Athens University of Economics and Business
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Industrial Economics — TA Session 1

Problem Sets I & II

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Academic Year 2025–2026

Exercise A (PS I, Problem 4)

A monopolist sells a good over two periods $t = 1, 2$. Demand: $q_t = 1 - p_t$. Marginal cost in $t = 1$: c, where $0 \leq c < 1$. Marginal cost in $t = 2$: $c - \lambda q_1$, where $0 < \lambda < 1$.

(i) Are demands and costs of the two periods dependent or independent?

(ii) Find the prices charged in each period (assume $\delta = 1$).

(iii) Compute the Lerner index and elasticity of demand in equilibrium for both periods. Discuss.

Q4 — Warm-Up: Static Monopoly

★ Your Turn

Before tackling the two-period problem, recall the static monopoly.

Demand: $q = 1 - p$ **MC:** c

Find the optimal price p^M, quantity q^M, and profit π^M.

$$\max_p \pi = (p - c)(1 - p) \Rightarrow \frac{d\pi}{dp} = 1 - 2p + c = 0 \Rightarrow p^M = \frac{1 + c}{2}$$

Static monopoly benchmark

$$p^M = \frac{1 + c}{2}, \quad q^M = \frac{1 - c}{2}, \quad \pi^M = \frac{(1 - c)^2}{4}$$

PS I, Q4 — Two-Period Monopoly with Learning-by-Doing

Setup

- Same linear demand both periods: $q_t = 1 - p_t$
- Period 1 marginal cost: $MC_1 = c$
- Period 2 marginal cost: $MC_2 = c - \lambda q_1, \quad \lambda \in (0, 1)$

Q4 — Are the Two Periods Linked?

★ Your Turn

Consider the model: $q_t = 1 - p_t, \;\; MC_1 = c, \;\; MC_2 = c - \lambda q_1$.

Are the two periods **independent** or **linked**?

Think about **(a) the demand side** and **(b) the cost side** separately.

Q4 — Structure of the Problem

Demands: unlinked

Period 1 demand depends only on p_1; period 2 demand depends only on p_2.

The two markets are **independent** on the demand side.

Costs: linked — learning-by-doing

$MC_2 = c - \lambda q_1$: higher output in period 1 reduces marginal cost in period 2.

The periods are **dependent** on the cost side.

Because costs are linked, the monopolist must solve a **joint** profit-maximisation problem — **not** two separate ones.

Q4 — Lifetime Profit Function & First-Order Conditions

Let π_t denote period-t profit and Π the present value of lifetime profits. Using $q_t = 1 - p_t$ and $\delta \to 1$:

$$\Pi = \underbrace{(p_1 - c)(1 - p_1)}_{\pi_1} + \underbrace{\delta}_{\to 1} \cdot \underbrace{\big(p_2 - c + \lambda(1 - p_1)\big)(1 - p_2)}_{\pi_2}$$

FOC with respect to p_1:

$$\frac{\partial \Pi}{\partial p_1} = \underbrace{(1 - 2p_1 + c)}_{\partial \pi_1 / \partial p_1} - \boldsymbol{\lambda(1 - p_2)} = 0 \qquad \leftarrow \text{cross-period: raising } p_1 \text{ reduces learning}$$

FOC with respect to p_2: $$\frac{\partial \Pi}{\partial p_2} = 1 - 2p_2 + c - \lambda(1 - p_1) = 0$$

Q4 — Solving the System: Equilibrium Prices

Rearranging the FOCs gives a linear system:

$$\begin{cases} 2p_1 - \lambda p_2 = 1 + c - \lambda & \text{(I)} \\ -\lambda p_1 + 2p_2 = 1 + c - \lambda & \text{(II)} \end{cases}$$

From (I): $p_1 = \frac{(1+c-\lambda)+\lambda p_2}{2}$. Substituting into (II) and simplifying yields $p_2^* = \frac{1+c-\lambda}{2-\lambda}$, and then $p_1^* = \frac{1+c-\lambda}{2-\lambda}$.

Equilibrium prices & quantities

$$p_1^* = p_2^* = \frac{1-\lambda+c}{2-\lambda}, \qquad q_1^* = q_2^* = \frac{1-c}{2-\lambda}$$

Since $\lambda \in (0,1)$: $p^* < p^M = \frac{1+c}{2}$ and $q^* > q^M = \frac{1-c}{2}$ — the cross-period cost term pushes p^* below the static monopoly price.

Q4 — Lerner Index: Computing L_1, L_2, η

Step 1: Equilibrium MC_2: $MC_2^* = c - \lambda q_1^* = \dfrac{2c - \lambda}{2 - \lambda}$

Step 2: Lerner indices and demand elasticity ($\eta \equiv -\frac{\partial q}{\partial p}\frac{p}{q} > 0$)

$$L_1 = \frac{p^* - c}{p^*} = \frac{(1-\lambda)(1-c)}{1-\lambda+c} \quad L_2 = \frac{p^* - MC_2^*}{p^*} = \frac{1-c}{1-\lambda+c}$$

$$\eta = \frac{1-\lambda+c}{1-c}$$

Q4 — Lerner Index: Interpretation

Recall: $L_1 = \dfrac{(1-\lambda)(1-c)}{1-\lambda+c}, \quad L_2 = \dfrac{1-c}{1-\lambda+c}, \quad \eta = \dfrac{1-\lambda+c}{1-c}$

Key finding

$L_2 = 1/\eta$ standard Lerner condition holds in period 2.

$L_1 < 1/\eta$ the firm **deliberately under-exploits market power in period 1** to sell more and reduce future costs.

Exercise B (PS I, Problem 7)

Purple Dream has a monopoly on purple LEDs. Markets A and B are geographically separated: $q_A = 1 - p_A$, $q_B = \frac{1}{2} - p_B$. Transport and production costs are **zero**.

(i) **Uniform price**: profit-maximising price and quantities sold.

(ii) **Price discrimination**: profit-maximising prices and quantities.

(iii) Compute CS and profit under each regime; compare and comment.

(iv) Does the result in (iii) hold generally? What if $q_B = \frac{1}{3} - p_B$?

Q7 — Think First

★ Your Turn

Before we solve: does price discrimination **increase** or **reduce** welfare?

Does your answer depend on anything?

PS I, Q7 — Third-Degree Price Discrimination

Setup (Purple Dream LEDs)

- Market A: $q_A = 1 - p_A$
- Market B: $q_B = \frac{1}{2} - p_B$
- $MC = 0$ throughout

Market B is **smaller and less willing to pay**.

Four parts

(i) Optimal uniform price
(ii) Optimal discriminating prices
(iii) Welfare comparison
(iv) Modified demand for B

Q7 — Part (i): Uniform Pricing

Case: both markets served ($p \le \frac{1}{2}$), so aggregate demand is
$Q(p) = (1 - p) + (\frac{1}{2} - p) = \frac{3}{2} - 2p$.

Monopolist's problem:

$$\max_p \ \pi(p) = p\left(\tfrac{3}{2} - 2p\right)$$

FOC: $\dfrac{d\pi}{dp} = \frac{3}{2} - 4p = 0 \ \Rightarrow \ p^* = \dfrac{3}{8}$

Optimal uniform outcome

$$p^* = \frac{3}{8}, \quad q_A^* = \frac{5}{8}, \quad q_B^* = \frac{1}{8}, \quad Q^* = \frac{3}{4}, \quad \pi^U = \frac{9}{32}$$

Corner check: serving A only at $p = \frac{1}{2}$ gives $\pi = \frac{1}{4} < \frac{9}{32} \Rightarrow$ optimal to serve **both** markets.

(Always verify the interior price lies in $[0, \frac{1}{2}]$.)

Q7 — Part (ii): Price Discrimination

The monopolist solves a separate problem in each market.

Market A:

$$\max_{p_A} \; p_A(1 - p_A)$$

FOC: $1 - 2p_A = 0$

Market A

$$p_A^* = \tfrac{1}{2}, \;\; q_A^* = \tfrac{1}{2}, \;\; \pi_A = \tfrac{1}{4}$$

Market B:

$$\max_{p_B} \; p_B\left(\tfrac{1}{2} - p_B\right)$$

FOC: $\frac{1}{2} - 2p_B = 0$

Market B

$$p_B^* = \tfrac{1}{4}, \;\; q_B^* = \tfrac{1}{4}, \;\; \pi_B = \tfrac{1}{16}$$

$$\pi^D = \pi_A + \pi_B = \tfrac{1}{4} + \tfrac{1}{16} = \tfrac{5}{16} > \tfrac{9}{32} = \pi^U$$

Q: *Which market gets the higher price, and why?*

→ Market A has **less elastic** demand ⇒ higher price.

Q7 — Part (iii): Welfare Comparison

	Uniform	Discrimination	Change
Consumer surplus	$\frac{13}{64}$	$\frac{5}{32}$	$\downarrow$
Profit	$\frac{9}{32}$	$\frac{5}{16}$	$\uparrow$
Total welfare	$\frac{31}{64}$	$\frac{15}{32}$	$\downarrow$

Interpretation

Both markets are served in both regimes $\Rightarrow$ total output is unchanged.

The welfare loss is **pure redistribution** from consumers to the firm — no deadweight loss is created or destroyed.

Q7 — Part (iv): General Result

Modified demand for B: $q_B = \frac{1}{3} - p_B$

Uniform pricing:

Monopolist's problem on A alone: $\max_p p(1-p)$, FOC: $p^* = \frac{1}{2}$.

Since $\frac{1}{2} > \frac{1}{3}$, market B is **excluded**.

Discrimination:

Market B: $\max_{p_B} p_B(\frac{1}{3} - p_B)$

FOC: $\frac{1}{3} - 2p_B = 0$

Market B

$p_B^* = \frac{1}{6}, \; q_B^* = \frac{1}{6}$

Q7 — Part (iv): Welfare Lesson

General lesson

Same markets served under both regimes: PD is **welfare-detrimental**.

PD opens a previously unserved market: PD is **welfare-enhancing**.

In this model, the welfare ranking is **unambiguous** in both cases.

Break

Resuming with oligopoly models

From Monopoly to Oligopoly

Monopoly recap:

- Single firm, no strategic interaction, sets $MR = MC$.
- Full market power: $L = 1/\eta$.

Oligopoly — strategic interdependence:

- Each firm's optimal decision depends on what rivals do.
- Two canonical models:
  - **Cournot** — quantities chosen simultaneously (*strategic substitutes*: $\partial BR_i/\partial q_j < 0$)
  - **Bertrand** — prices chosen simultaneously (*strategic complements*: $\partial BR_i/\partial p_j > 0$, both with homogeneous and with differentiated goods)

Monopoly $\longrightarrow$ Oligopoly $\longrightarrow$ Perfect competition

PS II, Q1 — Problem Statement

Exercise C (PS II, Problem 1)

Market demand: $p(Q) = 25 - Q$.

(i) Monopoly: cost $C(Q) = 4Q$. Find equilibrium Q, p, π.

(ii) Cournot duopoly: two firms, $C(q_i) = 4q_i$, choose quantities *simultaneously and separately*. Find each firm's output, the market price, and each firm's profit.

(iii) Compare (i) and (ii) in terms of total quantity, price, and consumer surplus.

PS II, Q1 — Monopoly Benchmark

Firm's profit: $\pi = (p - 4)\,Q$. Substitute $p = 25 - Q$:

$$\pi = (25 - Q - 4)\,Q = (21 - Q)\,Q$$

FOC:

$$\frac{\partial \pi}{\partial Q} = 21 - 2Q = 0 \;\Rightarrow\; Q^M = \frac{21}{2} = 10.5$$

Monopoly equilibrium

$Q^M = 10.5, \quad p^M = 25 - 10.5 = 14.5, \quad \pi^M = (14.5 - 4) \cdot 10.5 = 110.25, \quad CS^M = \frac{1}{2}(10.5)^2 \approx 55.1$

PS II, Q1 — Cournot: Best-Response Functions

Firm 1: $\pi_1 = (p - 4)\, q_1$. Substitute $p = 25 - q_1 - q_2$:

$$\pi_1 = (21 - q_1 - q_2)\, q_1 \quad \Rightarrow \quad \frac{\partial \pi_1}{\partial q_1} = 21 - 2q_1 - q_2 = 0 \quad \Rightarrow \quad BR_1 : \; q_1 = \frac{21 - q_2}{2}$$

Firm 2: $\pi_2 = (p - 4)\, q_2 = (21 - q_1 - q_2)\, q_2$

$$\frac{\partial \pi_2}{\partial q_2} = 21 - q_1 - 2q_2 = 0 \quad \Rightarrow \quad BR_2 : \; q_2 = \frac{21 - q_1}{2}$$

△ Watch Out

Derive BR_2 from Firm 2's own FOC — do not invoke cost symmetry to skip this step.

PS II, Q1 — Cournot: Equilibrium

Solving the system: Substitute BR_2 into BR_1:

$$q_1 = \frac{21 - \frac{21 - q_1}{2}}{2} \Rightarrow 4q_1 = 21 + q_1 \Rightarrow 3q_1 = 21 \Rightarrow q_1^* = 7$$

Back-substitute into BR_2: $q_2^* = \frac{21 - 7}{2} = \frac{14}{2} = 7$

Cournot equilibrium

$$q_1^* = q_2^* = 7, \quad Q^C = 14, \quad p^C = 25 - 14 = 11, \quad \pi_1^* = \pi_2^* = (11 - 4) \cdot 7 = 49$$

Q1 — Monopoly vs. Cournot: Comparison

	Q	p	CS	π (total)
Monopoly	10.5	14.5	55.1	110.3
Cournot	14	11	98	98
Direction of change	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$

Takeaway

Moving from monopoly to Cournot duopoly raises output, lowers price, benefits consumers, and increases total welfare — the first step along the competitive spectrum.

PS II, Q2 — Problem Statement

Exercise D (PS II, Problem 2)

Two firms, homogeneous good, choose quantities *simultaneously and separately*. Demand: $P = a - q_1 - q_2$. Costs: $C_1(q_1) = c_1 q_1$, $C_2(q_2) = c_2 q_2$, with $c_1 < c_2$ and $c_2 < \frac{a+c_1}{2}$.

(i) Find the Nash equilibrium. What are the market shares?

(ii) Find equilibrium profits, consumer surplus, and total welfare.

Q2 — Think First

★ Your Turn

Firm 1 has a **lower** marginal cost than Firm 2.

Which firm do you expect to **produce more**?

Which firm do you expect to **earn higher profit**?

PS II, Q2 — Profit Functions & FOCs

Firm 1: $\pi_1 = (p - c_1)\, q_1$. Substitute $p = a - q_1 - q_2$:

$$\pi_1 = (a - q_1 - q_2 - c_1)\, q_1 \qquad \Rightarrow \qquad \frac{\partial \pi_1}{\partial q_1} = a - 2q_1 - q_2 - c_1 = 0$$

Firm 2: $\pi_2 = (p - c_2)\, q_2$. Substitute $p = a - q_1 - q_2$:

$$\pi_2 = (a - q_1 - q_2 - c_2)\, q_2 \qquad \Rightarrow \qquad \frac{\partial \pi_2}{\partial q_2} = a - q_1 - 2q_2 - c_2 = 0$$

Q2 — Linear System & Solution

Rearrange the two FOCs into a 2×2 linear system:

$$\begin{cases} 2q_1 + q_2 = a - c_1 \\ q_1 + 2q_2 = a - c_2 \end{cases}$$

△ Watch Out

Do not solve one FOC and assume symmetry — firms have **different costs**. Always verify the feasibility condition $q_2^* > 0$, i.e. $c_2 < (a + c_1)/2$.

Q2 — Equilibrium, Profits & Market Shares

Equilibrium

$$q_1^* = \frac{a - 2c_1 + c_2}{3}, \qquad q_2^* = \frac{a - 2c_2 + c_1}{3}, \qquad p^* = \frac{a + c_1 + c_2}{3}$$

Profits: $\pi_i^* = (p^* - c_i)\, q_i^*$. Since $p^* - c_i = \frac{a+c_1+c_2}{3} - c_i = q_i^*$, we get $\pi_i^* = (q_i^*)^2$. **CS:** $CS^* = \frac{1}{2}(Q^*)^2$, $Q^* = \frac{2a-c_1-c_2}{3}$

Key insight

Since $c_1 < c_2$: $q_1^* > q_2^*$ and $\pi_1^* > \pi_2^*$

In Cournot, the low-cost firm produces more and earns more. Cost efficiency translates directly into market share advantage.

PS II, Q6 — Problem Statement

Exercise E (PS II, Problem 6)

Duopoly, firms choose prices *simultaneously and separately*. Market demand: $D(p) = 10 - p$. Lower price wins the whole market; equal prices split it equally.

Costs: $C_1(q_1) = q_1, \;\; C_2(q_2) = 4q_2$.

Find the Nash equilibrium prices, equilibrium profits, and output of each firm.

Q6 — Recall: Symmetric Bertrand

★ Your Turn

Same setup, but $MC_1 = MC_2 = 4$.

What is the Nash equilibrium? What are the profits?

$p_1^* = p_2^* = MC = 4, \quad \pi_1 = \pi_2 = 0$ — the **Bertrand paradox**.

Now: what changes when $MC_1 = 1 \neq MC_2 = 4$?

PS II, Q6 — Bertrand with Asymmetric Costs

Asymmetric costs: $MC_1 = 1, \; MC_2 = 4$

- Homogeneous good; consumers always buy from the cheapest firm.
- Symmetric benchmark: $p_1^* = p_2^* = 4, \; \pi_1 = \pi_2 = 0.$

Large vs. small asymmetry test:

$$p_1^M = \frac{10+1}{2} = 5.5 > MC_2 = 4 \; \Rightarrow \; \textbf{small asymmetry}$$

Equilibrium

$$p_1^* \approx MC_2 = 4, \quad q_1^* \approx 6, \quad q_2^* = 0, \quad \pi_1^* \approx 18, \quad \pi_2^* = 0, \quad CS^* \approx 18$$

Firm 1 prices just below $MC_2 \Rightarrow$ firm 2 exits $\Rightarrow$ firm 1 is a **de facto monopolist at $p \approx MC_2$**.

Q6 — Why the Paradox Breaks Down

△ Watch Out

Do not set $p_1^* = MC_1 = 1$. Pricing at own MC is the result only in the *symmetric* case.

Always perform the large vs. small asymmetry test first:

- $p_1^M > MC_2$ (small asymmetry): firm 1 prices at $p \approx MC_2$
- $p_1^M \leq MC_2$ (large asymmetry): firm 1 acts as an unconstrained monopolist

Intuition

Cost asymmetry breaks the Bertrand paradox: the low-cost firm has no incentive to price at its own MC when it can profitably undercut its rival and capture the entire market at $p \approx MC_2 > MC_1$.

PS II, Q7 — Problem Statement

Exercise F (PS II, Problem 7)

Two Bertrand competitors. Demands: $q_1 = 25 - 5p_1 + 2p_2$, $q_2 = 25 - 5p_2 + 2p_1$. Costs: $C(q_i) = 2 + q_i$.

(i) Find the best-response function of each firm. How does p_i change with p_j? Find the Nash equilibrium.

(ii) Compute the Lerner index for each good. Interpret. Do firms have market power?

(iii) Explain why the Bertrand paradox does not apply here.

Q7 — Think First

★ **Your Turn**

In the standard (homogeneous) Bertrand model, price competition drives profits to zero.

Goods here are **differentiated substitutes**.

Do you expect the Bertrand paradox to still hold?

PS II, Q7 — Bertrand with Differentiated Products

Setup: $q_1 = 25 - 5p_1 + 2p_2$, $q_2 = 25 - 5p_2 + 2p_1$, $MC = 1$. The positive cross-price term $(+2p_j)$ means goods are **substitutes**.

Firm 1's problem and FOC:

$$\max_{p_1} \pi_1 = (p_1 - 1)(25 - 5p_1 + 2p_2)$$

$$\frac{\partial \pi_1}{\partial p_1} = (25 - 5p_1 + 2p_2) - 5(p_1 - 1) = 0 \;\Rightarrow\; 30 - 10p_1 + 2p_2 = 0$$

Best-response function for firm 1

$$p_1 = 3 + \frac{p_2}{5}$$

Slope $= +\frac{1}{5} > 0 \Rightarrow$ **prices are strategic complements** (contrast with Cournot quantities: strategic substitutes).

Q7 — Nash Equilibrium

By analogous derivation from Firm 2's FOC: $BR_2: \; p_2 = 3 + \dfrac{p_1}{5}$

System: $\begin{cases} p_1 = 3 + \frac{p_2}{5} \\ p_2 = 3 + \frac{p_1}{5} \end{cases}$ Substitute BR_2 into BR_1:

$$p_1 = 3 + \frac{1}{5}\left(3 + \frac{p_1}{5}\right) = \frac{18}{5} + \frac{p_1}{25} \;\Rightarrow\; \frac{24}{25}p_1 = \frac{18}{5} \;\Rightarrow\; p_1^* = \frac{15}{4}$$

Back-substitute: $p_2^* = 3 + \dfrac{1}{5} \cdot \dfrac{15}{4} = 3 + \dfrac{3}{4} = \dfrac{15}{4}$

Nash equilibrium

$p_1^* = p_2^* = \frac{15}{4}$

$q_1^* = 25 - 5 \cdot \frac{15}{4} + 2 \cdot \frac{15}{4} = \frac{55}{4}, \quad q_2^* = 25 - 5 \cdot \frac{15}{4} + 2 \cdot \frac{15}{4} = \frac{55}{4}$

$\pi_i^* = \left(\frac{15}{4} - 1\right) \cdot \frac{55}{4} = \frac{11}{4} \cdot \frac{55}{4} = \frac{605}{16} \approx 37.81$

Q7 — Lerner Index & Market Power

Lerner index:

$$L = \frac{p^* - MC}{p^*} = \frac{\frac{15}{4} - 1}{\frac{15}{4}} = \frac{11}{15} \approx 0.73$$

Interpretation

$L \approx 0.73$ confirms that **market power exists**: the firm sets price 73% above marginal cost.

Contrast with homogeneous symmetric Bertrand: there, $p^* = MC \Rightarrow L = 0$.

The gap (0.73 vs. 0) is entirely due to **product differentiation**: undercutting does not steal the whole market, so firms can sustain positive markups.

Q7 — Why the Bertrand Paradox Fails

Key result

Goods are **imperfect substitutes**: undercutting by ε does *not* steal the whole market.

No firm has an incentive to price at MC $\Rightarrow$ **positive profit in equilibrium**.

Product differentiation restores market power even under price competition.

Summary

Topic	Take-away
PS I Q4	Learning-by-doing $\Rightarrow p^* < p^M$, $q^* > q^M$; $L_1 < 1/\eta$ (firm subsidises period-1 output).
PS I Q7	Higher price in the less elastic market; welfare $\downarrow$ when same markets served; welfare $\uparrow$ when PD opens a previously unserved market — no ambiguity.
Spectrum	Monopoly $\rightarrow$ Oligopoly $\rightarrow$ Perfect competition.
PS II Q1	Cournot duopoly: $\uparrow Q$, $\downarrow p$, $\uparrow CS$, $\uparrow W$ vs. monopoly.
PS II Q2	Asymmetric Cournot: low-cost firm gets larger share *and* higher profit.
PS II Q6	Asymmetric Bertrand: low-cost firm prices near rival's MC; paradox resolved.
PS II Q7	Differentiated Bertrand: $L \approx 0.73 > 0$; contrast with homogeneous symmetric Bertrand where $L = 0$.

Good luck with the problem sets!

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