ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
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# INDUSTRIAL ECONOMICS

## PRACTICE PROBLEM SET IV: COLLUSION

## Solution of 1

(i) In a period without tacit collusion we have three firms with homogeneous products, the same cost and price competition, thus, Bertrand with n firms. As we know (we have covered this case in class), in equilibrium all the firms set their prices equal to marginal cost, i.e., $P_i = 6$. It follows that the profits of each firm in equilibrium in a period without tacit collusion are equal to zero: $\pi_i^{NC} = 0$.

In a period with tacit collusion, the firms behave as a monopolist. Thus, they all set the price of the monopolist and they share the monopolist profits. In particular, the monopolist solves the following maximization problem:

$$\text{Max}_Q \ \pi = (20\text{-}Q)Q - 6Q$$

Differentiating $\pi$ in terms of Q, we have:

$$20 - 2Q - 6 = 0.$$

Thus: 14 – 2Q. It follows that the monopolist would produce: $Q^M = 7$.
In turn, the price would be (after substituting Q into P = 20 – Q): $P^M = 13$.
The resulting monopolist profits are:

$$(20 - 7)*7 - 6*7 = 13*7 - 6*7 = 7*7 = 49$$

Hence, the monopolist profits would be $\pi^M = 49$.
Since there are three firms in the market they will share these profits, i.e., in a period with tacit collusion each of the firms will get:

$$\pi_i^C = 49/3 = 16,33.$$

(ii) The condition for tacit collusion (we have seen this in class) is that $\delta > (\pi^D - \pi^C)/(\pi^D - \pi^P)$. We have already found $\pi^C = 16,33$. We have also found $\pi^D = \pi^M = 49$ and $\pi^P = \pi_i^{NC} = 0$. Thus:

$$\delta > (49 - 16,33)/(49 - 0) = 32,67/49 = 0.66$$

(iii) If they merge, then we have duopoly in the market. The profits of each firm in a period without tacit collusion will be the same as above, i.e., $\pi^P = \pi_i^{NC} = 0$.
The profits in a period of deviation will be again the ones of the monopolist, i.e., $\pi^D = \pi^M = 49$.
What will change though are the profits in a period of tacit collusion since 2 instead of 3 firms will be sharing the monopoly profits, i.e., $\pi^C = 49/2 = 24,5$.
It follows that the condition $\delta > (\pi^D - \pi^C)/(\pi^D - \pi^P)$ then becomes: $\delta > 0.5$.
So the answer changes; It is easier to have collusion in the market when there are 2 firms than when there are 3 firms because when there are fewer firms in the market each firm gets higher profits in a period of collusion (it splits the profits of monopoly with fewer firms) and thus its incentives for collusion are higher.

(iv) Now when the two firms merge we have duopoly in th market with asymmetric firms. The merged firm 12 has marginal cost 0 while firm 3 continues to have marginal cost 6. In this case, in a period of collusion they would behave as a monopolist. As we saw above this would mean that they will both set P = 13 and each of them will produce half of the monopoly quantity

Q/2 = 7/2 = 3,5. Their profits though will not be the same since they have different marginal costs. In particular, in a period with tacit collusion the profits of the merged firm 12 will be $\pi_{12}^C$ =13*3,5 = 45,5>24,5 while those of firm 3 will be $\pi_3^C$ 13*3,5 – 6*3,5 = 45,5 – 21 = 24,4 . In turn, the profits of firm 12 in case of deviation will be $\pi_{12}^D$ = 13*7 = 91.
The profits of firm 12 in case of punishment – no tacit collusion will be the ones in equilibrium with price competition and asymmetric costs (we have seen this in class):

It will set its monopoly price (the one it would set if it was a monopolist with marginal cost 0) if this price is lower than the marginal cost of its rival, i.e., lower than 6, while if it is higher than 6 it will set 6-ε where ε is a very small number. So, we need to check its monopoly price: $\text{Max}_Q\ \pi = (20-Q)Q$. Differentiating π in terms of Q, we have: 20 – 2Q = 0. Thus, the monopolist would produce $Q^M$ = 10. In turn, the price would be (after substituting Q into P = 20 – Q), $P^M$ = 10>6. It follows that firm 12 in equilibrium it would set in case of no tacit collusion $P_{12}$ = 6-ε and its rival would set $P_3$ = 6. In turn, the quantity and the profits of firm 12 in period of no tacit collusion (punishment period) would be: $q_{12}^P$ = 20-6 = 14 and $\pi_{12}^P$ = 6*14 = 84>0. It follows from this that for firm 2 its profits under collusion would be higher than in part b but its profits under deviation would be higher than in part b and its profits under punishment would be higher than in part b. Substituting for firm 12 into $\delta > (\pi^D - \pi^C)/(\pi^D - \pi^P)$, we find: δ > (91 – 45,5)/(91 - 84) = 45,5/7>1.

So in this setting firm 12 would ALWAYS deviate. The explanation has to do with the fact that as we discussed in class, when there is cost asymmetry, the firm with the lower cost (the most efficient) has high incentives to deviate because if it does so it will get higher profits (than if costs were symmetric) and it will not get punished severely (since the rival has a higher marginal cost). The higher is cost asymmetry thus, the less likely is tacit collusion.

## Solution of 2

(i)

- Tacit collusion

First we solve for monopoly:

$$\max_P \pi = (20-P)P - 4(20-P)$$

$$\frac{\partial \pi}{\partial P} = 20 - 2P + 4 = 0$$

$$\rightarrow \quad P_1^c = P_2^c = P_3^c = P_4^c = P_1^M = \frac{24}{2} = 12$$

$$Q^c = Q^M = 20 - 12 = 8$$

$$\rightarrow \quad q_i^c = \frac{Q^c}{4} = \frac{8}{4} = 2$$

$$\pi_i^c = 12 \cdot 2 - 4 \cdot 2 = 16$$

The profit of firm 1 in a period with tacit collusion are equal to $\pi_1^c$ =16.

- Deviation:

Firm 1 charges a price $p^d = p^c - \varepsilon = 12 - \varepsilon$, where ε a very small number.

It sells the monopoly quantity on its own and makes in this case monopoly profits:

$$\pi_1^d = (12 - 4)8 = 64$$

- Punishment:

When firm 1 deviates then firm 2, 3 and 4 react by setting the price equal to their marginal cost:

$$p^p = 5$$

So firm 1 sets in punishment period $p_1 = 5 - \varepsilon$ and sells all the quantity (since it has a lower price than the other firms). The quantity is:

$$Q = 20 - 5 = 15$$

So the profits that it makes in a period of punishment is:

$$\pi_1^p = (5 - 4)15 = 15$$

(ii)

In order to have tacit collusion, the following condition should be satisfied:

$$\delta > (\pi_1^d - \pi_1^c)/(\pi^{1d} - \pi_1^p) = (64 - 16)/(64 - 15) = 0{,}97$$

## Solution of 3

(i)

If the firms collude, they maximize their joint profits, which is equivalent to pricing as if they were a monopoly. Setting up a monopolist's profit maximization function in this regard,

$$\max_{Q \geq 0} (1 - Q)Q - cQ$$

with accompanying first-order condition

$$\frac{\partial \pi}{\partial Q} = 1 - 2Q - c = 0.$$

Solving this expression for $Q$ provides our equilibrium output level for a monopolist in this setting (i.e., our aggregate output)

$$Q^C = \frac{1 - c}{2}.$$

Since there are 2 identical firms, they each produce $q_i^C = \frac{Q^C}{2}$ of the aggregate quantity, or

$$q_i^C = \frac{Q^C}{2} = \frac{1 - c}{4}.$$

To find our equilibrium profit level, we must first derive our equilibrium price, which we obtain by inserting the aggregate output into the inverse demand function

$$p^C = 1 - Q^C = \frac{1 + c}{2}.$$

Lastly, we insert our equilibrium price and output into the profit function to find the equilibrium profit level of every firm $i$

$$\pi_i^C = (p^C - c)q_i^C = \left(\frac{1+c}{2} - c\right)\left(\frac{1-c}{4}\right) = \frac{(1-c)^2}{8}.$$

(ii)

- *Profits from collusion.* If each firm colludes, they receive profits of $\pi_i^* = \frac{(1-c)^2}{8}$ in each period, leading to a present discounted value of lifetime profits of

$$\begin{aligned} &\pi_i^C + \delta\pi_i^C + \delta^2\pi_i^C + \dots \\ &= \pi_i^C(1 + \delta + \delta^2 + \dots) \\ &= \pi_i^C\left(\frac{1}{1-\delta}\right) \\ &= \frac{(1-c)^2}{8(1-\delta)}. \end{aligned}$$

- *Profits from deviating.* To find our minimum discount factor, we must first determine the profit level of firm $i$'s most profitable deviation if the other firm colludes, $\pi_i^D$, and then the profit level of firm $i$ for every period after, $\pi_i^N$. Setting up firm $i$'s profit maximization problem,

$$\max_{q_i \geq 0} \ (1 - q_i - q_j)q_i - cq_i$$

with accompanying first-order condition

$$\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - q_j - c = 0,$$

which we can arrange into firm $i$'s best response function to the quantities produced by the other firm

$$q_i = \frac{1-c}{2} - \frac{1}{2}q_j.$$

If the other firm colludes, then $q_j^D = \frac{1-c}{4}$. Substituting this expression into firm $i$'s best response function, we obtain

$$q_i = \frac{1-c}{2} - \frac{1}{2}\left(\frac{1-c}{4}\right),$$

and simplifying, we find firm $i$'s optimal deviation

$$q_i^D = \frac{3(1-c)}{8}.$$

This leads to a price of

$$\begin{aligned} p^D &= 1 - Q = 1 - q_i^D - q_j^D \\ &= 1 - \frac{3(1-c)}{8} - \frac{1-c}{4} \\ &= \frac{3+5c}{8} \end{aligned}$$

and profits

$$\begin{aligned} \pi_i^D &= (p^D - c)q_i^D = \left(\frac{3+5c}{8} - c\right)\left(\frac{3(1-c)}{8}\right) \\ &= \frac{9(1-c)^2}{64}. \end{aligned}$$

*Profits from reversion to the Nash equilibrium.* Next, we calculate the profits that firm $i$ receives after each firm returns to the Nash equilibrium of the stage game. In a symmetric equilibrium, every firm produces the same output level, $q_i = q_j = q$. Inserting this property into firm $i$'s best response function,

$$q_i = \frac{1-c}{2} - \frac{1}{2}q_j,$$

we find

$$q = \frac{1-c}{2} - \frac{1}{2}q.$$

Solving this expression for $q$ yields each firm's equilibrium output level

$$q^N = \frac{1-c}{3}$$

with aggregate output

$$Q^N = 2q^N = \frac{2(1-c)}{3},$$

equilibrium price

$$p^N = 1 - Q^N = 1 - \frac{2(1-c)}{3} = \frac{1+2c}{3},$$

and lastly, equilibrium profits

$$\pi_i^N = (p^N - c)q^N = \left(\frac{1+2c}{3} - c\right)\left(\frac{1-c}{3}\right) = \frac{(1-c)^2}{9}.$$

Now, if firm $i$ deviates in the first period, it earns a profit of $\pi_i^D$ in that period, but a profit of $\pi_i^N$ in every period afterward as each firm reverts back to the Nash equilibrium of the stage game. Its present discounted value of lifetime profits is

$$\begin{aligned}
&\pi_i^D + \delta\pi_i^N + \delta^2\pi_i^N + \dots \\
&= \pi_i^D + \delta\pi_i^N(1 + \delta + \delta^2 + \dots) \\
&= \pi_i^D + \pi_i^N\left(\frac{\delta}{1-\delta}\right) \\
&= \frac{9(1-c)^2}{64} + \frac{\delta(1-c)^2}{9(1-\delta)}.
\end{aligned}$$

*Calculating the minimal discount factor,* $\underline{\delta}$. Firm $i$ has incentives to collude if its present discounted value of lifetime profits is at least as high as deviating, i.e.,

$$\begin{aligned}
\pi_i^C\left(\frac{1}{1-\delta}\right) &\geq \pi_i^D + \pi_i^N\left(\frac{\delta}{1-\delta}\right) \\
\frac{(1-c)^2}{8(1-\delta)} &\geq \frac{9(1-c)^2}{64} + \frac{\delta(1-c)^2}{9(1-\delta)}.
\end{aligned}$$

Rearranging, we find

$$\begin{aligned}
\frac{1}{8} &\geq \frac{9(1-\delta)}{64} + \frac{\delta}{9} \\
\left(\frac{9}{64} - \frac{1}{9}\right)\delta &\geq \frac{9}{64} - \frac{1}{8} \\
\frac{17}{576}\delta &\geq \frac{1}{64}.
\end{aligned}$$

Finally, solving for $\delta$, we obtain the minimal discount factor sustaining collusion as follows:

$$\delta \geq \frac{9}{17} \equiv \underline{\delta}.$$

Therefore, collusion can be sustained if firms assign a sufficiently high value to future profits. Otherwise, every firm wants to deviate and collusion cannot be sustained.

## Solution of 4

- Tacit collusion

First we solve for monopoly:

$$\max_Q \pi = (12-Q)Q - 2Q$$

$$\frac{\partial \pi}{\partial Q} = 12 - 2Q - 2 = 0$$

$$\rightarrow Q^m = \frac{10}{2} = 5$$

$$\rightarrow q_1^E = q_2^E = \frac{5}{2} = 2{,}5 \rightarrow p^E = 12 - 5 = 7$$

$$\rightarrow \pi_A^E = 7 \cdot 2{,}5 - 2(2{,}5) = 12{,}5$$

$$\pi_B^E = 7 \cdot 2{,}5 - 4(2{,}5) = 7{,}5$$

- Deviation:

If firm A deviates:

$$\max_{q_A} \pi_A = (12 - q_A - 2{,}5)\, q_A - 2 q_A$$

$$\frac{\partial \pi_A}{\partial q_A} = 9{,}5 - 2 q_A - 2 = 0 \rightarrow q_A^{\pi} = \frac{7{,}5}{2} = 3{,}75$$

$$\rightarrow p_A^{\pi} = 12 - 3{,}75 - 2{,}5 = 5{,}75$$

$$\rightarrow \pi_A^{\pi} = 5{,}75 \cdot 3{,}75 - 2(3{,}75) = 14{,}06$$

If firm B deviates:

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$$\max_{q_B} \pi_B = (12 - q_B - 2{,}5)\, q_B - 4 q_B$$

$$\frac{\partial \pi_B}{\partial q_B} = \ldots = 0 \rightarrow q_B^{\pi} = 2{,}75$$

$$\rightarrow p_B^{\pi} = 6{,}75$$

$$\pi_B^{\pi} = 7{,}56$$

- Punishment:

Cournot with asymmetric costs:

$$\max_{q_A} \pi_A = (12 - q_A - q_B)\, q_A - 2 q_A$$

$$\frac{\partial \pi_A}{\partial q_A} = \ldots = 0 \rightarrow q_A(q_B) = \ldots$$

$$\max_{q_B} \pi_B = (12 - q_B - q_A)\, q_B - 4 q_B$$

$$\frac{\partial \pi_B}{\partial q_A} = \ldots = 0 \quad q_B(q_A) = \ldots$$

Solving the system of the 2 Best-response functions:

$$q_A^T = 4 \quad q_B^T = 2 \rightarrow p^T = 6$$

$$\pi_A^T = 16 \quad \& \quad \pi_B^T = 4$$