



INDUSTRIAL ECONOMICS

PRACTICE PROBLEM SET II: STATIC OLIGOPOLY

1. Cournot Oligopoly vs. Monopoly

The market demand for a particular product is given by $p(Q) = 25 - Q$, where Q is the total quantity of the product.

- (i) Assume that the product is produced by a single firm. The total production cost faced by the firm is $C(Q) = 4Q$. Find the quantity that the firm will produce, the price that it will charge and the profit that it will make in equilibrium.
- (ii) Assume now that the same product is produced by two firms, firm 1 and firm 2. The total production cost faced by each firm i , with $i=1, 2$, is $C(q_i) = 4q_i$. Assume that the two firms choose the quantities that they produce simultaneously and separately. Find the quantity that each firm will produce, the price that it will charge and the profit that it will make in equilibrium.
- (iii) Compare cases (i) and (ii) above in terms of the total quantity in the market, the price in the market, and the consumer surplus.

2. Cournot Oligopoly with Asymmetric Marginal Costs

Consider two firms that produce a homogeneous good and choose their quantities simultaneously and separately. The demand function for the good is given by $P = a - q_1 - q_2$, where q_1 and q_2 are the quantities of firm 1 and firm 2 respectively. The cost functions of the two firms are $C_1(q_1) = c_1q_1$ and $C_2(q_2) = c_2q_2$, where $c_1 < c_2$ and $c_2 < (a + c_1)/2$.

- (i) Find the Nash equilibrium of the game. What are the market shares of the two firms?
- (ii) Given your answer to (i), find the equilibrium profits, consumer surplus and total surplus (welfare).

3. Cournot Oligopoly with $n=3$ Symmetric Firms

Consider a market with three firms producing a homogeneous good and facing demand function $p(Q) = 1 - Q$, where $Q = q_1 + q_2 + q_3$ denotes aggregate quantity. All firms face a constant marginal cost of production given by c . The three firms choose their quantities simultaneously and separately.

- (i) Find each firm's best response function and interpret it.
- (ii) Find the equilibrium quantities, price and profits.

4. Cournot Oligopoly with Differentiated Products

Consider two firms competing à la Cournot selling a differentiated good at a common marginal cost c , where $0 \leq c < 1$. Every firm i has an inverse demand function $p_i(q_i, q_j) = 1 - q_i - \gamma q_j$, where $\gamma \in [0, 1)$ denotes the inverse degree of horizontal product differentiation.

- (i) Find firm i 's best response function. How is it affected by increases in γ ?

- (ii) Find the equilibrium quantity and profit of each firm I and discuss how they are affected by γ and c .

5. Bertrand Oligopoly

Consider a market with duopoly in which the firms choose their prices simultaneously and separately. The market demand function is: $D(p) = 6 - p$. If a firm chooses a lower price than its competitor, then it supplies the whole market, if both firms choose the same price, then they share the market. The total cost functions are the same for both firms and are given by: $C(q_i) = 2q_i$, where $i = 1, 2$. Find the following:

- (i) Find firm 1's profits when it sets a price lower than the price of firm 2. Find also firm 1's profits when the firms choose the same price as firm 2.
- (ii) Explain why a situation in which the firms set the same price which is higher than the marginal cost $p_1 = p_2 > 2$ cannot be a Nash equilibrium.

6. Bertrand Oligopoly with Asymmetric Costs

Consider a market with duopoly in which the firms choose their prices simultaneously and separately. Market demand function is given by: $D(p) = 10 - p$. If a firm chooses a lower price than its competitor then it supplies the whole market. If the two firms choose the same price, then they share the market. The total cost of firm 1 and firm 2 is given by: $C_1(q_1) = q_1$ and $C_2(q_2) = 4q_2$ respectively. Find the Nash equilibrium prices as well as the equilibrium profits and output of each firm.

7. Bertrand Oligopoly with Differentiated Products

Suppose a market consists of two firms, firm 1 and firm 2. These firms are Bertrand competitors. Firm 1 makes and sells good 1 and firm 2 makes and sells good 2. The demand functions for each of these goods are:

$$q_1 = 25 - 5p_1 + 2p_2 \quad \text{and} \quad q_2 = 25 - 5p_2 + 2p_1$$

The total cost functions of these two firms are:

$$C(q_1) = 2 + q_1 \quad \text{and} \quad C(q_2) = 2 + q_2$$

- (i) Find the best response function of each firm. How does the price firm i sets change with increases in the price of its competitor's good? What is the Nash equilibrium?
- (ii) What is the Lerner Index for good 1 and for good 2? What is the interpretation? Do the firms have market power?
- (iii) Explain why the Bertrand Paradox of zero market power does not apply in this case.