ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INDUSTRIAL ECONOMICS

PRACTICE PROBLEM SET V: HORIZONTAL MERGERS

1. Bertrand with Horizontally Differentiated Products

Consider an industry with $N = 3$ firms competing à la Bertrand. Every firm i faces a demand function:

$$q_i\left(p_i, \overline{p}_{-i}\right) = 1 - p_i + \gamma \overline{p}_{-i},$$

where $\overline{p}_{-i} \equiv \frac{\sum_{j \neq i} p_j}{2}$ represents the average price of all i's rivals. When $\gamma = 0$, products are completely differentiated, but when $\gamma \to 1$, products are homogeneous. For simplicity, assume that all firms have zero marginal costs.

(a) *No merger.* If firms do not merge, find their best response functions, equilibrium prices, and profits.

(b) *Full merger.* Assume that all three firms merge. Find their equilibrium price and profits.

(c) *Full merger, profit comparison.* Compare the equilibrium profits of the merged firms, as found in part (b), against their profits when unmerged, as found in part (a), to show the full merger can be sustained in equilibrium.

2. Cournot with Horizontally Differentiated Products

Consider an industry with $N \geq 2$ firms competing à la Cournot. Every firm i, where $i \in \{1, \ldots, N\}$, faces an inverse demand function:

$$p_i\left(q_1, q_2, \ldots, q_N\right) = 1 - q_i - \gamma \sum_{j \neq i} q_j,$$

where $\gamma \in [0, 1]$ represents the degree of product differentiation. When $\gamma = 0$, products are completely differentiated, but when $\gamma = 1$, products are homogeneous. For simplicity, suppose that firms have constant marginal costs of production, $c = 0$.

(a) *Cournot competition.* Assume that every firm i independently and simultaneously chooses its output level, q_i. Find the equilibrium output, price, and profits for each firm.

(b) *Full merger.* Assume that all N firms merge to choose an output profile $(q_1, q_2, \ldots, q_N)$ that maximizes their joint profits. Find the equilibrium output, price, and profits in this setting.

(c) *Partial merger.* Assume that a subset of firms, k out of N, merge, while the remaining $N - k$

(d) Evaluate the equilibrium output in part (c) when $\gamma = 0$ and when $\gamma = 1$. Interpret.

(e) Find the equilibrium price and profits of merged and unmerged firms.

(f) Evaluate the equilibrium profits in part (e) when $\gamma = 0$ and when $\gamma = 1$:

(g) Under what condition on k can the partial merger in part (c) be sustained?

(h) Assume that $N = 10$. Find the minimum number of firms that merge, $\underline{k}$, which supports the merger when $\gamma = 0$, $\gamma = 0.25$, $\gamma = 0.5$, $\gamma = 0.75$, and $\gamma = 1$. Interpret.

3. Bertrand with Efficiency Gains

Consider a market with demand Q(p) = a - p, where 3 identical firms compete in prices. Each firm's total cost function is given by $TC_i(q_i) = cq_i$. Suppose two of the firms merge and that the total cost function of the firm that results from the merger is $TC_m(q_m)=c_mq_m$, with $c>(a+c_m)/2$.

(a) What is the market share of each firm before and after the merger?
(b) What is the H-index before and after the merger?

4. Cournot with and without Efficiency Gains

In an oligopolistic market, there are initially 4 firms, firm 1, firm 2, firm 3, and firm 4. The total cost of production of each firm i, with i=1,2,3,4 is equal to $C(q_i) = 3q_i$, where qi is the quantity of firm i. The firms choose their quantities simultaneously and independently. The total market demand for their product is p(Q) = 18 – Q, where Q is the total quantity of the product.

(a) Find the quantity and profits of each firm in equilibrium.
(b) Suppose now that firms 1, 2, and 3 are considering merging with each other. The total cost of production of the merged firm will be $C(q_{123}) = 3q_{123}$, where q_{123} is the quantity of output of the merged firm. Do the three firms have an incentive to merge or not?
(c) Suppose now that if firms 1, 2, and 3 merge, the total cost of production of the merged firm is $C(q_{123}) = 2q_{123}$, where q_{123} is the quantity of output of the merged firm. Do the three firms have an incentive to merge or not?