ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INDUSTRIAL ECONOMICS

PRACTICE PROBLEM SET III: DYNAMIC OLIGOPOLY – PRODUCT DIFFERENTIATION

1. Stackelberg (in quantities)

The market demand for a particular product is given by $Q = 37{,}5 - 0{,}25P$, where Q is the total quantity of the product. The product is produced by two firms, firm 1 and firm 2, which compete in the market. The total production cost faced by each firm i, with $i = 1, 2$, is $C(q_i) = 54q_i$.

(i) Assume that the two firms choose the quantities that they produce simultaneouly and separately. Find the quantity that each firm will produce and the price that it will charge in equilibrium.

(ii) Assume now that firm 1 chooses its quantity before firm 2 (and that firm 2 observes firm 1's quantity before choosing its own quantity). Find the quantity that each firm will produce and the price that it will charge in equilibrium.

2. Stackelberg (in prices with homogeneous product)

Consider a market with duopoly in which the firms choose their prices simultaneously and separately. The market demand function is: $D(p) = 1 - p$. If a firm chooses a lower price than its competitor, then it supplies the whole market, if both firms choose the same price, then they share the market. The total cost functions are the same for both firms and are given by: $C(q_i) = cq_i$, where $i = 1, 2$ and $0 < c < 1$.

(i) Assume that firm 1 chooses its price before firm 2 (and that firm 2 observes firm 1's price before choosing its own price). Find the price that each firm will charge in equilibrium.

(ii) Assume now that firms play the same game as in part (i) but that now the marginal cost of firm 1 is lower than the marginal cost of of firm 2, $c_1 < c_2$. Find the equilibrium prices when $c_1 > p_m$ as well as when $c_1 < p_m$, where p_m is the price that firm 1 would charge if it was a monopolist in the market.

3. Stackelberg (in prices with differentiated products)

Consider two firms competing à la Bertrand selling a differentiated good at a common marginal cost 0. Every firm i, where $i = 1, 2$, has demand function $q_i(p_i, p_j) = 10 - p_i + g\, p_j$.

(i) Assume that the two firms choose the prices that they produce simultaneouly and separately. Find the price and the profit of each firm in equilibrium.

(ii) Assume now that firm 1 chooses its price before firm 2 (and that firm 2 observes firm 1's price before choosing its own price). Find the price and the profit of each firm in equilibrium and compare it with that of part (i). Does a firm want to choose its price first or second?

4. Hoteling (with exogenous locations)

Consider a Hotelling linear market: consumers of total mass equal to 1 are uniformely distributed along a segment of length 1. Two firms, 1 and 2, located respectively in a and $(1-b)$, where $1-a-b \geq 0$, compete in prices.

(i) Give a graphical representation of the above situation. Discuss for which values of a and b the goods offered by the two firms are differentiated and for which values they are substitutes.

(ii) Assume that transportation costs equal t times the distance squared. Compute demands, as a function of the prices.

(iii) Find the equilibrium in prices, assuming marginal (and average) costs constant and equal to c for both firms.

5. Hoteling (with endogenous locations)

Consider the following model of horizontally differentiated products with two firms. In the first stage, every firm i chooses its location, l_i, in the interval [0, 1], where $i = \{1,2\}$. In the second stage, every firm, observing the location pair (l_1, l_2) from the first stage, responds setting a price p_i. In the third stage, given firms' location and prices, consumers buy one unit of the good from either firm 1 or 2. Consumers are, for simplicity, uniformly distributed in the unit line. Assume that consumers suffer quadratic transportation costs, and both firms' marginal production cost is $c > 0$.

(i) Find the demand faced by each firm.

(ii) Find the prices in equilibrium for given locations.

(iii) Find the equilibrium locations.