



## INDUSTRIAL ECONOMICS

### PRACTICE PROBLEM SET III: DYNAMIC OLIGOPOLY – PRODUCT DIFFERENTIATION

#### Solution of 1

(i)

$P(Q) = 150 - 4(q_1 + q_2)$ , then  $MR_a = 150 - 4q_b - 8q_a$ . By equating  $MR_a = MC = 54$ , we get

$$q_a = 12 - \frac{1}{2}q_b$$

Analogously,

$$q_b = 12 - \frac{1}{2}q_a$$

Using both reaction curves, we substitute one into the other to get the equilibrium outputs. For firm A,

$$\begin{aligned} q_a &= 12 - \frac{1}{2}q_b \\ &= 12 - \frac{1}{2}\left(12 - \frac{1}{2}q_a\right) \\ \Rightarrow q_a^* &= 8 \end{aligned}$$

Likewise,

$$q_b^* = 8$$

Recalling that  $P(Q) = 150 - 4(q_a + q_b)$ , then

$$P^* = 150 - 4(8 + 8) = 86$$

(ii)

Solving using backward induction, firm B's reaction curve is again:

$$q_b = 12 - \frac{1}{2}q_a$$

Firm A internalizes firm B's reaction curve into its demand, to find its (inverse) effective demand,  $q_a^e = Q - q_b$ . Notice that  $Q = 37.5 - \frac{1}{4}P$ . Then

$$\begin{aligned} q_a^e &= \left(37.5 - \frac{1}{4}P\right) - \left(12 - \frac{1}{2}q_a^e\right) \\ q_a^e &= 25.5 - \frac{1}{4}P + \frac{1}{2}q_a^e \\ \frac{1}{4}P &= 25.5 - \frac{1}{2}q_a^e \\ P &= 102 - 2q_1 \end{aligned}$$

Then, the effective marginal revenue for firm 1 is  $MR_1 = 102 - 4q_1$ . By equating  $MR_1 = MC$ , we get:

$$\begin{aligned} 102 - 4q_1 &= 54 \\ \Rightarrow q_1^* &= 12 \end{aligned}$$

Replacing  $q_1^* = 12$  into firm 2's reaction curve:

$$q_2^* = 12 - \frac{1}{2}(12) = 6$$

Recalling that  $P(Q) = 150 - 4(q_1 + q_2)$ , then

$$P^* = 150 - 4(12 + 6) = 78$$

## Solution of 2

(i)

### • Stage 2

Firm 2 observes  $p_1$  & chooses  $p_2$ .

Its best response/reply is:

-if  $p_1 > c$ , the best response is  $p_2 = p_1 - \epsilon > c$

-if  $p_1 = c$ , " " " "  $p_2 = c$

-if  $p_1 < c$ , " " " "  $p_2 = c$

### • Stage 1

Firm 1 knows that:

-if it sets  $p_1 > c \rightarrow p_2 = p_1 - \epsilon$   
 $\rightarrow \pi_1 = 0 \rightarrow$  no sales

-if it sets  $p_1 = c \rightarrow p_2 = c$   
 $\rightarrow \pi_1 = 0 \rightarrow$  sales

-if it sets  $p_1 < c \rightarrow p_2 = c$   
 $\rightarrow \pi_1 < 0$

$\rightarrow$  A SINE is  $(p_1^*, p_2^*) = (c, c)$

This is the only/unique SINE if a firm prefers making sales than no making sales. Otherwise, there are multiple equilibria in which  $p_1 \geq c$ .

(ii)

• If  $c_2 > p_1^M$ :

Stage 2:

Firm 2 observes  $p_1$  & chooses  $p_2$ .

Its best response is:

- if  $p_1 > c_2 \rightarrow p_2 = p_1 - \epsilon > c_2$

- if  $p_1 = c_2 \rightarrow p_2 = c_2$

- if  $p_1 < c_2 \rightarrow p_2 = c_2$

Stage 1:

Firm 1 knows that  $p_1^M$  is the monopoly price, i.e., it is the price that max. the monopolist profits (max. profits)

It also knows that if it sets  $p_1 = p_1^M$

then the best reply of firm 2 will be in

stage 2 to set  $p_2 = c_2$  since  $p_1 = p_1^M < c_2$ .

Thus, the optimal strategy of firm 1 is to

set  $p_1 = p_1^M \rightarrow$  it will obtain monopoly profits

so the SWE is  $(p_1^*, p_2^*) = (p_1^M, c_2)$ .

• If  $c_2 < p_1^M$ : Stage 2 is the same as above. But now if firm 1 sets in stage 1,  $p_1 = p_1^M$  it knows that firm 2 will set in stage 2:  $p_2 = p_1 - \epsilon$  since  $p_1 = p_1^M > c_2 \rightarrow \pi_2 = 0$ . So, now it is not optimal to set  $p_1 = p_1^M$ .

The SWE is now:  $(p_1^*, p_2^*) = (c_2 - \epsilon, c_2)$ . There is no profitable deviation from this.

### Solution of 3

(i)

$$(a) \quad \text{Max}_{P_i} \pi_i = (10 - P_i + s P_j) P_i$$

$$\frac{\partial \pi_i}{\partial P_i} = 10 - 2P_i + s P_j = 0$$

$$\rightarrow P_i = R_i(P_j) = \frac{10 + s P_j}{2}$$

$$\rightarrow P_1 = \frac{10 + s P_2}{2} \quad \& \quad P_2 = \frac{10 + s P_1}{2}$$

$$\rightarrow 2P_1 = 10 + s \frac{10 + s P_2}{2} \quad \rightarrow \quad 4P_1 = 20 + 10s + s^2 P_2$$

$$\rightarrow P_1 (4 - s^2) = 10(2 + s)$$

$$\rightarrow P_1 = \frac{10(2+s)}{(2-s)(2+s)}$$

$$\rightarrow P_1^* = P_2^* = \frac{10}{2-s}$$

$$\rightarrow q_1 = 10 - \frac{10}{2-s} + s \frac{10}{2-s} = 10 - \frac{10}{2-s} (1-s)$$

$$= \frac{10(2-s) - 10(1-s)}{2-s} = \frac{10(2-s-1+s)}{2-s}$$

$$\rightarrow q_1^* = q_2^* = \frac{10}{2-s}$$

$$\rightarrow \pi_1 = q_1^* P_1^* = \left(\frac{10}{2-s}\right) \frac{10}{2-s} \quad \rightarrow \quad \pi_1^* = \pi_2^* = \left(\frac{10}{2-s}\right)^2$$

(ii)

(b) Stage 2

$$\text{Max}_{P_2} \pi_2 = (10 - P_2 + 5P_1) P_2$$

$$\frac{\partial \pi_2}{\partial P_2} = 10 - 2P_2 + 5P_1 = 0$$

$$\rightarrow P_2 = R_2(P_1) = \frac{10 + 5P_1}{2}$$

Stage 1

$$\text{Max}_{P_1} \pi_1 = \left( 10 - P_1 + 5 \frac{10 + 5P_1}{2} \right) P_1$$

$$= \frac{20 - 2P_1 + 105 + 25P_1}{2} P_1$$

$$\frac{\partial \pi_1}{\partial P_1} = \frac{1}{2} (20 - 4P_1 + 105 + 25P_1) = 0$$

$$\rightarrow P_1 (4 - 25) = 20 + 105$$

$$\rightarrow 2P_1 (2 - 5) = 10 (2 + 5)$$

$$P_1^* = \frac{10(2+5)}{2(2-5)} = \frac{5(2+5)}{2-5}$$

$$\rightarrow P_2 = \frac{10 + 5 \frac{10(2+5)}{2(2-5)}}{2} = \dots$$

$$P_2^* = \frac{5(4+25-5^2)}{2(2-5^2)}$$

$$\rightarrow q_1 = 10 - P_1^* - 5P_2^* \rightarrow \dots \quad q_1^* = \frac{5(2+5)}{2}$$

$$\rightarrow q_2 = 10 - P_2^* - 5P_1^* \rightarrow \dots \quad q_2^* = \frac{5(4+25-5^2)}{2(2-5^2)}$$

$$\rightarrow \pi_1 = p_1^* q_1^* = \dots$$

$$\pi_1^* = \frac{25(2+g)^2}{2(2-g^2)}$$

$$\rightarrow \pi_2 = p_2^* q_2^* = \dots$$

$$\pi_2^* = \frac{25[4 + (2-g)g]^2}{4(2-g^2)^2}$$

$$\rightarrow \pi_1^* - \pi_2^* = \dots = \frac{-25g^3(4+3g)}{4(2-g^2)^2} < 0$$

$$\rightarrow \pi_1^* < \pi_2^*$$

So the leader would prefer to be follower instead of leader. //

## Solution of 4

(i)

1) Look at the following graph:

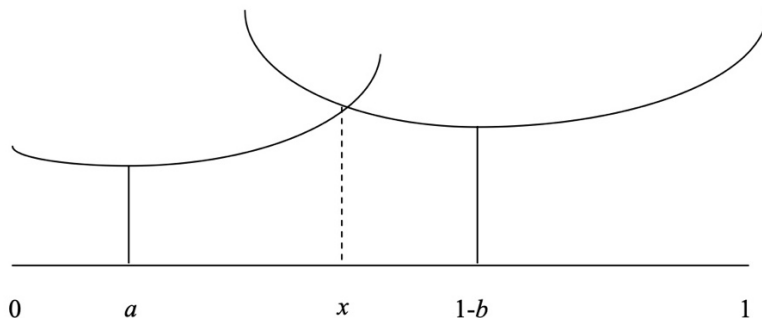


Given that  $1-a-b \geq 0$ , firm 1 is located on the left of firm 2.

If  $a=1-b$ , (i.e.  $a+b=1$ ) the two locations coincide: this is the case of perfect substitutes (minimum differentiation); if  $a=b=0$ , firms are located at the extremes of the unitary segment and there is maximal differentiation.

(ii)

2) We first compute the indifferent consumer (called  $x$ ):



$$p_1 + t(x-a)^2 = p_2 + t(x-1+b)^2$$

This yields:

$$x = \frac{p_2 - p_1}{2t(1-b-a)} + \frac{1-b+a}{2}$$

Firm 1 faces the following demand, as a function of the prices:

$$D_1 = x = \frac{p_2 - p_1}{2t(1-b-a)} + \frac{1-b+a}{2}$$

Firm 2 faces the following demand, as a function of the prices:

$$D_2 = 1-x = 1 - \frac{p_2 - p_1}{2t(1-b-a)} - \frac{1-b+a}{2}$$

(iii)

3) To find the Nash equilibrium in prices, we maximize firms' profits with respect to the prices:

$$\text{Firm 1: } \max_{p_1} \pi_1 = (p - AC_1)D_1 = (p_1 - c) \left( \frac{p_2 - p_1}{2t(1-b-a)} + \frac{1-b+a}{2} \right)$$

From  $FOC_{p_1}$  we derive firm 1's best reply function:

$$p_1^*(p_2) : p_1 = \frac{p_2 + c + t(1-b-a)(1-b+a)}{2}$$

$$\text{Firm 2: } \max_{p_2} \pi_2 = (p - AC_2)D_2 = (p_2 - c) \left( 1 - \frac{p_2 - p_1}{2t(1-b-a)} - \frac{1-b+a}{2} \right)$$

$$\text{Firm 2: } \max_{p_2} \pi_2 = (p - AC_2)D_2 = (p_2 - c) \left( 1 - \frac{p_2 - p_1}{2t(1-b-a)} - \frac{1-b+a}{2} \right)$$

From  $FOC_{p_2}$  we derive firm 2's best reply function:

$$p_2^*(p_1) : p_2 = \frac{p_1 + c + t(1-b-a)(1+b-a)}{2}$$

Solving the system of best replies we get:

$$p_2^* = c + t(1-b-a) \left( 1 + \frac{b-a}{3} \right)$$

$$p_1^* = c + t(1-b-a) \left( 1 + \frac{a-b}{3} \right)$$

## Solution of 5

(i)

(a) *Third stage—Finding demand.* For given locations from the first stage, and given prices from the second stage, find the demand that each firm has in the third stage.

- If a consumer purchases from firm 1, his utility is  $r - p_1 - t(x - l_1)^2$ , while purchasing from firm 2 yields  $r - p_2 - t(x - l_2)^2$ . Therefore, the indifferent consumer  $\hat{x}$  solves

$$r - p_1 - t(\hat{x} - l_1)^2 = r - p_2 - t(\hat{x} - l_2)^2,$$

which yields

$$\hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2t(l_2 - l_1)}.$$

Firm 1's demand is  $\hat{x}$ , while firm 2's demand is  $1 - \hat{x}$ .

- When both firms set the same prices,  $p_1 = p_2$ , firm 1's demand simplifies to  $\hat{x} = \frac{l_1 + l_2}{2}$ , while firm 2's demand becomes  $1 - \hat{x} = 1 - \frac{l_1 + l_2}{2}$ .

(ii)

(b) *Second stage—Prices.* For given locations from the first stage, find the price that each firm sets in the second stage.

- *Finding firm 1's best response function.* Firm 1 chooses the price  $p_1$  that solves

$$\max_{p_1 \geq 0} (p_1 - c) \underbrace{\left( \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2t(l_2 - l_1)} \right)}_{\text{Demand, } \hat{x}}.$$

Differentiating with respect to  $p_1$ , we obtain

$$\left( \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2t(l_2 - l_1)} \right) + (p_1 - c) \left( -\frac{1}{2t(l_2 - l_1)} \right) = 0.$$

Solving for  $p_1$ , we find firm 1's best response function

$$p_1(p_2) = \frac{t(l_1 + l_2)(l_2 - l_1) + c}{2} + \frac{1}{2}p_2$$

with vertical intercept at  $\frac{t(l_1 + l_2)(l_2 - l_1) + c}{2}$  and slope  $\frac{1}{2}$ . Intuitively, when firm 2 increases its price by \$1, firm 1 responds by increasing its own by \$0.5.

- In addition, a marginal increase in firm 1's location,  $l_1$ , or in firm 2's location,  $l_2$ , yields the following change in the above best response functions:

$$\frac{\partial p_1(p_2)}{\partial l_1} = -2tl_1 < 0 \quad \text{and} \quad \frac{\partial p_1(p_2)}{\partial l_2} = 2tl_2 > 0,$$

respectively. Therefore, when firm 1 moves its position rightward, its best response function shifts downward in a parallel fashion, indicating that the firm charges less for its product. Intuitively, its position is closer to firm 2's, attenuating product differentiation and ultimately decreasing the price that firm 1 can charge. In contrast, when firm 2 moves its position rightward, both firms move further away from each other, entailing more differentiated products. In this case, firm 1's best response function shifts upward, indicating a higher price  $p_1$ .

- *Finding firm 2's best response function.* Operating similarly for firm 2, we have that this firm chooses price  $p_2$  to solve

$$\max_{p_2 \geq 0} (p_2 - c) \underbrace{\left( 1 - \left( \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2t(l_2 - l_1)} \right) \right)}_{\text{Demand, } 1 - \hat{x}}.$$

Differentiating with respect to  $p_2$ , we obtain

$$\left( 1 - \left( \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2t(l_2 - l_1)} \right) \right) + (p_2 - c) \left( -\frac{1}{2t(l_2 - l_1)} \right) = 0.$$

Solving for  $p_2$ , we find firm 2's best response function

$$p_2(p_1) = \frac{t(l_2 - l_1)(2 - l_1 - l_2) + c}{2} + \frac{1}{2}p_1$$

with vertical intercept at  $\frac{t(l_2 - l_1)(2 - l_1 - l_2) + c}{2}$  and slope  $\frac{1}{2}$ , thus exhibiting the same intuition as the positively sloped best response function of firm 1.

- Similar to the case of firm 1, a marginal increase in firm 1's location,  $l_1$ , or in firm 2's location,  $l_2$ , yields

$$\frac{\partial p_2(p_1)}{\partial l_1} = -2t(1 - l_1) < 0 \quad \text{and} \quad \frac{\partial p_2(p_1)}{\partial l_2} = 2t(1 - l_2) > 0,$$

respectively. Therefore, when firm 1 moves its position rightward, it becomes closer to firm 2, attenuating product differentiation and ultimately decreasing the price that firm 2 charges (downward shift in firm 2's best response function). In contrast, when firm 2 moves its position rightward, both firms move further away from each other, entailing more differentiated products. In this case, firm 2's best response function shifts upward.

- *Finding equilibrium prices.* Simultaneously solving for  $p_1$  and  $p_2$  in the above best response functions, we find equilibrium prices

$$p_1^*(l_1, l_2) = c + \frac{t}{3}(l_2 - l_1)(2 + l_1 + l_2), \text{ and}$$

$$p_2^*(l_1, l_2) = c + \frac{t}{3}(l_2 - l_1)(4 - l_1 - l_2).$$

Following with our above discussion, note that when both firms are located at the same position,  $l_1 = l_2$ , equilibrium prices simplify to marginal cost pricing

$$p_1^*(l_1, l_2) = p_2^*(l_1, l_2) = c.$$

Our results also help us examine the case of pricing under exogenous product differentiation (e.g.,  $l_1 = 0$  and  $l_2 = 1$ ). In this setting, equilibrium prices become

$$p_1^*(l_1, l_2) = c + \frac{t}{3}(1 - 0)(2 + 0 + 1) = c + t, \text{ and}$$

$$p_2^*(l_1, l_2) = c + \frac{t}{3}(1 - 0)(4 - 0 - 1) = c + t.$$

Therefore, second-stage profits are

$$\pi_1^*(l_1, l_2) = \frac{t}{18}(l_2 - l_1)(2 + l_1 + l_2)^2 \text{ and}$$

$$\pi_2^*(l_1, l_2) = \frac{t}{18}(l_2 - l_1)(4 - l_1 - l_2)^2$$

which also collapse to zero when both firms are located at the same position,  $l_1 = l_2$ , and to  $\pi_1^*(l_1, l_2) = \pi_2^*(l_1, l_2) = \frac{t}{2}$  when firms' locations are exogenously determined at  $l_1 = 0$  and  $l_2 = 1$ .

(iii)

(c) *First stage—Equilibrium location.* Anticipating equilibrium behavior in the second and third stages, find the equilibrium location choice of each firm in the first stage of the game.

- *Finding firm 1's best response function.* In the first stage, firm 1 anticipates the equilibrium prices that firms charge in the second stage and chooses its location  $l_1$  to solve

$$\max_{l_1} \frac{t}{18}(l_2 - l_1)(2 + l_1 + l_2)^2.$$

Differentiating with respect to  $l_1$ , we find

$$-\frac{t}{18}(2 + l_1 + l_2)^2 + \frac{t}{9}(l_2 - l_1)(2 + l_1 + l_2) = 0,$$

which, solving for  $l_1$ , yields firm 1's best response function

$$l_1(l_2) = -\frac{2}{3} + \frac{1}{3}l_2.$$

- *Finding firm 2's best response function.* Similarly, firm 2 chooses location  $l_2$  to solve

$$\max_{l_2} \frac{t}{18}(l_2 - l_1)(4 - l_1 - l_2)^2.$$

Differentiating with respect to  $l_2$ , we find

$$\frac{t}{18}(4 - l_1 - l_2)^2 - \frac{t}{9}(l_2 - l_1)(4 - l_1 - l_2) = 0,$$

which, solving for  $l_2$ , yields firm 2's best response function

$$l_2(l_1) = \frac{4}{3} + \frac{1}{3}l_1.$$

- *Finding equilibrium location.* Simultaneously solving for  $l_1$  and  $l_2$  in the above best response functions yields

$$l_1 = -\frac{2}{3} + \frac{1}{3} \underbrace{\left( \frac{4}{3} + \frac{1}{3}l_1 \right)}_{l_2(l_1)}.$$

Solving for  $l_1$ , we obtain

$$l_1 = -\frac{1}{4}.$$

Inserting this result into firm 2's best response function, we find that

$$l_2 = \frac{4}{3} + \frac{1}{3} \left( -\frac{1}{4} \right) = \frac{5}{4}.$$

Therefore, firms differentiate their products as much as possible. In the interval  $[0, 1]$ , they locate at the endpoints of the line,  $l_1 = 0$  and  $l_2 = 1$ . From our above discussion, we know that these positions yield equilibrium prices

$$p_1^*(0, 1) = p_2^*(0, 1) = c + t$$

and equilibrium profits

$$\pi_1^*(0, 1) = \pi_2^*(0, 1) = \frac{t}{2}.$$