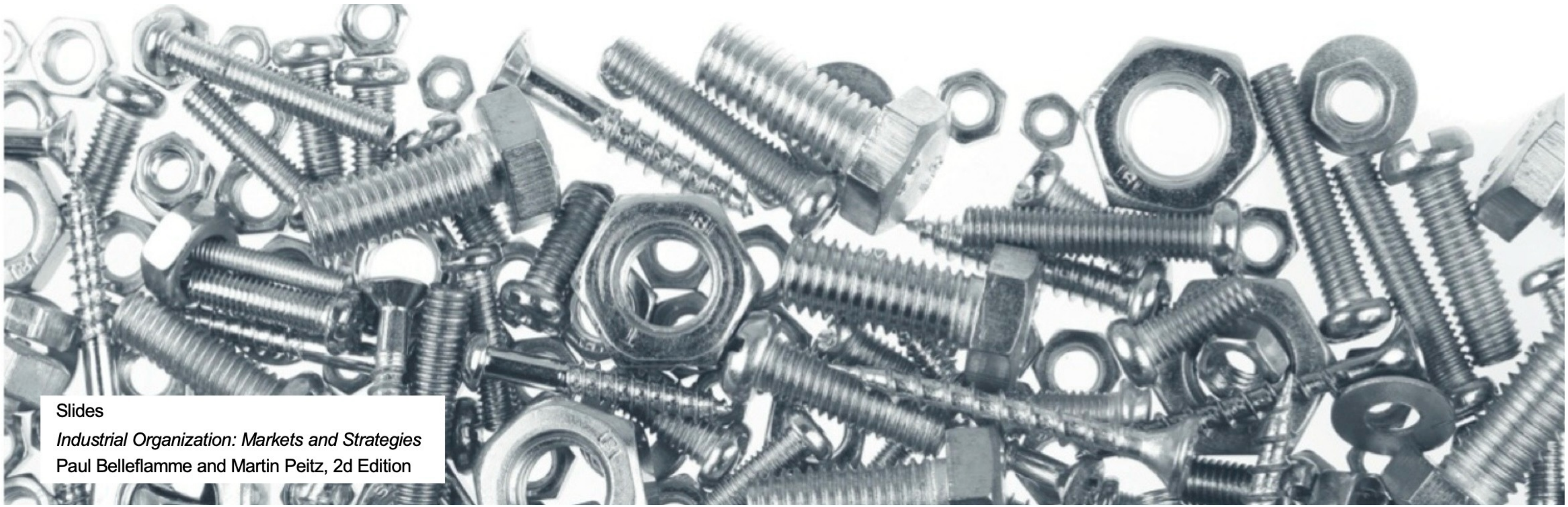
Industrial Economics

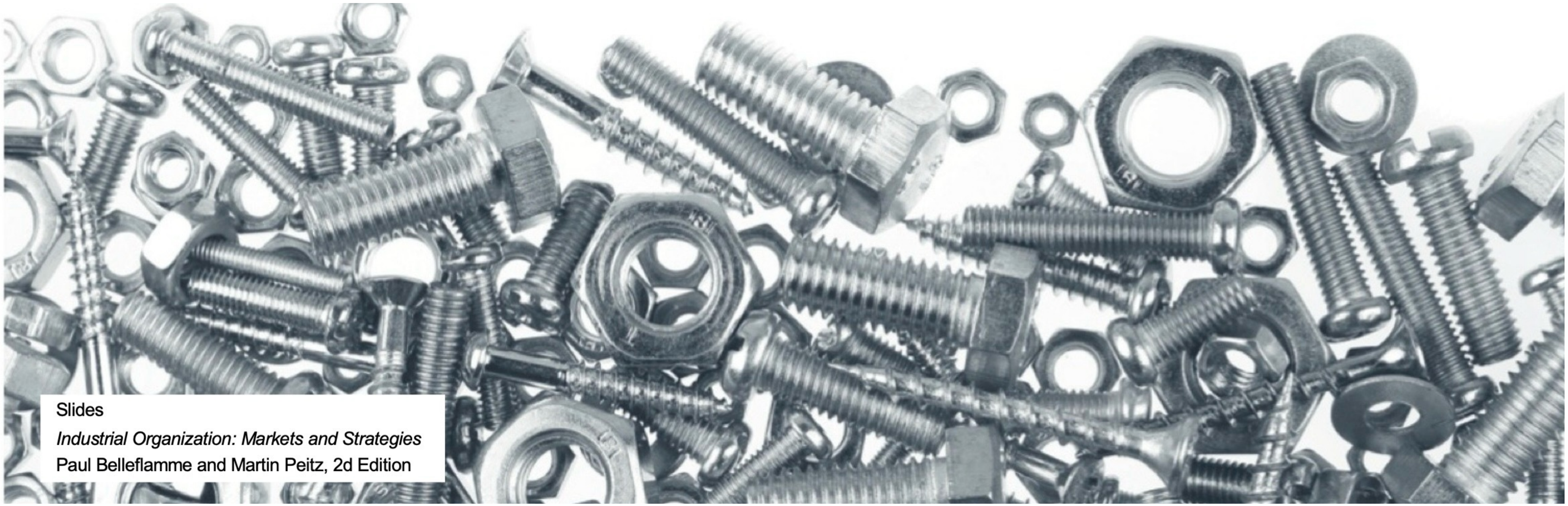
Chrysovalantou Milliou

AUEB – Erasmus Program



Slides
Industrial Organization: Markets and Strategies
Paul Belleflamme and Martin Peitz, 2d Edition

Practice exercises on HORIZONTAL MERGERS



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Exercise 2

In an oligopolistic market, there are initially four firms (1,2,3 and 4). The total cost of production of each firm i, with $i = 1, 2, 3, 4$ is equal to $C(q_i) = 3q_i$, where q_i is the quantity of firm i. The firms choose their quantities simultaneously and independently. The total market demand for their product is $p(Q) = 18 - Q$, where Q is the total quantity of the product.

1. Find the quantity and profits of each firm in equilibrium.

Solution

(1) Like in exercise (1), we need to solve the profit maximization problem of each firm, in order to find their best response functions

$$\max_{q_i \geq 0} \Pi_i(q_i) = (18 - q_i - q_{-i})q_i - 3q_i$$

After a little algebra, the problem becomes

$$\max_{q_i \geq 0} \Pi_i(q_i) = (15 - q_i - q_{-i})q_i$$

The first order condition is

$$\frac{\partial \Pi_i(q_i)}{\partial q_i} = 0 \Rightarrow 15 - 2q_i - q_{-i} = 0 \Rightarrow \boxed{q_i(q_{-i}) = \frac{15 - q_{-i}}{2}}$$

This is firm i's best response function. We have a linear system of 4 equations in 4 unknowns. As in exercise (1) the system is again symmetric, so we can just set $q_1^* = q_2^* = q_3^* = q_4^* = q^*$ to find $\boxed{q^* = 3.}$ Then by plugging this into the demand function we get the equilibrium price $\boxed{p^* = 6,}$ and finally we can find the profits of each firm in equilibrium $\boxed{\Pi^* = 9.}$ $\square$

2. Suppose now that firms 1,2, and 3 are considering merging with each other. The total cost of production of the merged firm will be $C(q_m) = 3q_m$, where q_m is the quantity of output of the merged firm. Do the three firms have an incentive to merge or not?

(2) After the merger we have two firms in the market, the merged firm (m) and firm 4. The merged firm solves

$$\max_{q_m} \Pi_m(q_m) = (18 - q_m - q_4)q_m - 3q_m = 15q_m - q_m^2 - q_m q_4$$

The first order condition is

$$\frac{\partial \Pi_m(q_m)}{\partial q_m} = 0 \Rightarrow 15 - 2q_m - q_4 = 0 \Rightarrow \boxed{q_m(q_4) = \frac{15 - q_4}{2}}$$

Firm 4 solves

$$\max_{q_4} \Pi_4(q_4) = (18 - q_4 - q_m)q_4 - 3q_4 = 15q_4 - q_4^2 - q_4 q_m$$

The first order condition is

$$\frac{\partial \Pi_4(q_4)}{\partial q_4} = 0 \Rightarrow 15 - 2q_4 - q_m = 0 \Rightarrow \boxed{q_4(q_m) = \frac{15 - q_m}{2}}$$

As we can see (both from the profit function and the best responses), the firms are symmetric when it comes to their quantity setting behavior. Their only difference is their ownership structure, i.e. that the profits of the merged firm have to be shared between firms 1, 2, and 3, while firm 4 is entitled to all of its profits. Ownership structure does not play a role in how the firm decides its quantity. What matters is joint profit maximization. (Maximizing the size of the pie in this model has nothing to do with how the pie is shared.)

Due to symmetry, we can easily solve the system to find

$$\boxed{q_m^* = q_4 = 5^* \Rightarrow p^* = 8 \Rightarrow \Pi_m^* = \Pi_4^* = 25}$$

The merger will take place if the profits of the merged firm are higher than the profits firm 1, 2, and 3 independently make before the merger. Here, the merger does not take place, because before the merger the profit of an independent firm is 9. Hence, firms 1, 2, and 3 collectively realize profit equal to 27 before the merger, which is higher than 25, their profit if they merge.

This result is known in the literature as the *merger paradox*. The merger we examined is a merger between firms that have a very high share in the market, 75%, so it is not desirable that the model predicts this merger is unprofitable, because in reality we observe mergers between firms with much less share in the market. The reason for this paradox is attributed to the only effect of this merger being the reduction of the number of firms, which leads to higher profits for each firm due to higher concentration. However, as those increased profits have to be shared between three partners, they are not enough to incentivize the merger. Therefore, we need to find other ways in which the merger gives advantages to the merged firm. (This is the point of the following question.) □

3. Suppose now that if firms 1,2, and 3 merge, the total cost of production of the merged firm is $C(q_m) = 2q_m$, where q_m is the quantity of the merged firm. Do the three firms have an incentive to merge or not?

(3) Now the merged firm has reduced costs, because the merger is assumed to be efficiency enhancing. The new maximization problem of the merged firm is

$$\max_{q_m} \Pi_m(q_m) = (18 - q_m - q_4)q_m - 2q_m = 16q_m - q_m^2 - q_m q_4$$

The first order condition is

$$\frac{\partial \Pi_m(q_m)}{\partial q_m} = 0 \Rightarrow 16 - 2q_m - q_4 = 0 \Rightarrow \boxed{q_m(q_4) = \frac{16 - q_4}{2}}$$

Firm 4's problem is the same as in (2),

$$\max_{q_4} \Pi_4(q_4) = (18 - q_4 - q_m)q_4 - 3q_4 = 15q_4 - q_4^2 - q_4 q_m$$

The first order condition is

$$\boxed{q_4(q_m) = \frac{15 - q_m}{2}}$$

The best response functions form a linear system of 2 equations in 2 unknowns, which is unfortunately not symmetric. We solve by substitution (plug one of the best response functions into the other) to find

$$q_m^* = 5.67, q_4^* = 4.66, p^* = 7.67, \Pi_m^* = 32.35$$

Notice that $32.35 \geq 27$ hence the merger will take place. The reduction in costs is large enough to incentivize the merger. ■

Exercise 3

Consider an industry with 3 firms competing à la Cournot. Every firm $i = 1, 2, 3$ faces an inverse demand function:

$$p_i(q_i, q_{-i}) = 1 - q_i - \gamma q_{-i}$$

where

$$q_{-i} = \sum_{j \neq i} q_j$$

is the total quantity produced by every firm except for i, and $\gamma \in [0, 1]$ represent the degree of product differentiation. When $\gamma = 0$, products are completely differentiated, but when $\gamma = 1$, products are homogeneous. For simplicity, suppose that firms have constant marginal costs of production, $c = 0$.

1. *Cournot competition.* Assume that every firm i independently and simultaneously chooses its output level q_i. Find the equilibrium output, price, and profits for each firm.

Firm $i = 1, 2, 3$ solves

$$\max_{q_i \geq 0} \Pi_i(q_i) = (1 - q_i - \gamma q_{-i}) q_i$$

The first order condition is

$$\frac{\partial \Pi_i(q_i)}{\partial q_i} = 0 \Rightarrow 1 - 2q_i - \gamma q_{-i} = 0 \Rightarrow \boxed{q_i(q_{-i}) = \frac{1 - \gamma q_{-i}}{2}}$$

This is the best response function of firm i. The best response functions form a symmetric linear system of 3 equations in 3 unknowns. Due to symmetry, we can quickly find the equilibrium by setting $q_1^* = q_2^* = q_3^* = q^*$.

$$q^* = \frac{1 - 2\gamma q^*}{2} \Rightarrow \boxed{q^* = \frac{1}{2(1 + \gamma)}}$$

This is the best response function of firm i. The best response functions form a symmetric linear system of 3 equations in 3 unknowns. Due to symmetry, we can quickly find the equilibrium by setting $q_1^* = q_2^* = q_3^* = q^*$.

$$q^* = \frac{1 - 2\gamma q^*}{2} \Rightarrow \boxed{q^* = \frac{1}{2(1+\gamma)}}$$

Plugging this to the demand function we get the equilibrium price

$$p^* = 1 - \frac{1}{2(1+\gamma)} - \frac{2\gamma}{2(1+\gamma)} \Rightarrow \boxed{p^* = \frac{1}{2(1+\gamma)}}$$

Finally the profits of each firm in equilibrium are

$$\boxed{\Pi^* = \frac{1}{4(1+\gamma)^2}} \square$$

3. *Partial merger.* Assume that firm 1 and firm 2 merge to choose an output profile (q_1, q_2) that maximizes their joint profits. Find the equilibrium output.

(3) If firms 1 and 2 merge, and 3 stays independent, then we have two firms in the market, the merged firm and firm 3. The merged firm is a multi-product firm, while firm 3 is a single-product firm. The merged firm solves

$$\max_{\{q_1, q_2\}} \Pi_m = \sum_{i=1}^{2} [(1 - q_i - \gamma q_{-i})q_i] = \sum_{i=1}^{2} [(1 - q_i - \gamma q_j - \gamma q_3)q_i]$$

where $i, j = 1, 2$ and $j \neq i$. Here we have two first order conditions, one for q_1 and one for q_2.

$$\frac{\partial \Pi_m(q_1, q_2)}{\partial q_i} = 0 \Rightarrow \boxed{q_i = \frac{1 - 2\gamma q_j - \gamma q_3}{2}}$$

The two first order conditions form a symmetric linear system of 2 equations with 2 unknowns. We set $q_1^* = q_2^* = q_m^*$ to find

$$\boxed{q_m^* = \frac{1 - \gamma q_3^*}{2(1+\gamma)}}$$

This is the best response function of the merged firm.

Firm 3 solves

$$\max_{q_3} \Pi_3(q_3) = (1 - q_3 - \gamma q_1 - \gamma q_2) q_3$$

The first order condition is

$$\frac{\partial \Pi_3(q_3)}{\partial (q_3)} = 0 \Rightarrow 1 - 2q_3 - \gamma q_1 - \gamma q_2 = 0 \Rightarrow \boxed{q_3 = \frac{1 - \gamma q_1 - \gamma q_2}{2}}$$

This is the best response function of firm 3. In equilibrium, this best response function becomes

$$\boxed{q_3^* = \frac{1 - 2\gamma q_m^*}{2}}$$

since $q_1^* = q_2^* = q_m^*$.

The two best response functions form a linear system of 2 equations in 2 unknowns. Unfortunately it is not symmetric so we solve by substitution to find

$$\boxed{q_m^* = \frac{2-\gamma}{2(2+2\gamma-\gamma^2)}}$$

$$\boxed{q_3^* = \frac{2}{2(2+2\gamma-\gamma^2)}} \quad \square$$

4. Evaluate the equilibrium output in (3) when $\gamma = 0$ and when $\gamma = 1$. Interpret.
5. Find the equilibrium price and profits of merged and unmerged firms in (3).
6. Evaluate the equilibrium profits in (5) when $\gamma = 0$ and when $\gamma = 1$. Interpret.

(4) If $\gamma = 0$ the goods are completely differentiated, i.e. there is no degree of substitution between them for the consumers. We find

$$q_m^* = q_3^* = \frac{1}{2}$$

which is the typical monopoly solution (demands are independent of each other).

If $\gamma = 1$ the goods are homogeneous, i.e. they are perfect substitutes. We find

$$q_m^* = \frac{1}{6}, \; q_3^* = \frac{1}{3}$$

Recall that $q_m^* = q_1^* = q_2^*$ hence the total quantity produced by the merged firm $(q_1^* + q_2^*)$ is $1/3$, exactly the same as firm 3 (as we expect from exercises (1) and (2) of the problem set). $\square$

(5) We substitute the quantities we found in (3) into the demand function of each firm. First for the merged firm

$$p_m^* = 1 - q_m^* - \gamma q_m^* - \gamma q_3^* - = 1 - \frac{2-\gamma}{2(2+2\gamma-\gamma^2)} - \frac{\gamma(2-\gamma)}{2(2+2\gamma-\gamma^2)} - \frac{2\gamma}{2(2+2\gamma-\gamma^2)} \Rightarrow$$

$$\Rightarrow \boxed{p_m^* = \frac{2+\gamma-\gamma^2}{2(2+2\gamma-\gamma^2)}}$$

Then for firm 3,

$$p_3^* = 1 - q_3^* - \gamma q_m^* - \gamma q_m^* = 1 - q_3^* - 2\gamma q_m^* \Rightarrow$$

$$\Rightarrow p_3^* = 1 - \frac{2}{2(2+2\gamma-\gamma^2)} - \frac{2\gamma(2-\gamma)}{2(2+2\gamma-\gamma^2)} \Rightarrow$$

$$\Rightarrow \boxed{p_3^* = \frac{2+\gamma-\gamma^* 2}{2(2+2\gamma-\gamma^2)}}$$

The profits of the merged firm are

$$\Pi_m^* = p_1^* q_1^* + p_2^* q_2^* = 2 p_m^* q_m^* = 2\frac{2+\gamma-\gamma^2}{2(2+2\gamma-\gamma^2)}\frac{2-\gamma}{2(2+2\gamma-\gamma^2)} \Rightarrow$$

$$\Rightarrow \boxed{\Pi_m^* = \frac{(2+\gamma-\gamma^2)(2-\gamma)}{2(2+2\gamma-\gamma^2)^2}} \square$$

(6) If $\gamma = 0$ the goods are completely differentiated. We find

$$\Pi_m^* = \frac{1}{2}, \ \Pi_3^* = \frac{1}{4}$$

which is the typical monopoly solution (demands are independent of each other if $\gamma = 0$). Note that the merged firm operates two separate monopolies, and this is why it has double the profits of firm 3.

If $\gamma = 1$ the goods are homogeneous. We find

$$\Pi_m^* = \Pi_3^* = \frac{1}{9}$$

which is expected, since being a multi-product firm in a homogeneous good environment is irrelevant - the merged firm and firm 3 are symmetric.

Exercise

Merger to Monopoly Price Competition with Differentiated Products

Consider two differentiated products, each produced by a different firm. The inverse demand curve for product 1 is: $q_1(p_1,p_2)=12 - p_1+0.5p_2$. The inverse demand function for product 2 is: $q_2(p_1,p_2)=12 - p_2+0.5p_1$. The total production cost of each good is 0. Suppose that the two firms choose their prices simultaneously and separately.

(a) What is the price of each good?

Solution:

We start with the "no merger" case, where each firm i, i =1, 2, solves:

$$Max_{pi}\ \pi_i = p_i\ (12 - p_i+0.5p_j)$$

FOC:

$$\partial\pi_i/\partial p_i = 12 - 2p_i + 0,5p_j = 0$$

From this, we obtain firm i's best- response function:

$$p_i(p_j) = (12+0,5p_j)/2$$

Exercise

Merger to Monopoly Price Competition with Differentiated Products

Firm j solves a similar problem, so we also obtain its best- response function:

$$p_j(p_i) = (12+0,5p_i)/2$$

Thus, we have two unknowns, p1 & p2, and two equations, the two best response functions. We can find the two unknowns by solving the system of the two equations:

$$p_i(p_j) = (12+0,5p_j)/2 \;\Rightarrow\; 2p_i = 12+0,5p_j$$

$$2p_i = 12+0,5\times[(12+0,5p_i)/2]$$

$$4p_i = 24+0,5\times(12+0,5p_i)$$

$$4p_i = 24+6+0,25p_i$$

$$(4 - 0,25)p_i = 30$$

$$p_i = 30/3,75 = 8 = p_1^N = p_2^N$$

Exercise

Merger to Monopoly Price Competition with Differentiated Products

(b) Suppose that the two firms merge and the merged firm produces now the two product varieties. What is the price of each product?

Solution: We move now to the "merger" case, where we have a multi-product monopolist which solves the following:

$$\text{Max}_{p_1 \,\&\, p_2}\ \pi_{12} = p_1 \times (12 - p_1 + 0.5p_2) + p_2 \times (12 - p_2 + 0.5p_1)$$

FOCs:

$$\partial \pi_1 / \partial p_1 = 12 - 2p_1 + 0{,}5p_2 + 0{,}5p_2 = 0$$

$$\partial \pi_2 / \partial p_2 = 12 - 2p_2 + 0{,}5p_1 + 0{,}5p_1 = 0$$

Thus:

$$p_1 = (12 + p_2)/2 \quad \& \quad p_2 = (12 + p_1)/2$$

Exercise

Merger to Monopoly Price Competition with Differentiated Products

Solving the system of the two equations $p_1 = (12+ p_2)/2$ & $p_2 = (12+ p_1)/2$, we have:

$$p_1 = (12+p_2)/2 \rightarrow 2p_1 = 12+p_2$$

$$2p_i = 12+(12+p_1)/2$$

$$4p_1 = 24+(12+p_1)$$

$$4p_1 = 24+12+p_1$$

$$(4-1)p_1 = 36$$

$$p_1^M = p_2^M = 36/3 = 12$$

So clearly, the merger results in an increase in the price:

$$p_i^M = 12 > p_i^N = 8$$

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

Consider a market with demand $Q(p) = a - p$, where 3 identical firms compete in prices. Each firm's total cost function is given by $TC_i(q_i) = cq_i$. Suppose two of the firms merge and that the total cost function of the firm that results from the merger is $TC_m(q_m)=c_m q_m$, with $c>(a+c_m)/2$.

(a) What is the market share of each firm before and after the merger?

Solution:

Before the merger, every firm sets in equilibrium:

$$P_i^N = MC = c$$

The total market quantity in equilibrium is: $Q^N = a - c$

Since the 3 firms are symmetric, the quantity of each firm i is:

$$q_i^N = Q^N/3 = (a - c)/3$$

The market share of each firm is thus:

$$s_i^N = q_i^N/Q^N = 1/3 = 0{,}33$$

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

After the merger, the merged firm 12 has lower marginal cost than firm 3.

Is the cost asymmetry large or small?

If it behaved as a monopolist, it would solve the following problem:

$$\text{Max } \pi = (a - p)\, p - c_m(a - p)$$

This would lead to:

$$d\pi/dp = a\text{-}2p+c_m=0$$

Thus, if firm 12 was a monopolist it would charge: $p_m = (a+c_m)/2$

According to the description of the exercise $c>(a+c_m)/2$. Thus, the cost asymmetry is large. This means that in equilibrium (as we have saw in class in the oligopoly theory):

$$(p_{12}^M, p_3^M) = ((a+c_m)/2,\ c)$$

It follows that firm 12, having smaller price than its rival, will be the only one selling the good. Thus, $s_{12}^M = 1$

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

(b) What is the H-index before and after the merger?

Solution:

Before the merger:

$$H^N = (0{,}33)^2+(0{,}33)^2+(0{,}33)^2 = 0{,}32$$

After the merger:

$$H^M = (1)^2+(0)^2 = 1$$

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

Consider a market in which two firms compete in prices. Market demand is given by $Q(p) = 22 - 2p$. The marginal cost that each firm faces is equal to 6. Suppose that the two firm merge and their marginal cost reduces to 2. What is the price in equilibrium before and after the merger? What is the impact of the merger on consumer surplus and on total welfare?

Solution:

Before the merger, both firms in equilibrium set their price:

$$p_1 = p_2 = MC = 6$$

The total market quantity is:

$$Q = 22 - 2p = 22 - 2*6 = 22 - 12 = 10$$

The profits of each firm are equal to zero.

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

<u>After the merger</u>, there is a monopolist in the market with MC = 2. The monopolist chooses its price in the following way:

$$\text{Max } \pi = (22 - 2p)p - 2\,(22\text{-}2p)$$

FOC:

$$d\pi/dp = 22 - 4p + 4 = 0$$

Thus, the monopolist (merged firm) will set:

$$p^m = 26/4 \;= 6{,}5$$

Thus, the total quantity in the case of the merger will be:

$$Q^m = 22 - 2\,(6{,}5) = 9$$

In turn, the profits of the merged firm will be:

$$\pi^m \;= 6{,}5*9 - 2*9 = 40{,}5$$

Exercise

Merger Price Competition (Homogeneous product) with Efficiency Gains

<u>Before the merger:</u>

Consumer surplus (it is easier if before calculating it you draw the diagram) is:

$$CS = (½)*(11-6)*(10) =25$$

Profits are zero, so total welfare is equal to consumer surplus:

$$TS = CS = 25$$

<u>After the merger</u>:

Consumer surplus is:

$$CS^m = (1/2) * (11-6,5)*(9)= 20,25$$

Total surplus:

$$TS^m = CS^m + \pi^m = 20,25 + 40,5 = 60,75$$

Clearly, the merger reduces consumer surplus but increases total welfare due to efficiency gains (reduction of MC of the merged firm).