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8.3.1 Product market competition with homogenous goods

In this section I assume that the firms' products are homogenous, that is are seen as perfect substitutes by consumers. I will analyse in turn the three benchmark cases where firms compete in prices, in quantities, and where they choose their actions so as to maximise joint profits.

8.3.1.1 Price competition (Bertrand model)

Consider two firms (but the extension to n firms would be straightforward and offer the same results) that:

A1) sell *homogenous* goods;

A2) play a *one-shot game*;

A3) independently and simultaneously choose the *price* at which they want to sell their product;

A4) have *no capacity constraints*, that is, they are able to serve all demand that is addressed to them;

A5) have the same *identical marginal cost*, c, and no fixed costs.

Consumers address the firms according to the following demand function:

$$D_i(p_i, p_j) = \begin{cases} D(p_i), & \text{if } p_i < p_j \\ D(p_i)/2, & \text{if } p_i = p_j \\ 0, & \text{if } p_i > p_j \end{cases}, \qquad (8.26)$$

that is: if a firm sets a price lower than its rival, then all consumers will address it (and vice versa: nobody addresses the firm setting the higher price); if both firms set the same price, consumers are indifferent between one or the other, and it is assumed that they equally split their demand between the two firms.

This being a one-shot game, the appropriate solution concept is the Nash equilibrium. In this model, a Nash equilibrium in prices, or Bertrand equilibrium, is a pair of prices (p_i^*, p_j^*) such that $\pi_i(p_i^*, p_j^*) \geq \pi_i'(p_i, p_j^*)$, for any $i, j = 1, 2$, with $i \neq j$ and any p_i in the real numbers. am going to show the following result:

(Bertrand equilibrium) The unique price equilibrium of this game is given by $p_i^* = p_j^* = c$, *with* $\pi_i(p_i^*, p_j^*) = \pi_j(p_i^*, p_j^*) = 0$.

Proof. To prove this result, we need to prove first that the proposed one is a Nash equilibrium of the game, and then that its is the only one.

Step 1. To see that $p_1^* = p_2^* = c$ is a Nash equilibrium is straightforward. To be a Nash equilibrium, no firm must have incentives to deviate from it given that the other plays according to the equilibrium strategy. Suppose then that $p_1^* = c$, does firm 2 have an incentive to deviate and sell at a different price than $p_2^* = c$? By selling at marginal cost, firm 2 serves half of the market but makes zero profit. If it deviates and sets a *lower* price than c, it will serve all the market, but make losses; if it deviates and sets a *higher* price, no consumers will address it, and therefore will make zero profits, thus not improving its situation relative to playing the candidate equilibrium. Therefore, firm 2 will have no incentive to deviate. Likewise, given the perfect symmetry between the two firms, firm 1 has no incentives to deviate.

Step 2. Let us reason by contradiction. Suppose that there is a different candidate equilibrium, and show there is at least one deviation that would make better off a firm, thereby breaking the candidate equilibrium.

- $p_i^* = p_j^* = p^* > c$: at this candidate equilibrium, firms share the market equally and make positive profits $\pi(p^*, p^*) = (p^* - c)D(p^*)/2$. However, given the price p^* of firm i, its rival firm j can set a price p_j' which is a shade less than p^* (that is, $p_j' = p^* - \varepsilon$) and get the whole market. It would therefore make a profit $\pi_j'(p_i^*, p_j') = (p^* - c - \varepsilon)D(p^* - \varepsilon)$ that, for ε small enough, is clearly higher than $\pi(p^*, p^*)$ (since a marginal reduction in the price results in a disproportionate increase in demand, which doubles). Therefore, this cannot be a Nash equilibrium of the game.

- $p_i^* = p_j^* = p^* < c$: at the candidate equilibrium, both firms make losses. Trivially, this cannot be an equilibrium since a firm would have an incentive to deviate and set a price at or above marginal cost: it would not sell anything and therefore make zero profits (better than negative profits).

- $p_i^* > p_j^* \geq c$: if $p_j^* = c$, firm j makes zero profits, but given that firm i sets a price above cost, it can improve its position by charging any price between c and firm i's price, since it would still get the whole demand but would command a positive margin. In particular, the optimal deviation would be to set the price $p_i^* - \varepsilon$ and get profits $(p_i^* - c - \varepsilon)D(p_i^* - \varepsilon) > 0$. Therefore, this candidate equilibrium, and by similar reasoning all pairs with asymmetric prices, cannot be a Nash equilibrium of the game. If $p_j^* > c$, firm i would also have an incentive to deviate, and charge a price just below p_j^*.

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The Bertrand result is a striking one. Despite the fact that there are only two firms in the industry, they will end up selling at marginal cost, and get zero profits. As we shall see below, this is not a robust result, as it crucially depends on a series of strong assumptions: by relaxing, in turn, each of assumptions A1)-A5), one obtains equilibria where prices are above marginal cost and firms make positive profits. Nevertheless, this case provides a useful benchmark, corresponding to the lower bound that equilibrium prices can take. In other words, the Bertrand equilibrium corresponds to the toughest possible degree of product market competition.

Before introducing the other main benchmark case, that of Cournot competition, let me briefly describe the Bertrand game in two interesting cases: (1) under asymmetric firms; (2) under capacity constraints.

Bertrand under cost asymmetric firms Consider exactly the same game as above, but relax assumption A5) and assume that the two firms have asymmetric costs. Firm 1 and 2 have respectively marginal cost c_1 and c_2 with $c_1 < c_2$. There are two possible solutions to this game. In the first case, firm 1 is so much more efficient than firm 2 (firm 1's costs are much lower), that it can behave as if it was a monopolist: firm 2's cost is so high that firm 1's monopoly price, p_1^m is below c_2. In the second case, if the firms' costs are close enough to each other (how close is to be specified below), then firm 1 will set a price slightly below firm 2's marginal cost and get the whole market.

To keep the analysis simple, I develop these two cases under the assumption that firm i's demand function is given by $D_i = 1 - p_i$ if $p_i < p_j$, $D_i = 0$ if $p_i > p_j$, and $D_i = (1 - p_i)/2$ if $p_i = p_j$.

Large asymmetries Consider the case where firm 2's costs are large enough relative to firm 1's (more precisely, $c_2 \geq (1 + c_1)/2$, as will be seen below). If firm 1 were alone in the market, it would choose its price so as to maximise $\pi = (1 - p)(p - c_1)$. We have seen that the monopolist's problem is easily solved by taking $d\pi/dp = 0$ and re-arranging, which gives as result:

$$p_1^m = \frac{1 + c_1}{2}. \tag{8.27}$$

As long as p_1^m is below the marginal cost of firm 2, firm 1 can charge the monopoly price and get the whole market undisturbed, as firm 2 would have no incentive to undercut it: if it did set a price below p_1^m, all consumers would address it and it would accumulate losses. Better then to charge a price higher than p_1^m and get zero profits.

Therefore, if $c_2 \geq (1 + c_1)/2$, at equilibrium firm 1 will set $p_1^m = (1 + c_1)/2$, firm 2 a price $p > p_1^m$. Firm 1 will get monopoly profits $(1 - c_1)^2/4$, and firm 2 zero profit.

Therefore, if $c_2 \geq (1 + c_1)/2$, at equilibrium firm 1 will set $p_1^m = (1 + c_1)/2$, firm 2 a price $p > p_1^m$. Firm 1 will get monopoly profits $(1 - c_1)^2/4$, and firm 2 zero profit.

In the study of innovations, this case corresponds to the case where a firm gets a *drastic innovation*, that is an innovation that makes it so much more competitive than its rival, that it can behave as a monopolist (see chapter 2 for some applications of this case).

Small asymmetries Consider now the case where the firms' costs are close enough: $c_1 < c_2 < (1 + c_1)/2$. In this case, firm 1 setting its monopoly price cannot be the equilibrium of the game: if firm 1 sets $p_1^m = (1 + c_1)/2$, firm 2 will charge a price $p = (1 + c_1)/2 - \varepsilon$, get all the market, and make a positive profit (on each unit sold, it will make a gain equal to $(1 + c_1)/2 - \varepsilon - c_2$).

The following is instead a Nash equilibrium of the game: $(p_1^*, p_2^*) = (c_2 - \varepsilon, c_2)$, that is, firm 2 charges a price equal to marginal cost and firm 1 a price which is a shade below that. It is easy to check that firm 2 has no incentive to deviate from this pair, as undercutting $p_1^* = c_2 - \varepsilon$ would leave it with losses rather than zero profits, whereas increasing the price above c_2 leaves it again with zero profits. Firm 1 has no incentive to deviate either: by setting a higher price, say $p_1 = c_2$ it would have to share the market with the rival (or lose it completely to firm 2 if the price was above c_2),[737] whereas by setting a lower price it would continue to get all the market but it would sell at a lower margin, and thus making lower profits.

At this equilibrium, firm 1 makes profits $(c_1 - c_2)(1 - c_2)$, and firm 2 gains zero.

In principle, there exist other equilibria of this game, but they are less "reasonable". Consider for instance a price $p \in (c_1, c_2)$. It is easy to see that the pair $(p_1^{**}, p_2^{**}) = (p - \varepsilon, p)$ represents an equilibrium of the game.

However, such an equilibrium looks less appealing than the equilibrium (p_1^*, p_2^*) identified above, and indeed the main criteria of equilibrium selection would select the latter over the former. Under Pareto-dominance, for instance, (p_1^*, p_2^*) would be chosen because it gives higher profits for firm 1 while keeping the same profits (zero) for firm 2. Elimination of weakly dominated strategies also selects (p_1^*, p_2^*) as the only equilibrium of the game. To see this, note that when player 2 chooses action $p_2^* = c_2$ it gets $\pi_2(c_2 - \varepsilon, c_2) = 0$ and $\pi_2(p - \varepsilon, c_2) = 0$, where $p \in (c_1, c_2)$; when it chooses action $p_2^{**} = p$, it gets $\pi_2(p - \varepsilon, p) = 0$ if player 1 chooses $p - \varepsilon$, but it gets $\pi_2(p + \varepsilon, p) < 0$ if player 1 chooses $p + \varepsilon$. The equilibrium (p_1^{**}, p_2^{**}) contains a strategy that is weakly dominated for player 2.