ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

INDUSTRIAL ECONOMICS

SOLUTIONS TO PRACTICE PROBLEM SET I

Solution of 1

1) The monopolist solves the following maximization problem:

$$\max_{p} \pi = (p - AC)q = (p-c)(a-bp)$$

from which we derive first order conditions *(FOC)* with respect to the price:

$FOC_p : a - bp - b(p\text{-}c) = 0$

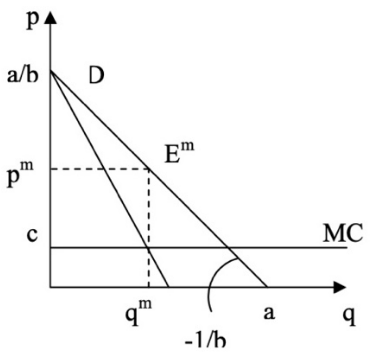
Solving the *FOC*, we obtain the monopoly equilibrium price:

$$p^m = \frac{a+bc}{2b}$$

Now, substitute the monopoly equilibrium price into the demand function, to obtain the monopoly equilibrium quantity:

$$q^m = \frac{a-bc}{2}$$

The graphical representation is as follows:



2) The elasticity of demand (to price), as a positive number, is given by the following formula:

$$\varepsilon = -D(p)'\frac{p}{D(p)}$$

which yields, in this case:

$$\varepsilon = \frac{bp}{a-bp}$$

It is clear that, the lager b, the lager the numerator and the smaller the denominator; the demand elasticity is thus an increasing function of the parameter b.
This result can be derived also at a more intuitive level.

Solution of 2

(i)

Total Revenue is given by $p(q) \cdot q$, that is, the revenue that Las-O-Vision receives when it sells q units. To get $p(q)$, we invert the demand function $q = 10,200 - 100p$ by solving for p in terms of q, or $p(q) = 102 - \frac{q}{100}$.
Substituting this into our total revenue equation, we obtain
$TR(q) = (102 - \frac{q}{100}) \cdot q = 102q - \frac{q^2}{100}$.

Since our demand equation is linear in q, we can obtain the marginal revenue using the "twice-as-steep" rule, so $MR(q) = 102 - \frac{q}{50}$. Alternatively, we can obtain it by realling that marginal revenue is the derivative of Total Revenue with respect to q.

(ii)

The profit maximizing quantity, q^* is that quantity at which marginal cost and marginal revenue are equal. Setting $MR(q) = MC$, we have

$$102 - \frac{q^*}{50} = q^*$$
$$q^* = 100$$

The profit maximizing price is that which generates $q^* = 100$ in sales or, substituting into the inverse demand function calculated in (a),

$$p(100) = 102 - \left(\frac{100}{100}\right) = 101$$

When selling 100 units, Las-O-Vision generates Total revenues equal to

$$TR(100) = (102)(100) - \frac{100^2}{100} = \$10,100$$

Its total cost is $TC = \frac{100^2}{2} = 5,000$. Therefore its total profit when it sells 100 units is $10,100 - 5,000 = \$5,100$.

Solution of 3

Scope economies exist when

$$C(q_1, q_2) < C(q_1, 0) + C(0, q_2)$$

Then, we have scope economies when

$$6 + q_1 + 3q_2 + q_1 q_2 \; < \; 4 + 3q_2 + 3 + q_1$$
$$\Rightarrow q_1 q_2 < 1.$$

Solution of 4

1) Demands are independent because the quantity in each period depends only on that period price. Costs are, instead, dependent because the first period quantity influences both TC_1 and TC_2. Thsi is an example of the so called *learning by doing*: the more a firm produces at time 1, the lower its costs at time 2.
2) The monopolist maximizes the present value of its profits, i.e. the sum of the two periods profits (recall that the discount factor is normalized to 1).

$$\max_{p_1, p_2} \pi = (p_1 - AC_1)q_1 + (p_2 - AC_2)q_2 = (p_1 - c)(1 - p_1) + [p_2 - c - \lambda(1 - p_1)](1 - p_2)$$

First order conditions are as follows:

$$FOC_{p_2} : 1 - p_2 - p_2 + c - \lambda(1 - p_1) = 0$$

$$FOC_{p_1} : 1 - p_1 - p_1 + c - \lambda(1 - p_2) = 0$$

From which we compute the equilibrium prices in both periods:

$$p_2^m(p_1) = \frac{1 + c - \lambda(1 - p_1)}{2}$$

$$p_1^m(p_2) = \frac{1 + c - \lambda(1 - p_2)}{2}$$

There is a typo above: it should be (p1-c)(1-p1q)+[p2-c+λ(1-p1)](1-p2), i.e. the sign before of the λ(1-p1) term should be a plus, not a minus.

3) Now we calcolate the equilibrium Lerner index and demand elasticity at time 1 and time 2:

$$I(L)_2 = \frac{p_2^m - MC_2}{p_2^m} = \frac{1 - c + \lambda(1 - p_1)}{1 + c - \lambda(1 - p_1)}$$

$$\varepsilon_2 = -D_2'(p_2)\frac{p_2}{D_2} = \frac{1 + c - \lambda(1 - p_1)}{1 - c + \lambda(1 - p_1)}$$

Notice that $I(L)_2 = 1/\varepsilon_2$, as expected from the theory.

$$I(L)_1 = \frac{p_1^m - MC_1}{p_1^m} = \frac{1 - c - \lambda(1 - p_2)}{1 + c - \lambda(1 - p_2)}$$

$$\varepsilon_1 = -D_1'(p_1)\frac{p_1}{D_1} = \frac{1 + c - \lambda(1 - p_2)}{1 - c + \lambda(1 - p_2)}$$

Differently from before, $I(L)_1 < 1/\varepsilon_1$ in the first period, because the denominator is the same in the two ratios, but the numerator of $I(L)_1$ is smaller. This means that the relative *mark up* is lower compared to the one expected on good 1 only-monopoly. Put another way, the monopolist produces more in the first period because it internalizes the positive effect of q_1 on the second period costs.

Solution of 5

1) Demands are dependent. Notice that the first period quantity depends only on the first period price, while the second period quantity depends on both prices. Costs are independent and constant. This is an exemple of goodwill.
2) The monopolist maximizes the present value of its profits, i.e. the sum of the two period profits (recall that the discount factor is normalized to 1).

$$\max_{p_1, p_2} \pi = (p_1 - AC_1)q_1 + (p_2 - AC_2)q_2 = (p_1 - c)(1 - p_1) + (p_2 - c)(1 - p_1 - p_2)$$

First order conditions are as follows:

$$FOC_{p_2} : 1 - p_2 - p_2 + c - p_1 = 0$$

$$FOC_{p_1} : 1 - p_1 - p_1 + c - p_2 + c = 0$$

Therefore, the optimal prices in the two periods are:

$$p_2^m(p_1) = \frac{1 + c - p_1}{2}$$

$$p_1^m(p_2) = \frac{1 - p_2 + 2c}{2}$$

3) Now we compute the Lerner index and the demand elasticity in equilibrium, for both periods:

$$I(L)_2 = \frac{p_2^m - MC_2}{p_2^m} = \frac{1 - c - p_1}{1 + c - p_1}$$

$$\varepsilon_2 = -D_2'(p_2)\frac{p_2}{D_2} = \frac{1 + c - p_1}{1 - c - p_1}$$

Notice that $I(L)_2 = 1/\varepsilon_2$, as expected from the theory.

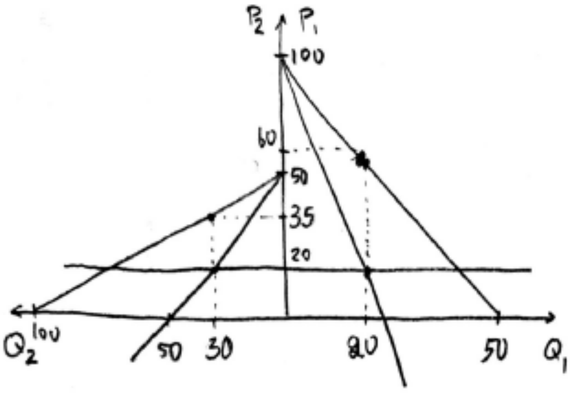
$$I(L)_1 = \frac{p_1^m - MC_1}{p_1^m} = \frac{1 - p_2}{1 + 2c - p_2}$$

$$\varepsilon_1 = -D_1'(p_1)\frac{p_1}{D_1} = \frac{1 + 2c - p_2}{1 - 2c + p_2}$$

Differently from before, $I(L)_1 < 1/\varepsilon_1$ in the first period, because the denominator is the same in the two ratios, but the numerator of $I(L)_1$ is smaller for $c < p_2$, which is always satisfied (no firm charges a price lower than its marginal cost). This means that the relative *mark up* is lower compared to the one expected on good 1 only-monopoly. Put another way, the monopolist produces more in the first period because it internalizes the positive effect of q_1 (an increase in q_1 means a decrease in p_1) on the second period demand.

Solution of 6

(Q4) 1) First Class: $P = 100 - 2Q \Rightarrow MR = 100 - 4Q$, Economy Class: $P = 50 - 0.5Q \Rightarrow MR = 50 - Q$



First Class: $MR = MC \Leftrightarrow 100 - 4Q = 20$

$Q^* = 20$

$P^* = 60$

Economy Class: $MR = MC \Leftrightarrow 50 - Q = 20$

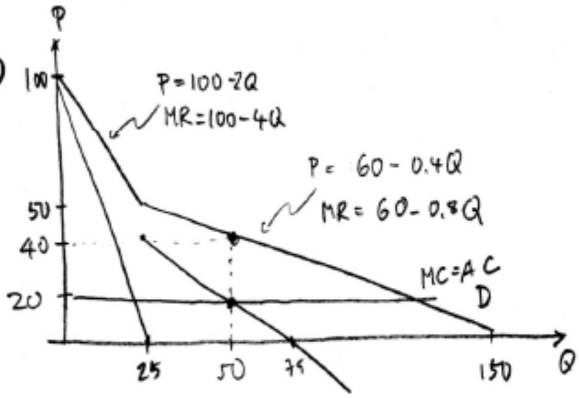
$Q^* = 30$

$P^* = 35$

Profit $= [20 \times 60] + [30 \times 35] - [20 \times 50]$

$= 1200 + 1050 - 1000 = 1250.$

2)



Single price: $P_M^* = 40$

$Q_M^* = 50.$

Profit $= 2000 - [20 \times 50] = 1000$

Solution of 7

1. The firm chooses a uniform price p to maximize $\pi = p(q_A(p) + q_B(p)) = p(3/2 - 2p)$. The profit-maximizing price is easily found as $p_u = 3/8$. The quantities sold are $q_A(3/8) = 5/8$ and $q_B(3/8) = 1/8$.
2. Here, the firm chooses p_A to maximize $\pi_A = p_A(1 - p_A)$ and p_B to maximize $\pi_B = p_B(1/2 - p_B)$. The optimal prices are found from the first-order conditions, respectively as $p_A^* = 1/2$ and $p_B^* = 1/4$; the corresponding quantities are $q_A(1/2) = 1/2$ and $q_B(1/4) = 1/4$.

3. Profits under uniform pricing and under third-degree price-discrimination are respectively equal to

$$\pi^{uni} = \frac{3}{8}\left(\frac{5}{8}+\frac{1}{8}\right) = \frac{9}{32},$$
$$\pi^{3rd} = \frac{1}{2}\frac{1}{2}+\frac{1}{4}\frac{1}{4} = \frac{10}{32},$$

which clearly shows that the firm is better off under price discrimination. As for consumer surplus, it is computed as $CS_A = (1/2)(1-p)^2$ on market A and as $CS_B = (1/2)(1/2-p)^2$ on market B for a given price p. We thus check that, on both markets, consumer surplus is a decreasing function of price. As $p_A^* > p_u > p_B^*$, we see that consumers in market A (resp. B) are worse (resp. better) off under price discrimination than under uniform pricing. As for total consumer surplus, we see that uniform pricing is globally preferred:

$$CS^{uni} = \frac{1}{2}\left(1-\frac{3}{8}\right)^2 + \frac{1}{2}\left(\frac{1}{2}-\frac{3}{8}\right)^2 = \frac{13}{64} > CS^{3rd}$$
$$= \frac{1}{2}\left(1-\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)^2 = \frac{10}{64}.$$

4. If $q_B = 1/3 - p_B$, then $p_u = 1/3$, while $p_B^* = 1/6$. Here, the optimum for the monopolist is to sell only in market A. We still have that consumers in market A prefer uniform pricing whereas consumers in market B prefer price discrimination. Although consumers in market B make no surplus under uniform pricing, it is still the case that uniform pricing is preferred globally:

$$CS^{uni} = \frac{1}{2}\left(1-\frac{1}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{3}-\frac{1}{3}\right)^2 = \frac{8}{36} > CS^{3rd}$$
$$= \frac{1}{2}\left(1-\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{3}-\frac{1}{6}\right)^2 = \frac{5}{36}.$$