ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INDUSTRIAL ECONOMICS

PRACTICE PROBLEM SET II: STATIC OLIGOPOLY

Solution 1

(i)

$$\max_{Q} \ \pi = (25-Q)Q - 4Q$$

$$MC = \frac{\partial C}{\partial Q} = 4$$

$$MR = \frac{\partial[(25-Q)Q]}{\partial Q} = 25 - 2Q$$

$$MR = MC \rightarrow 25 - 2Q = 4 \rightarrow 2Q = 21 \rightarrow Q^M = 10{,}5$$

$$p^M = 25 - 10{,}5 = 14{,}5$$

$$\pi^M = 14{,}5 * 10{,}5 - 4 * 10{,}5 = 152{,}25 - 42 = 110{,}25$$

(ii)

$$\max_{q_1} \ \pi_1 = (25 - q_1 - q_2)q_1 - 4q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 25 - 2q_1 - q_2 - 4 = 0$$

$$\rightarrow q_1 = BR_1(q_2) = \frac{21 - q_2}{2}$$

Similarly (since firms are symmetric): $q_2 = BR_2(q_1) = \frac{21 - q_1}{2}$

Solving the system of the 2 best response functions:

$$2q_1 = 21 - \frac{21 - q_1}{2} \rightarrow 4q_1 = 42 - 21 + q_1 \rightarrow$$

$$3q_1 = 21 \rightarrow q_1^c = 7$$

$$\rightarrow q_2^c = q_1^c = 7$$

$$\rightarrow p^c = 25 - 7 - 7 = 25 - 14 = 11$$

$$\pi_1^c = \pi_2^c = 11 \cdot 7 - 4 \cdot 7 = 77 - 28 = 49$$

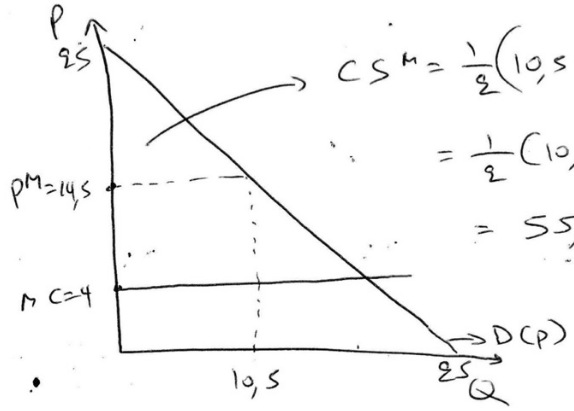
(iii)

$p^M = 14{,}5 > p^C = 11$

$Q^M = 10{,}5 < Q^C = 7 + 7 = 14$

Consumers surplus:

MONOPOLY

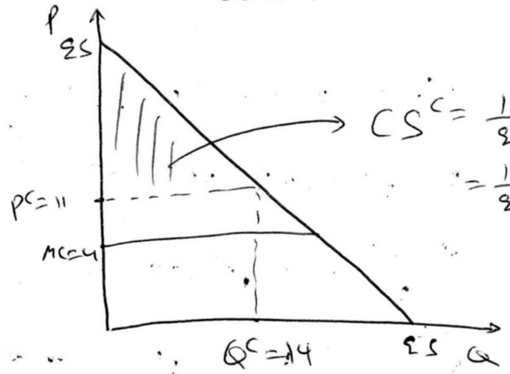


$CS^M = \frac{1}{2}\left(10{,}5 * (25 - 14{,}5)\right)$

$= \frac{1}{2}(10{,}5 * 10{,}5)$

$= 55{,}125$

OLIGOPOLY



$CS^C = \frac{1}{2}(14 * (25 - 11))$

$= \frac{1}{2}(14 * 14) =$

$= 98$

Thus: $CS^C > CS^M$

More consumer surplus in case (b) - Oligopoly than in case (a) - Monopoly

Solution 2

1. The Nash equilibrium of the game is obtained by solving the following system of equations:

$$\frac{\partial \pi_1}{\partial q_1} = (a - q_1 - q_2) - q_1 - c_1 = 0,$$
$$\frac{\partial \pi_1}{\partial q_2} = (a - q_1 - q_2) - q_2 - c_2 = 0.$$

The equilibrium is given by the pair $q_1^* = \frac{1}{3}(a - 2c_1 + c_2)$ and $q_2^* = \frac{1}{3}(a - 2c_2 + c_1)$. Given q_1^* and q_2^*, the equilibrium market shares are:

$$\alpha_1 = \frac{q_1^*}{q_1^* + q_2^*} = \frac{a - 2c_1 + c_2}{2a - c_1 - c_2},$$
$$\alpha_2 = \frac{q_2^*}{q_1^* + q_2^*} = \frac{a - 2c_2 + c_1}{2a - c_1 - c_2}.$$

2. It is easily seen from the first-order conditions that the equilibrium profits are equal to the square of the equilibrium quantities: $\pi_1^* = \frac{1}{9}(a - 2c_1 + c_2)^2$ and $\pi_2^* = \frac{1}{9}(a - 2c_2 + c_1)^2$. The consumer surplus is given by

$$CS^* = \int_0^{q_1^* + q_2^*} (a - q)dq - (a - q_1^* - q_2^*)(q_1^* + q_2^*)$$
$$= \frac{1}{2}(q_1^* + q_2^*)^2 = \frac{1}{18}(2a - c_1 - c_2)^2.$$

Adding the three together, social welfare is given by

$$W^* = CS^* + \pi_1^* + \pi_2^* = \frac{1}{2}(q_1^* + q_2^*)^2 + (q_1^*)^2 + (q_2^*)^2.$$

Solution 3

(i)

- Firm 1's profit maximization problem is

$$\max_{q_1 \geq 0} \ \pi_1 = (1 - q_1 - q_2 - q_3)q_1 - cq_1.$$

Differentiating with respect to q_1, we obtain

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - q_3 - c = 0$$

rearranging, we find $2q_1 = 1 - q_2 - q_3 - c$, and solving for q_1, we obtain firm 1's best response function

$$q_1(q_2, q_3) = \frac{1-c}{2} - \frac{1}{2}(q_2 + q_3)$$

which originates at a vertical intercept of $\frac{1-c}{2}$ and decreases at a rate of $\frac{1}{2}$ when either firm 2 or 3 marginally increases its output.

Repeat the process for firms 2 and 3 to obtain their best response functions. [*Hint*: You should find that all firms have symmetric best response functions.]

- *Firm 2.* Firm 2's profit maximization problem is

$$\max_{q_2 \geq 0} \ \pi_2 = (1 - q_1 - q_2 - q_3)q_2 - cq_2.$$

  Differentiating with respect to q_2, we obtain

$$\frac{\partial \pi_2}{\partial q_2} = 1 - q_1 - 2q_2 - q_3 - c = 0$$

  rearranging, we find $2q_2 = 1 - q_1 - q_3 - c$, and solving for q_2, we obtain firm 2's best response function

$$q_2(q_1, q_3) = \frac{1-c}{2} - \frac{1}{2}(q_1 + q_3)$$

  which is symmetric to firm 1's best response function. This comes at no surprise since all firms face the same inverse demand function and total cost function.

- *Firm 3.* Firm 3's profit maximization problem is

$$\max_{q_3 \geq 0} \ \pi_3 = (1 - q_1 - q_2 - q_3)q_3 - cq_3.$$

  Differentiating with respect to q_3, we obtain

$$\frac{\partial \pi_3}{\partial q_3} = 1 - q_1 - q_2 - 2q_3 - c = 0$$

rearranging, we find $2q_3 = 1 - q_1 - q_2 - c$, and solving for q_3, we obtain firm 3's best response function

$$q_3(q_1, q_2) = \frac{1-c}{2} - \frac{1}{2}(q_1 + q_2)$$

which is symmetric to firm 1's best response function. Again, this is not surprising since all firms face the same inverse demand function and total cost function.

Interpret firm 1's best response function: if firm 2 where to marginally increase its output, does firm 1 increase or decrease its own output? By how much?

- To find this, we differentiate firm 1's best response function with respect to firm 2's output q_2:

$$\frac{\partial q_1(q_2, q_3)}{\partial q_2} = -\frac{1}{2}.$$

  For each unit increase in firm 2's output, firm 1 decreases their output by half unit.

(ii)

Using the above three best response functions, find the point where they cross, the triplet (q_1^*, q_2^*, q_3^*) that characterizes the Nash equilibrium of this Cournot game.

- In a symmetric equilibrium, all firms produce the same output level, $q_1^* = q_2^* = q_3^* = q^*$. We can insert this property into firm 2's best response function to obtain

$$q^* = \frac{1-c}{2} - \frac{1}{2}\left(q^* + q^*\right)$$

which is equivalent to dropping the subscripts of all output levels. Simplifying, we find

$$2q^* = 1 - c - 2q^*,$$

or $4q^* = 1 - c$. Solving for q^*, we find that every firm produces an equilibrium output of

$$q^* = \frac{1-c}{4}.$$

- The price each firm faces is

$$\begin{aligned} p^* &= 1 - q_1 - q_2 - q_3 \\ &= 1 - 3q^* \\ &= 1 - 3\frac{(1-c)}{4} \\ &= \frac{1+3c}{4}. \end{aligned}$$

- Each firm earns the same profit in equilibrium, that is,

$$\pi = pq - cq$$

or, plugging in for p^* and q^*,

$$\begin{aligned} \pi^* &= \left(\frac{1+3c}{4}\right)\frac{1-c}{4} - c\frac{1-c}{4} \\ &= \frac{(1-c)^2}{16} \end{aligned}$$

Solution 4

(i)

- Every firm i chooses q_i to solve the profit maximization problem, as follows:

$$\max_{q_i \geq 0} \ \pi(q_i) = \left(1 - q_i - \gamma q_j\right) q_i - c q_i$$

Taking the first-order condition with respect to q_i, and assuming interior solutions,

$$1 - 2q_i - \gamma q_j - c = 0$$

Solving for q_i, we find that firm i's best response function is

$$q_i\left(q_j\right) = \frac{1-c}{2} - \frac{\gamma}{2} q_j$$

meaning that the best response function originates at a vertical intercept of $\frac{1-c}{2}$ and decreases in q_j at a rate $\frac{\gamma}{2}$. Graphically, an increase in parameter γ (goods are more homogeneous) produces an inward rotation of the best response function, leaving its vertical intercept unchanged at $\frac{1-c}{2}$ but making this function steeper. Intuitively, that means that, as firms sell more homogeneous products, a given increase in firm j's output yields a larger reduction in firm i's output level, so that goods become stronger strategic substitutes.

(ii)

- In a symmetric equilibrium, every firm i produces the same output level, $q_i = q_j$. Inserting this property in the above best response function, we obtain that

$$q_i = \frac{1-c}{2} - \frac{\gamma}{2} q_i.$$

Rearranging and solving for q_i yields an equilibrium output of

$$q_i^* = \frac{1-c}{2+\gamma}.$$

This output is decreasing in γ indicating that, as firms' output become more homogeneous (higher γ), each firm produces fewer units. As expected, equilibrium output is also decreasing in firms' common marginal cost, c.

- Inserting the equilibrium output, $q_i^* = \frac{1-c}{2+\gamma}$, into firm i's objective function, we find that its equilibrium profits are

$$\begin{aligned}
\pi_i^* &= \left(1 - \frac{1-c}{2+\gamma} - \frac{\gamma(1-c)}{2+\gamma} - c\right)\frac{1-c}{2+\gamma} \\
&= \left(\frac{(2+\gamma)(1-c) - (1+\gamma)(1-c)}{2+\gamma}\right)\frac{1-c}{2+\gamma} \\
&= \left(\frac{1-c}{2+\gamma}\right)^2
\end{aligned}$$

which is decreasing in the parameter measuring product homogeneity, γ, and in marginal cost, c.

Solution 5

(i) If $p_1<p_2$, then it follows:

$$q_1= D_1(p_1,p_2) = D(p_1) = 6 - p_1$$

Thus:

$$\pi_1 = p_1\times (6 - p_1) – 2 \times (6 - p_1)$$
$$= (p_1 – 2) \times (6 – p_1).$$

If $p_1 = p_2$, then it follows:

$$q_1= D_1(p_1,p_2) = (1/2) \times D(p_1) = (1/2) \times (6 – p_1)$$

Thus:

$$\pi_1 = p_1\times (1/2) \times (6 – p_1)$$
$$– 2 \times (1/2) \times (6 – p_1)$$
$$= (p_1 – 2) \times (1/2) \times (6 – p_1)$$

(ii) We know from part (b) that when $p_1 = p_2 =p^c >2$, then:

$$\pi_1^c = (p^c – 2) \times (1/2) \times (6 – p^c) = \pi_2^c > 0$$

Does any of the two firms have incentives to deviate unilaterally (i.e., change its price)?
If one of the firms, e.g., firm 1, changes its price to $p_1^d = p^c – \varepsilon > 2$, then:

$$q_1^d = (6 – p_1^d)$$
$$\pi_1^d = (p_1^d – 2) \times (6 – p_1^d) > \pi_1^c = (p^c – 2) \times (1/2) \times (6 – p^c)$$

So, firm 1 has incentives to deviate, and thus, $p_1 = p_2 = p^c >2$ cannot be a Nash equilibrium.

Solution 6

The firms have asymmetric costs: $MC_1 = 1 < MC_2 = 4$.
Is the asymmetry "small" or "large"? To answer this, we need first to find the price that firm 1 (the more efficient one) would charge if it was a monopolist in the market:

If firm 1 was a monopolist in the market:

$$\text{Max}_{p_1}\ \pi_1 = p_1(10 - p_1) – 1(10 - p_1)$$

The first order condition (foc) is:

$$\partial\pi_1/\partial p_1 = 10 - 2p_1 + 1 = 0$$

It follows from the above that the monopoly price would be:

$$p_1^M = 11/2 = 5{,}5$$

Clearly, since $p_1^M = 5{,}5 > MC_2 = 4$, it follows that the asymmetry is small, and thus, that firm 1 cannot behave as a monopolist.

Thus, the Nash equilibrium is:

$$(p_1, p_2) = (4 -\varepsilon, 4)$$

The resulting quantity of each firm is:

$$q_1 = 10 – p_1 = 10 – (4 – \varepsilon) = 6 + \varepsilon \cong 6$$
$$q_2 = 0$$

The resulting profits of each firm is:

$$\pi_1 = (4 – \varepsilon)\times 6 - 1\times 6 =18$$
$$\pi_2 = 0$$

Solution 7

(i)

$\pi_1(p_1,p_2) = TR_1 - TC_1$

Write in terms of the choice variables p_1 and p_2:

$$\pi_1(p_1,p_2) = p_1 * (25 - 5p_1 + 2p_2) - 2 - (25 - 5p_1 + 2p_2)$$

$$\frac{\partial}{\partial p_1}\pi_1(p_1,p_2) = (25 - 5p_1 + 2p_2) + p_1 * (-5) + 5$$

Set derivative equal to zero:

$$(25 - 5p_1 + 2p_2) + p_1 * (-5) + 5 = 0$$

Solve for p_1 to find best response function:

$$30 + 2p_2 = 10p_1$$

$$p_1(p_2) = \frac{15 + p_2}{5}$$

Firm 1's best response (profit maximizing response) is given by its best response function.
Firm 1 responds to its competitor's price increase by increasing its own price.
Following the same procedure for firm 2, we find:

$$30 + 2p_1 = 10p_2$$

$$p_2(p_1) = \frac{15 + p_1}{5}$$

(When firms are symmetric no need to solve separately for the best response function for each firm if the firms are symmetric. For symmetric firms, you only need solve the profit maximization problem once because you know the answer will be the same for all of the symmetric firms.)

Solve for the intersection of the best response functions:

$$p_1(p_2) = \frac{15 + p_2}{5}$$

$$p_1\left(\frac{15 + p_1}{5}\right) = \frac{15 + \frac{15+p_1}{5}}{5}$$

$$p_1 = \frac{75 + 15 + p_1}{25}$$

$$24p_1 = 90$$

$$p_1^* = \$3.75$$

$$p_2(p_1) = \frac{15 + p_1}{5}$$

$$p_2(p_1^*) = \frac{15 + 3.75}{5}$$

$$p_2^* = \$3.75$$

This corresponds to the following quantities:

$$q_1 = 25 - 5 * 3.75 + 2 * 3.75 = 13.75 \quad q_2 = 25 - 5 * 3.75 + 2 * 3.75 = 13.75$$

(ii)

$$L_1 = \frac{3.75 - 1}{3.75} = 0.73$$

$$L_2 = \frac{3.75 - 1}{3.75} = 0.73$$

This shows that these firms have substantial market power: 73% of the price charged to consumers is pure mark-up.

(iii)

In this case we have allowed for heterogenous goods that are not perfect substitutes for each other. Hence neither firm can capture the entire market by slightly undercutting the other.