Industrial Economics

TA Session 2 — Problem Handout

Athens University of Economics and Business
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Session Overview

This handout contains the problems we will work through in TA Session 2, 21 May 2026 (11:15–14:00).

- **Problem Set III (Dynamic Oligopoly):** Problems 2 and 3 (Stackelberg in prices, homogeneous and differentiated products).
- **Problem Set IV (Collusion):** Problems 1, 2, and 3 (tacit collusion in prices and in quantities).

Problem Set III — Dynamic Oligopoly

Exercise A (PS III, Problem 2) — Stackelberg in Prices, Homogeneous Product

Consider a duopoly market in which the firms choose their prices *sequentially*. The market demand function is $D(p) = 1 - p$. If a firm chooses a lower price than its competitor, then it supplies the whole market; if both firms choose the same price, they share the market equally.

(i) Assume the two firms have the **same** cost function $C(q_i) = c\,q_i$, where $i = 1, 2$ and $0 < c < 1$. Firm 1 chooses its price before firm 2 (and firm 2 observes p_1 before choosing p_2). Find the equilibrium prices.

(ii) Assume now that firms play the same game as in part (i) but the marginal cost of firm 1 is **lower** than the marginal cost of firm 2, $c_1 < c_2$. Find the equilibrium prices when $c_2 > p_1^m$ and when $c_2 < p_1^m$, where p_1^m is the price that firm 1 would charge if it were a monopolist.

Exercise B (PS III, Problem 3) — Stackelberg in Prices, Differentiated Products

Consider two firms competing à la Bertrand in differentiated goods at a common marginal cost of zero. Each firm i ($i = 1, 2$) faces the demand function

$$q_i(p_i, p_j) = 10 - p_i + g\,p_j, \qquad 0 < g < 1, \quad j \neq i.$$

(i) Assume the two firms choose their prices *simultaneously and separately*. Find the price and the profit of each firm in equilibrium.
(ii) Assume now that firm 1 chooses its price before firm 2 (and firm 2 observes p_1 before choosing p_2). Find the equilibrium price and profit of each firm and compare with part (i). Does a firm want to choose its price first or second?

Problem Set IV — Collusion

Exercise C (PS IV, Problem 1) — Tacit Collusion in Prices (Three Firms & Mergers)

In an oligopolistic market there are initially three firms (1, 2, and 3) that choose their prices *simultaneously and independently* in each period $t = 1, 2, \ldots, T$. The lowest-price firm supplies the whole market; if firms charge the same price, they share the market equally; a firm charging a strictly higher price than the others has zero demand. Marginal cost is 6 for each firm. The market demand for the product is $P = 20 - Q$. The discount factor is $\delta \in [0, 1]$. Assume $T \to \infty$.

(i) What are the price and the profits of each firm in a period *without* tacit collusion and in a period *with* tacit collusion (when firms charge the monopoly price)?
(ii) Under what condition does tacit collusion arise in equilibrium?
(iii) Assume now that firms 1 and 2 merge and that the merged firm's marginal cost remains equal to 6. Would your answer in part (ii) change? Yes or no, and why?
(iv) Assume instead that firms 1 and 2 merge and the merged firm's marginal cost is now equal to 0. Suppose that under tacit collusion firms charge the price a monopolist with marginal cost 6 would charge. Is it now *easier* or *harder* to sustain tacit collusion relative to part (iii)? Why?

Exercise D (PS IV, Problem 2) — Tacit Collusion in Prices with Asymmetric Costs

In an oligopolistic market there are four firms (1, 2, 3, and 4) that choose their prices *simultaneously and independently* in each period $t = 1, 2, \ldots, T$. The lowest-price firm supplies the whole market (with the usual tie-breaking rule). Marginal cost is $c_1 = 4$ for firm 1 and $c_i = 5$ for $i = 2, 3, 4$. Market demand is $P = 20 - Q$. Discount factor $\delta \in [0, 1]$, and $T \to \infty$. Assume that in a collusive period firms set the price that a monopolist with marginal cost 4 would set.

(i) What are firm 1's profits in a period *without* tacit collusion, in a period in which firm 1 *deviates* from tacit collusion, and in a period in which it is *punished* for its deviation?
(ii) Under what condition does tacit collusion arise in equilibrium?

Exercise E (PS IV, Problem 3) — Tacit Collusion in Quantities (Cournot)

Consider a market with two identical firms that compete in quantities and face the demand function $p(Q) = 1 - Q$, where Q is total quantity. Each firm i ($i = 1, 2$) faces a constant marginal cost c, with $0 < c < 1$. The horizon is infinite and firms discount future profits at rate $\delta \in [0, 1]$.

(i) Find the equilibrium output and profit of each firm if they (tacitly) collude.
(ii) Under what condition does tacit collusion arise in equilibrium?

Quick Reminders

Trigger strategy (infinite-horizon): firms play the collusive price p^c as long as no one has deviated; if any firm deviates, all firms revert to the static Nash price p^p forever.

Critical discount factor (general formula):

$$\delta \geq \frac{\pi^d - \pi^c}{\pi^d - \pi^p},$$

where π^c is the collusion period payoff, π^d is the one-shot deviation payoff, and π^p is the per-period punishment payoff. Tacit collusion is more likely when π^c is high, π^d is low, and the punishment π^p is severe.

In-Class Practice

Exercise F — Stackelberg in Quantities, Homogeneous Product

Consider a duopoly market in which two firms produce a homogeneous good with zero marginal costs. Market inverse demand is $P = 100 - Q$, where $Q = q_1 + q_2$. Firm 1 is the quantity *leader*: it chooses q_1 first, and firm 2 observes q_1 before choosing q_2.

(i) Find the Stackelberg equilibrium: output, price, and profit of each firm.
(ii) Would you prefer to be the leader or the follower? Explain briefly.

Exercise G — Tacit Collusion with Four Symmetric Firms

Consider a market with **four identical firms** competing in prices (*Bertrand*). The good is homogeneous, each firm has zero marginal cost, and market inverse demand is $P = 20 - Q$. The horizon is infinite and all firms share discount factor $\delta \in [0, 1]$.

(i) Find the per-period profit of each firm under: *(a)* the static Nash equilibrium, *(b)* a collusive agreement at the monopoly price shared equally, and *(c)* a

one-shot deviation from the collusive agreement.

(ii) Explain in your own words how *grim trigger strategies* work. Then derive the condition on δ under which tacit collusion arises in equilibrium.