ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INDUSTRIAL ECONOMICS

PRACTICE PROBLEM SET V: HORIZONTAL MERGERS

Solution of Exercise 1

(a) *No merger.* If firms do not merge, find their best response functions, equilibrium prices, and profits.

- Every firm i chooses p_i to solve the following profit maximization problem:

$$\max_{p_i>0} \pi\left(p_i\right) = p_i\left(1 - p_i + \gamma \overline{p}_{-i}\right)$$

Differentiating with respect to p_i, we find:

$$1 - 2p_i + \gamma \overline{p}_{-i} = 0$$

Rearranging, the best response of firm i is

$$p_i\left(\overline{p}_{-i}\right) = \frac{1}{2} + \frac{\gamma}{2}\overline{p}_{-i}$$

which originates at a vertical intercept of $1/2$ and increases in the average price of all its rivals at a rate of $\gamma/2$. Intuitively, when goods are completely differentiated, $\gamma = 0$, this best response function collapses to $p_i\left(\overline{p}_{-i}\right) = \frac{1}{2}$, implying that firm i's pricing decision does not depend on its rivals' prices, and sets a price equal to $\frac{1}{2}$, as in standard monopolies with this linear demand curve. However, when goods are homogeneous, $\gamma \to 1$, the best response function becomes $p_i\left(\overline{p}_{-i}\right) = \frac{1}{2} + \frac{1}{2}\overline{p}_{-i}$.

Graphically, when $\gamma = 0$, the best response function is flat (unaffected by the pricing decisions of firm i's rivals, $\overline{p}_{-i}$). When γ increases, however, the best response function pivots upward, counterclockwise, thus exhibiting a positive slope, $\frac{\gamma}{2}$, which becomes the steepest when $\gamma \to 1$. This happens because prices are strategic complements, and price competition is softened when goods become more homogeneous (higher γ).

- In a symmetric equilibrium, prices satisfy $\overline{p}_{-i} = p_i$, entailing

$$p_i = \frac{1}{2} + \frac{\gamma}{2}p_i$$

which, solving for p_i, yields an equilibrium price

$$p^* = \frac{1}{2-\gamma}$$

which is increasing in γ. When goods are completely differentiated, $\gamma = 0$, this price becomes $p_i^* = \frac{1}{2}$, which coincides with the standard result in profit-maximizing monopolists. In contrast, when goods are homogeneous, $\gamma \to 1$, the price increases to $p_i^* = 1$.

- Inserting this price, $p_i^* = \frac{1}{2-\gamma}$, into the profit function of firm i, we obtain an equilibrium price of

$$\begin{aligned}
\pi^* &= p^*\left(1 - p^{NM} + \gamma p^{NM}\right) \\
&= \frac{1}{2-\gamma}\left(1 - \frac{1}{2-\gamma} + \gamma\frac{1}{2-\gamma}\right) \\
&= \frac{1}{(2-\gamma)^2}
\end{aligned}$$

which is increasing in γ since firms charge a higher price on more homogeneous goods, softening price competition and, thus, increasing equilibrium profits.

(b) *Full merger.* Assume that all three firms merge. Find their equilibrium price and profits.

- Assume that all three firms merge, then it chooses p_i^M for every firm i to solve

$$\max_{p_i^M > 0} \pi\left(p_i^M\right) = \sum_{i=1}^{3} p_i^M \left[1 - p_i^M + \frac{\gamma}{2}\sum_{j \neq i} p_j^M\right]$$

where p_j^M is the price of every other merged firm $j \neq i$.

Differentiating with respect to p_i^M, and assuming interior solutions, we find:

$$1 - 2p_i^M + \gamma \sum_{j \neq i} p_j^M = 0$$

Since every merged firm is symmetric, in equilibrium we must have $p^M = p_i^M = p_j^M$, yielding the best response function of every merged firm, as follows:

$$p^M = \frac{1}{2(1-\gamma)}$$

- When goods are completely differentiated, $\gamma = 0$, the best response function simplifies to $p^M = \frac{1}{2}$. In contrast, when γ increases, goods become more homogeneous, and we obtain that

$$\frac{\partial p^M}{\partial \gamma} = \frac{1}{2(1-\gamma)^2} > 0$$

so that p^M increases in γ.

(c) *Full merger, profit comparison.* Compare the equilibrium profits of the merged firms, as found in part (b), against their profits when unmerged, as found in part (a), to show the full merger can be sustained in equilibrium.

- Substituting p^M into π^M, every merged firm earns profits of

$$\pi^M = \frac{1}{2(1-\gamma)}\left[1 - \frac{1-\gamma}{2(1-\gamma)}\right] = \frac{1}{4(1-\gamma)}$$

- Every firm i has incentives to merge if and only if $\pi^M \geq \pi^*$, where

$$\frac{1}{4(1-\gamma)} \geq \frac{1}{(2-\gamma)^2}$$

simplifies to

$$\gamma^2 \geq 0$$

that holds, so that the full merger is supported for all values of $\gamma \in [0, 1)$.

Solution of Exercise 2

(a) *Cournot competition.* Assume that every firm i independently and simultaneously chooses its output level, q_i. Find the equilibrium output, price, and profits for each firm.

- Every firm i simultaneously chooses q_i to solve

$$\max_{q_i \geq 0} \pi_i(q_i) = (1 - q_i - \gamma q_{-i}) q_i$$

where $q_{-i} \equiv \sum_{j \neq i} q_j$ represents the sum of output of all firm i's rivals.

Taking the first-order condition with respect to q_i, and assuming interior solutions

$$1 - 2q_i - \gamma q_{-i} = 0$$

Rearranging and solving for q_i, we obtain the best response function of firm i:

$$q_i(q_{-i}) = \frac{1}{2} - \frac{\gamma}{2} q_{-i}$$

which originates at $1/2$ and decreases in q_{-i} at a rate of $\frac{\gamma}{2}$. Intuitively, the more homogeneous are the products (higher γ), the more responsive is firm i's output to the output of the other $N - 1$ firms.

Since firms are symmetric, in equilibrium we must have $q = q_i = q_j$, yielding

$$q = \frac{1}{2} - \frac{\gamma}{2}(N-1) q$$

Rearranging and solving for q, we find:

$$q^* = \frac{1}{2 + \gamma(N-1)}$$

Substituting $q = \frac{1}{2+\gamma(N-1)}$ into the inverse demand function, equilibrium price becomes:

$$p^* = 1 - q^* - (N-1)\gamma q^* = \frac{1}{2 + \gamma(N-1)}$$

Finally, equilibrium profits of every firm i are

$$\pi^* = p^* q^* = \frac{1}{[2 + \gamma(N-1)]^2}$$

(b) *Full merger.* Assume that all N firms merge to choose an output profile $(q_1, q_2, \ldots, q_N)$ that maximizes their joint profits. Find the equilibrium output, price, and profits in this setting.

- The merged firm chooses $(q_1, q_2, \ldots, q_N)$ to solve

$$\max_{q_1,q_2,\ldots,q_N \geq 0} \pi(q_1, q_2, \ldots, q_N) = \sum_{i=1}^{N} (1 - q_i - \gamma q_{-i}) q_i$$

Taking first-order condition with respect to q_i, and assuming interior solutions

$$1 - 2q_i - 2\gamma q_{-i} = 0$$

Rearranging and solving for q_i, we obtain the best response function of firm i:

$$q_i(q_{-i}) = \frac{1}{2} - \gamma q_{-i}.$$

This best response function originates at $1/2$, as that in part (a) when firms independently choose their output levels, but decreases faster in q_{-i}, thus indicating that a given increase in the aggregate output of firm i's rivals yields twice as much reduction in i's output level. This happens because output is a strategic substitute, and by merging, every firm i internalizes the externality that increasing its own output induces all other firms to reduce their output.

Since firms are symmetric, in equilibrium we must have $q = q_i = q_j$, yielding

$$q = \frac{1}{2} - \gamma(N-1)q$$

Rearranging and solving for q, we find

$$q^{FM} = \frac{1}{2[1 + \gamma(N-1)]}$$

where the superscript *FM* stands for full merger.

Substituting $q^{FM} = \frac{1}{2[1+\gamma(N-1)]}$ into the inverse demand function, equilibrium price becomes:

$$p^{FM} = 1 - q - (N-1)\gamma q = \frac{1}{2}$$

which is same as the profit-maximizing price of a monopolist subject to an inverse demand function $p(Q) = 1 - Q$.

Finally, equilibrium profits of every firm i are

$$\pi^{FM} = p^{FM} q^{FM} = \frac{1}{4[1 + \gamma(N-1)]}$$

It is straightforward to verify that the every firm i enjoys higher profits when merged than not merged, since $\pi^{FM} \geq \pi^*$ simplifies to

$$\frac{1}{4[1 + \gamma(N-1)]} \geq \frac{1}{[2 + \gamma(N-1)]^2}$$

which is rearranged to yield

$$\gamma^2 (N-1)^2 \geq 0$$

that holds since $\gamma \in [0, 1]$ by definition, and $N \geq 2$, so that every firm i has incentives to merge into one firm.

(c) *Partial merger.* Assume that a subset of firms, k out of N, merge, while the remaining $N-k$ firms stay out of the merger. Find the equilibrium output in this context.

- The merged firm chooses q_i to solve the profit maximization problem:

$$\max_{q_i \geq 0} \pi_i(q_i) = \sum_{i=1}^{k} \left(1 - q_i - \gamma \sum_{j \neq i} q_j - \gamma Q^{NM}\right) q_i$$

where q_j is the output of every other merged firm $j \neq i$ and Q^{NM} represents the total output of all unmerged firms.

Differentiating with respect to q_i, and assuming interior solutions, we obtain:

$$1 - 2q_i - 2\gamma \sum_{j \neq i} q_j - \gamma Q^{NM} = 0$$

Since every merged firm is symmetric, in equilibrium, we must have $q^M = q_i = q_j$, yielding the best response function of the merged firm as follows:

$$q^M\left(Q^{NM}\right) = \frac{1}{2\left[1 + \gamma(k-1)\right]} - \frac{\gamma}{2\left[1 + \gamma(k-1)\right]} Q^{NM}$$

When all firms merge, $k = N$, we have $Q^{NM} = 0$ for there are no outsiders, yielding $q^M(0) = \frac{1}{2[1+\gamma(N-1)]}$ as in part (b).

- Every unmerged firm chooses q_i to solve the profit maximization problem:

$$\max_{q_i \geq 0} \pi_i (q_i) = \left(1 - q_i - \gamma \sum_{l \neq i} q_l - \gamma Q^M\right) q_i$$

where q_l is the output of every other unmerged firm $l \neq i$ and Q^M represents the total output of all merged firms.

Differentiating with respect to q_i, and assuming interior solutions, we find:

$$1 - 2q_i - \gamma \sum_{l \neq i} q_l - \gamma Q^M = 0$$

Since every unmerged firm is symmetric, in equilibrium, we must have $q^{NM} = q_i = q_l$, yielding the best response function of every unmerged firm as follows:

$$q^{NM}\left(Q^M\right) = \frac{1}{2 + \gamma (N - k - 1)} - \frac{\gamma}{2 + \gamma (N - k - 1)} Q^M$$

- Since $Q^M = kq^M$ and $Q^{NM} = (N - k) q^{NM}$, the best response functions become:

$$q^M\left(q^{NM}\right) = \frac{1}{2[1 + \gamma (k - 1)]} - \frac{\gamma (N - k)}{2[1 + \gamma (k - 1)]} q^{NM}$$

$$q^{NM}\left(q^M\right) = \frac{1}{2 + \gamma (N - k - 1)} - \frac{\gamma k}{2 + \gamma (N - k - 1)} q^M$$

Substituting the first best response function into the second, we obtain

$$q^M = \frac{1}{2[1 + \gamma(k - 1)]} - \frac{\gamma(N - k)}{2[1 + \gamma(k - 1)]}\left[\frac{1}{2 + \gamma(N - k - 1)} - \frac{\gamma k}{2 + \gamma(N - k - 1)} q^M\right]$$

Simplifying, the equilibrium output of every merged firm becomes

$$q^M = \frac{2 - \gamma}{2[2 + \gamma (N + k - 3)] + \gamma^2 [(k - 2)(N - k) - 2(k - 1)]}$$

Inserting q^M into $q^{NM}\left(q^M\right)$, the equilibrium output of every unmerged firm is

$$q^{NM} = \frac{2 + \gamma (k - 2)}{2[2 + \gamma (N + k - 3)] + \gamma^2 [(k - 2)(N - k) - 2(k - 1)]}$$

(d) Evaluate the equilibrium output in part (c) when $\gamma = 0$ and when $\gamma = 1$. Interpret.

- When $\gamma = 0$, goods are completely differentiated, so $q^M = q^{NM} = 1/2$ as in standard monopolies where demands are independent of each other.
- When $\gamma = 1$, goods are homogeneous, so that every unmerged firm produces $q^{NM} = \frac{1}{N-k+2}$ which is k times the output of every merged firm, $q^M = \frac{1}{k(N-k+2)}$. This happens because the merged firm behaves as one firm producing one homogeneous good competing

with the other $N-k$ unmerged firms, in contrast with a multiproduct firm that has as many product lines as the number of merged firms when goods are nonhomogeneous, $\gamma \in (0,1)$.

(e) Find the equilibrium price and profits of merged and unmerged firms.

- Substituting q^M and q^{NM} into p^{NM}, every unmerged firm charges a price of

$$\begin{aligned} p^{NM} &= 1-[1+\gamma(N-k-1)]\,q^{NM}-\gamma k q^M \\ &= 1-\frac{[1+\gamma(N-k-1)][2+\gamma(k-2)]-\gamma k(2-\gamma)}{2[2+\gamma(N+k-3)]+\gamma^2[(k-2)(N-k)-2(k-1)]} \\ &= \frac{2+\gamma(k-2)}{2[2+\gamma(N+k-3)]+\gamma^2[(k-2)(N-k)-2(k-1)]} \end{aligned}$$

- Substituting p^{NM} and q^{NM} into π^{NM}, every unmerged firm earns a profit of

$$\pi^{NM} = \frac{[2+\gamma(k-2)]^2}{\left[2(2+\gamma(N+k-3))+\gamma^2((k-2)(N-k)-2(k-1))\right]^2}$$

- Substituting q^M and q^{NM} into p^M, every merged firm charges a price of

$$\begin{aligned} p^{M} &= 1-[1+\gamma(k-1)]\,q^{M}-\gamma(N-k)\,q^{NM} \\ &= 1-\frac{(2-\gamma)[1+\gamma(k-1)]+\gamma(N-k)[2+\gamma(k-2)]}{2[2+\gamma(N+k-3)]+\gamma^2[(k-2)(N-k)-2(k-1)]} \\ &= \frac{(2-\gamma)[1+\gamma(k-1)]}{2[2+\gamma(N+k-3)]+\gamma^2[(k-2)(N-k)-2(k-1)]} \end{aligned}$$

- Substituting p^M and q^M into π^M, every merged firm earns a profit of

$$\pi^{M} = \frac{(2-\gamma)^2[1+\gamma(k-1)]}{\left[2(2+\gamma(N+k-3))+\gamma^2((k-2)(N-k)-2(k-1))\right]^2}$$

(f) Evaluate the equilibrium profits in part (e) when $\gamma=0$ and when $\gamma=1$:

- When $\gamma=0$, goods are completely differentiated, so $\pi^M=\pi^{NM}=1/4$ as in standard monopolies where demands are independent of each other.
- When $\gamma=1$, goods are homogeneous, so that every unmerged firm earns $\pi^{NM}=\frac{1}{(N-k+2)^2}$ which is k times the profits of every merged firm, $\pi^M=\frac{1}{k(N-k+2)^2}$. Since the merged firm is in every aspect homogeneous to the unmerged firm, this firm evenly splits its profits among k member firms.

(g) Under what condition on k can the partial merger in part (c) be sustained?

- For the partial merger to be sustained, we need the profits of every firm that merges, π^M, to be greater than the profits that it earns if not merged, π^*, that is,

$$\frac{(2-\gamma)^2[1+\gamma(k-1)]}{\left[2(2+\gamma(N+k-3))+\gamma^2((k-2)(N-k)-2(k-1))\right]^2} \geq \frac{1}{[2+\gamma(N-1)]^2}$$

(h) Assume that $N=10$. Find the minimum number of firms that merge, $\underline{k}$, which supports the merger when $\gamma=0$, $\gamma=0.25$, $\gamma=0.5$, $\gamma=0.75$, and $\gamma=1$. Interpret.

- Substituting $N = 10$ and different values of γ into the results of part (g) yields:

$$\underline{k}(0) = 1$$
$$\underline{k}(0.25) = 4.39$$
$$\underline{k}(0.5) = 6.3$$
$$\underline{k}(0.75) = 7.38$$
$$\underline{k}(1) = 8.15$$

When goods are completely differentiated ($\gamma = 0$), firms are monopolies, so a merger between two or more firms does not create any spillover effects on rival firms. However, when goods become more homogeneous (γ increases), firms compete with each other more intensively, so a larger number of firms are required to merge in offsetting the competitive effect of the outsiders. In the limit where goods are homogeneous ($\gamma = 1$), the merger requires the largest number of participating firms when outputs are strongest strategic substitutes.

Solution of Exercise 3

(a)

Before the merger, every firm sets in equilibrium:

$$P_i^N = MC = c$$

The total market quantity in equilibrium is: $Q^N = a - c$

Since the 3 firms are symmetric, the quantity of each firm i is:

$$q_i^N = Q^N/3 = (a - c)/3$$

The market share of each firm is thus:

$$s_i^N = q_i^N/Q^N = 1/3 = 0,33$$

After the merger, the merged firm 12 has lower marginal cost than firm 3.

Is the cost asymmetry large or small?

If it behaved as a monopolist, it would solve the following problem:

$$\text{Max } \pi = (a - p)\, p - c_m(a - p)$$

This would lead to:

$$d\pi/dp = a - 2p + c_m = 0$$

Thus, if firm 12 was a monopolist it would charge: $p_m = (a+c_m)/2$

According to the description of the exercise $c > (a+c_m)/2$. Thus, the cost asymmetry is large.

This means that in equilibrium (as we have saw in class in the oligopoly theory):

$$(p_{12}^M, p_3^M) = ((a+c_m)/2,\ c)$$

It follows that firm 12, having smaller price than its rival, will be the only one selling the good.

Thus, $s_{12}^M = 1$

(b)

Before the merger:

$$H^N = (0,33)^2+(0,33)^2+(0,33)^2 = 0,32$$

After the merger:

$$H^M = (1)^2+(0)^2 = 1$$

Solution of Exercise 4

(a)

Each firm i solves:

$$Max_{qi}\ \pi_i = q_i(18 - q_i - Q_{-i}) - 3q_i$$

FOC:

$$d\pi_i/dq_i = 18 - 2q_i - Q_{-i} - 3 = 0$$

Thus, the best-response function is:

$$q_i = (15 - Q_{-i})/2$$

Since the 4 firms are symmetric, we know that in equilibrium:

$$q_1 = q_2 = q_3 = q_4$$

Taking the above into aacount we kan rewrite the bestpreposn function as follows:

$$q_i = (15 - 3q_i)/2$$

$$2q_i = 15 - 3q_i$$

$$q^{X\Sigma}_i = 3$$

Thus:

$$P = 18 - 4(3)$$

$$\mathbf{P^{X\Sigma} = 6}$$

$$\pi_i = 6(3) - 3(3) = 18 - 9$$

$$\mathbf{\pi^{X\Sigma}_i = 9}$$

(b)

The merged firm solves:

$$Max_{q123}\ \pi_{123} = q_i(18 - q_{123} - q_4) - 3q_{123}$$

FOC:

$$d\pi_{123}/dq_{123} = 18 - 2q_{123} - q_4 - 3 = 0$$

The resulting best-response function is:

$$q_4 = (15 - q_{123})/2$$

Taking into account the symmetry, we can rewrite the best-response functions:

$$q_{123} = (15 - (15 - q_{123})/2)/2$$

Thus:

$$2q_{123} = 15 - (15 - q_{123})/2$$

$$4q_{123} = 30 - 15 + q_{123}$$

$$\mathbf{q^{\Sigma}_{123} = 5}$$

$$P^{\Sigma} = 18 - 2(5) = 18 - 10$$

$$\mathbf{P^{\Sigma} = 8}$$

The profit of the merged firm is:

$$\pi_{123} = 8(5) - 3(5) = 40 - 15$$

$$\mathbf{\pi^{\Sigma}_{123} = 25}$$

Withou the merger their profits of the 3 firms which consider merging are:

$$3\pi^{X\Sigma}_i = \mathbf{3(9) = 27}$$

Since $\pi^{\Sigma}_{123} = \mathbf{25} < 3\pi^{X\Sigma}_i = \mathbf{27}$, firms have NO merger incentive.

(c)

Merged firm solves:

$$Max_{q123}\ \pi_{123} = q_i(18 - q_{123} - q_4) - 2q_{123}$$

FOC:

$$d\pi_{123}/dq_{123} = 18 - 2q_{123} - q_4 - 2 = 0$$

Best-response function:

$$q_{123} = (16 - q_4)/2$$

Best-response function of firm 4:

$$q_4 = (15 - q_{123})/2$$

Solving:

$$q_{123} = (16 - (15 - q_{123})/2)/2$$
$$2q_{123} = 16 - (15 - q_{123})/2$$
$$4q_{123} = 32 - 15 + q_{123}$$
$$q_{\Sigma 123} = 5{,}67$$

Thus:

$$q_4 = (15 - 5{,}67)/2 = 4{,}66$$
$$P_{\Sigma} = 18 - 5{,}67 - 4{,}66 = 7{,}67$$

The profits of the merged firm are:

$$\pi_{123} = 7{,}67(5{,}67) - 2(5{,}67) = 43{,}49 - 11{,}34$$
$$\pi_{\Sigma 123} = 32{,}35$$

Withou the merger their profits of the 3 firms which consider merging are:

$$3\pi^{X\Sigma}{}_{i} = \mathbf{3(9)=27}$$

Since $\pi_{\Sigma 123} = 32{,}35 < 3\pi_{X\Sigma i} = 27$, firms do have merger incentive – they will merge.
Why? Because of the efficiency gains (synergy) of the merger – the reduction of MC.