leader's Problem

Firm 1 substitutes firm 2's BR into its demand:

$$q_1 = 10 - p_1 + g\left(5 + \tfrac{g}{2}\,p_1\right) = 10 + 5g - p_1\left(1 - \tfrac{g^2}{2}\right).$$

Profit (MC $= 0$): $\pi_1 = p_1\left[(10 + 5g) - p_1(1 - g^2/2)\right]$. FOC:

$$(10 + 5g) - 2p_1\left(1 - \tfrac{g^2}{2}\right) = 0 \;\Rightarrow\; p_1^S = \frac{10 + 5g}{2 - g^2} = \frac{5(2 + g)}{2 - g^2}.$$

Then $p_2^S = 5 + \tfrac{g}{2}\,p_1^S = \dfrac{5(4 + 2g - g^2)}{2(2 - g^2)}$.

B (ii) — Equilibrium Quantities and Profits

Substituting back:

$$q_1^S = \frac{5(2+g)}{2}, \qquad q_2^S = p_2^S = \frac{5(4+2g-g^2)}{2(2-g^2)}.$$

Equilibrium profits (MC = 0)

$$\pi_1^S = p_1^S\, q_1^S = \frac{25(2+g)^2}{2(2-g^2)}, \qquad \pi_2^S = (p_2^S)^2 = \frac{25(4+2g-g^2)^2}{4(2-g^2)^2}.$$

Numerical check with $g = 1$:
$p_1^S = 15,\ p_2^S = 12.5,\ q_1^S = 7.5,\ q_2^S = 12.5,\ \pi_1^S = 112.5,\ \pi_2^S \approx 156.25.$

B (ii) — Who Wants to Move First?

Compare with the simultaneous game at $g = 1$: $p^* = 10$, $\pi^* = 100$ each.

	Leader	Follower
Price	15	12.5
Quantity	7.5	12.5
Profit	112.5	156.25
vs. simultaneous	$\uparrow$	$\uparrow\uparrow$

Second-mover advantage

Both firms gain from Stackelberg vs. simultaneous — prices are higher because the leader **commits to a high price**. But the follower undercuts *slightly*, gaining more market share at still-high prices. **The follower's profit exceeds the leader's.**

B — The Big Picture: Strategic Substitutes vs. Complements

Classic Stackelberg

Strategic **substitutes**.

Leader raises q; follower lowers q in response.

$\Rightarrow$ **First-mover advantage**: commit to large quantity.

Stackelberg in Prices

Strategic **complements**.

Leader raises p; follower also raises p.

$\Rightarrow$ **Second-mover advantage**: undercut a known high price.

The sign of the slope of the best-response function is the only thing that matters for the leader/follower ranking. Memorise it: **slope $<$ 0 favours leader; slope $>$ 0 favours follower**.

Exercise C — Problem Statement

Exercise C (PS IV, Problem 1)

Three firms ($i = 1, 2, 3$) compete in prices in each period $t \to \infty$. Common MC $= 6$. Market demand: $P = 20 - Q$. Discount factor δ.

(i) Static Nash vs. collusive prices and profits.

(ii) Condition for tacit collusion in equilibrium.

(iii) Firms 1 & 2 merge, MC still 6 — does answer change?

(iv) Same merger but merged firm has MC $= 0$, collusion price set as if MC $= 6$ — easier or harder than (iii)?

C (i) — Static Nash and Collusion Benchmarks

Static Nash (Bertrand, 3 symmetric firms):

$$p^N = c = 6, \qquad \pi_i^N = 0.$$

Collusion at the monopoly price: $\max_p (p-6)(20-p) \Rightarrow p^m = 13,\ Q^m = 7,$

$$\pi^m = 7 \times (13-6) = 49, \qquad \pi_i^c = \frac{\pi^m}{n} = \frac{49}{3} \approx 16.33.$$

Period payoffs (firm i, trigger strategy)

Collusion: $\pi^c = 49/3$. Deviation (undercut to $13-\varepsilon$): $\pi^d \approx 49$. Punishment (revert to Nash forever): $\pi^p = 0$.

C (ii) — Critical Discount Factor (3 Firms)

Trigger strategy — the *no-deviation condition*:

$$\frac{\pi^c}{1-\delta} \;\geq\; \pi^d + \frac{\delta\,\pi^p}{1-\delta} \;\Leftrightarrow\; \delta \;\geq\; \frac{\pi^d - \pi^c}{\pi^d - \pi^p}.$$

Plugging in $\pi^c = 49/3,\ \pi^d = 49,\ \pi^p = 0$:

$$\delta \;\geq\; \frac{49 - 49/3}{49 - 0} = \frac{98/3}{49} = \frac{2}{3}.$$

Result

With three firms, tacit collusion is sustainable iff $\delta \geq 2/3$.

C (iii) — After Symmetric Merger (Firms 1 & 2)

Now $n = 2$ symmetric firms (merged firm + firm 3), MC = 6. Collusion price unchanged: $p^m = 13$, $\pi^m = 49$.

- Collusion: $\pi^c = 49/2 = 24.5$.
- Deviation: $\pi^d \approx 49$ (still capture the whole market).
- Punishment: $\pi^p = 0$ (Bertrand with 2 symmetric firms).

$$\delta \geq \frac{49 - 24.5}{49 - 0} = \frac{1}{2}.$$

Comparison & the general *n*-firm result

From $\delta \geq 2/3$ (3 firms) to $\delta \geq 1/2$ (2 firms): the merger makes collusion **easier** to sustain. In general, with n symmetric firms the condition is $\delta \geq 1 - \frac{1}{n}$ — the collusive pie is split among fewer firms, so each keeps a bigger slice and has less incentive to undercut.

C (iv) — Asymmetric Merger: Setup

Firms 1 & 2 merge with $\text{MC}_M = 0$; firm 3 stays with $\text{MC}_O = 6$. Collusion price is fixed at $p^m = 13$ (monopoly with $\text{MC} = 6$), market shared 50/50.

- **Merged firm (M)**: margin at $p = 13$ is $13 - 0 = 13$. Selling $Q/2 = 3.5$, $\pi_M^c = 45.5$.
- **Outsider (O)**: margin $13 - 6 = 7$, selling 3.5, $\pi_O^c = 24.5$.

Deviation payoffs (undercut to $13 - \varepsilon$, get $Q = 7$):

- $\pi_M^d \approx 13 \times 7 = 91$.
- $\pi_O^d \approx 7 \times 7 = 49$.

C (iv) — Punishment Payoffs Under Bertrand with Asymmetric MC

Reverting to static Nash with $\text{MC}_M = 0 < \text{MC}_O = 6$:

- M's monopoly price $= (20+0)/2 = 10 > 6 = \text{MC}_O \Rightarrow$ **small asymmetry**.
- Equilibrium: $p_M \approx 6 - \varepsilon$, M captures market. $\pi^p_M = 6 \times 14 = 84$. $\pi^p_O = 0$.

△ Watch Out

Notice the problem: under static Nash, the merged firm earns $\pi^p_M = 84$ — **more than its collusion payoff of** 45.5. Punishment is not really a punishment for M.

C (iv) — Why Collusion Is Impossible

Firm M's no-deviation condition:

$$\frac{45.5}{1-\delta} \;\geq\; 91 + \frac{\delta \cdot 84}{1-\delta}.$$

Multiply through by $(1-\delta)$:

$$45.5 \;\geq\; 91 - 7\delta \;\Leftrightarrow\; 7\delta \geq 45.5 \;\Leftrightarrow\; \delta \geq 6.5.$$

Since $\delta \leq 1$, the constraint cannot be satisfied.

Bottom line

Asymmetric mergers — where the merger gives a cost advantage — make collusion **harder, and in this case impossible**. The low-cost firm has both a bigger deviation payoff *and* a more comfortable static-Nash payoff than the collusion payoff.

Exercise D — Problem Statement

Exercise D (PS IV, Problem 2)

Four firms in prices, $t \to \infty$. $c_1 = 4$, $c_2 = c_3 = c_4 = 5$. Demand: $P = 20 - Q$. Collusion at the monopoly price for $\text{MC} = 4$.

(i) Firm 1's profit in a Nash period, a deviation period, and a punishment period.

(ii) Condition for tacit collusion in equilibrium.

D (i) — Firm 1's Three Period Payoffs

Static Nash (no collusion). Limit-pricing à la asymmetric Bertrand: firm 1 prices just below $c_{-1} = 5$. Market $Q = 15$.

$$\pi_1^p = (5 - 4) \times 15 = 15.$$

Collusion period. Monopoly price for MC $= 4$: $p^m = (20 + 4)/2 = 12$, $Q^m = 8$, $\pi^m = 8 \times (12 - 4) = 64$. Shared equally across 4 firms:

$$\pi_1^c = 12 \times 2 - 4 \times 2 = 16.$$

Deviation period. Undercut to $12 - \varepsilon$, capture all 8 units:

$$\pi_1^d = (12 - 4) \times 8 = 64.$$

D (ii) — Critical Discount Factor for Firm 1

Firm 1's no-deviation condition (the binding constraint, since firm 1 has the largest cost advantage and the biggest static-Nash payoff):

$$\frac{\pi_1^c}{1-\delta} \;\geq\; \pi_1^d + \frac{\delta\,\pi_1^p}{1-\delta},$$

i.e. $16 \geq 64(1-\delta) + 15\delta = 64 - 49\delta$, so

$$\delta \;\geq\; \frac{48}{49} \approx 0.98.$$

Comparison with the high-cost firms

For any $i \in \{2,3,4\}$: $\pi_i^c = 14$, $\pi_i^d = 56$, $\pi_i^p = 0$. Their condition gives $\delta \geq 3/4$.

Firm 1's condition ($\delta \geq 48/49$) is much tighter — **cost-asymmetric collusion is hard to sustain because the low-cost firm has the largest incentive to deviate**.

D — Why Asymmetry Makes Collusion Fragile

Restating the general formula:

$$\delta_i^* = \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p}.$$

For firm 1, two effects work against collusion simultaneously:

- $\pi_1^d - \pi_1^c$ is **large**: undercutting wins the full monopoly profit.
- $\pi_1^d - \pi_1^p$ is **small**: under punishment, firm 1 still earns the limit-pricing rent (15 of the 64).
- Ratio close to 1 $\Rightarrow$ critical δ close to 1.

△ Watch Out

Real-world implication: cartels of identical firms (steel, cement) are easier to sustain than cartels with one efficient maverick. Antitrust authorities look for the maverick.

Exercise E — Problem Statement

Exercise E (PS IV, Problem 3)

Two identical Cournot firms, demand $p(Q) = 1 - Q$, $\text{MC} = c$ with $0 < c < 1$. Infinite horizon, discount factor δ.

(i) Find the equilibrium output and profit per firm under (tacit) collusion.

(ii) Condition for tacit collusion to arise in equilibrium.

E (i) — Collusive Output (Joint Monopoly)

Treating the cartel as a single monopolist:

$$\max_Q (1 - Q - c)Q \;\Rightarrow\; Q^m = \frac{1-c}{2}, \quad p^m = \frac{1+c}{2}, \quad \pi^m = \frac{(1-c)^2}{4}.$$

Each firm produces half:

$$q_i^c = \frac{Q^m}{2} = \frac{1-c}{4}, \qquad \pi_i^c = \frac{\pi^m}{2} = \frac{(1-c)^2}{8}.$$

Static Cournot benchmark

For comparison: $q_i^N = (1-c)/3$, $\pi_i^N = (1-c)^2/9$.
Collusion produces less and earns more.

E (ii) — The Deviation Payoff (Cournot)

Take the other firm's collusive output as given: $q_j = (1-c)/4$. My best response is

$$q_i^d = \frac{1-c-q_j}{2} = \frac{1-c-(1-c)/4}{2} = \frac{3(1-c)}{8}.$$

Then $Q = q_i^d + q_j = 5(1-c)/8$, and

$$\pi_i^d \; = \; (1-Q-c)\, q_i^d \; = \; \frac{3(1-c)}{8} \cdot \frac{3(1-c)}{8} \; = \; \frac{9(1-c)^2}{64}.$$

Note: $\pi_i^d/\pi_i^c = (9/64)/(1/8) = 72/64 = 9/8$. The deviation gain is only 12.5% above the collusive payoff — much smaller than the Bertrand case.

E (ii) — Critical Discount Factor

Punishment payoff = static Cournot Nash: $\pi_i^p = (1-c)^2/9$.
No-deviation condition:

$$\frac{\pi_i^c}{1-\delta} \geq \pi_i^d + \frac{\delta\,\pi_i^p}{1-\delta}.$$

Divide through by $(1-c)^2$:

$$\frac{1/8}{1-\delta} \;\geq\; \frac{9}{64} + \frac{\delta/9}{1-\delta}.$$

Multiply by $576(1-\delta)$ to clear fractions: $72 \geq 81 - 17\delta$, so

$$\boxed{\delta \;\geq\; \frac{9}{17} \approx 0.529.}$$

E — Comparison with Bertrand Collusion

	Bertrand (C, $n=2$)	Cournot (E, $n=2$)
Deviation payoff π^d	π^m	$\frac{9}{64}(1-c)^2$
Punishment payoff π^p	0	$\frac{(1-c)^2}{9}$
Critical δ	$1/2$	$9/17$

Why Cournot collusion is harder ($9/17 > 1/2$)

- **Less to gain from deviating**: a quantity deviation cannot capture the whole market.
- **Less to lose from punishment**: Cournot Nash profits are positive ($(1-c)^2/9 > 0$); Bertrand Nash profits are zero.
- The second effect dominates: although the deviation gain is smaller, the punishment threat is also weaker.

Break

Back in 20 minutes

Quick Quiz — Q1 of 6

Stackelberg in Quantities

In Stackelberg *quantity* competition, compared to the simultaneous Cournot game, the leader produces ______ and earns ______.

(a) More; more

(b) The same; the same

(c) More; less — the follower ends up capturing the leader's profit

(d) Less; less

Answer: (a)

Quick Quiz — Q2 of 6

Stackelberg in Prices, Homogeneous Goods

In Stackelberg *price* competition with *homogeneous* goods and symmetric costs (Exercise A), moving first...

(a) Gives a strict advantage: the leader captures the whole market

(b) Makes no difference: the Bertrand paradox still delivers $p = c$

(c) Hurts the leader: the follower always undercuts to steal the market

(d) Doubles both firms' profits compared to simultaneous Bertrand

Answer: (b)

Quick Quiz — Q3 of 6

Stackelberg in Prices, Differentiated Goods

In Exercise B, the follower earns *more* than the leader. The core reason is:

(a) The follower has a lower marginal cost

(b) The follower produces more and benefits from economies of scale

(c) The leader sets the monopoly price, leaving all demand to the follower

(d) Prices are strategic complements: the follower undercuts a known high price

Answer: (d)

Quick Quiz — Q4 of 6

Collusion and Number of Firms

As the number of symmetric Bertrand firms n increases, the minimum discount factor δ^* required for tacit collusion...

(a) Increases — each firm's collusive share shrinks but the deviation gain stays the same

(b) Decreases — more firms means a stronger collective punishment

(c) Stays at $\delta^* = \frac{1}{2}$ regardless of n

(d) First increases, then decreases once $n > 5$

Answer: (a)

Quick Quiz — Q5 of 6

Asymmetric Costs (Exercise D)

In Exercise D, which firm has the *tightest* no-deviation condition (highest δ^*)?

(a) All firms face the same condition by symmetry

(b) Firms 2, 3, or 4 — they earn less under collusion so gain more from deviating

(c) The question cannot be answered without knowing δ

(d) Firm 1 — it has the largest deviation gain *and* the most comfortable punishment payoff

Answer: (d)

Quick Quiz — Q6 of 6

Cournot vs. Bertrand Collusion

Comparing Exercise C (Bertrand, $n = 2$, $\delta^* = 1/2$) and Exercise E (Cournot, $n = 2$, $\delta^* = 9/17$), Cournot collusion is *harder* to sustain because...

(a) A Cournot deviation gains more than a Bertrand deviation

(b) Cournot punishment leaves firms with *positive* Nash profits, weakening the threat

(c) Cournot firms produce more, so the collusive price is lower

(d) There are always more firms in Cournot markets

Answer: (b)

Exercise F — Your Turn

★ Your Turn

Two firms, homogeneous good, $\mathrm{MC} = 0$. Inverse demand: $P = 100 - Q, \quad Q = q_1 + q_2$.
Firm 1 is the quantity leader.

(i) Find the Stackelberg equilibrium: output, price, and profit of each firm.

(ii) Would you prefer to be the leader or the follower? Explain briefly.

Exercise F — Solution

Stage 2 (follower). Firm 2 maximises $(100 - q_1 - q_2)q_2$:

$$q_2^{BR}(q_1) = \frac{100 - q_1}{2} = 50 - \tfrac{q_1}{2}.$$

Stage 1 (leader). Substituting:

$$\pi_1 = \left(100 - q_1 - 50 + \tfrac{q_1}{2}\right) q_1 = \left(50 - \tfrac{q_1}{2}\right) q_1. \quad \Rightarrow \quad q_1^* = 50.$$

$$q_2^* = 25, \quad Q^* = 75, \quad P^* = 25, \quad \pi_1^* = 1250, \quad \pi_2^* = 625.$$

Leader or follower? (Part ii)

Leader earns 1,250 $>$ 625 $=$ follower $\Rightarrow$ **first-mover advantage**. Contrast with Exercises A and B: in *price* Stackelberg the follower benefits. The difference is the sign of the best-response slope: negative (quantities) favours the leader; positive (prices) favours the follower.

Exercise G — Your Turn

★ **Your Turn**

Four identical Bertrand firms. Homogeneous good, $\mathrm{MC} = 0$. Inverse demand: $P = 20 - Q$. Infinite horizon, discount factor δ, grim trigger strategies.

(i) Find each firm's per-period profit under: (a) static Nash, (b) collusion at the monopoly price (shared equally), (c) one-shot deviation.

(ii) Explain how grim trigger strategies work. Derive the condition on δ for tacit collusion to arise in equilibrium.

Exercise G — Solution (i): The Three Payoffs

(a) Static Nash (Bertrand):

$$p^N = 0, \qquad \pi_i^N = 0.$$

(b) Collusion (joint monopoly, equal shares):

$$\max_P (20 - P)P \;\Rightarrow\; P^m = 10,\; Q^m = 10,\; \pi^m = 100. \qquad \pi_i^c = \frac{100}{4} = 25.$$

(c) One-shot deviation (undercut to $P^m - \varepsilon$, capture whole market):

$$\pi_i^d \approx 100.$$

Summary

$\pi_i^N = 0, \quad \pi_i^c = 25, \quad \pi_i^d \approx 100.$

Exercise G — Solution (ii): Grim Trigger & δ^*

Grim trigger: each firm prices at P^m as long as no deviation has ever occurred; if *any* firm deviates, all revert to $p = 0$ (Nash) *forever*.

No-deviation condition (collusion payoff $\geq$ deviation payoff):

$$\underbrace{\frac{25}{1-\delta}}_{\text{cooperate forever}} \geq \underbrace{100}_{\text{deviate today}} + \underbrace{\frac{\delta \cdot 0}{1-\delta}}_{\text{Nash forever}}.$$

$$25 \geq 100(1-\delta) \Leftrightarrow \delta \geq \frac{3}{4}.$$

Connecting to the general formula

$$\delta^* = \frac{\pi^d - \pi^c}{\pi^d - \pi^N} = \frac{100 - 25}{100 - 0} = \frac{3}{4}.$$

Consistent with $\delta^* = 1 - \frac{1}{n} = 1 - \frac{1}{4} = \frac{3}{4}$. **Four firms make collusion hard.**

Wrap-Up — Key Takeaways

Stackelberg (PS III + F):

- **Quantities** (F): best-response slopes are negative ⇒ **first-mover advantage**.
- **Prices, homogeneous** (A): Bertrand paradox persists; sequential moves change nothing.
- **Prices, differentiated** (B): best-response slopes are positive ⇒ **second-mover advantage**.

Collusion (PS IV + G):

- $\delta^* = (\pi^d - \pi^c)/(\pi^d - \pi^p)$. Three payoffs, one formula.
- **More firms** ⇒ higher δ^* (harder). $\delta^* = 1 - 1/n$ in the symmetric Bertrand case.
- **Cost asymmetry**: the low-cost firm is the binding constraint; can make collusion impossible.
- **Cournot** harder than **Bertrand**: Nash punishment leaves positive profits.

Thank you for your attention

Questions?

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