

Industrial Economics

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AUEB – Erasmus Program



Slides

Industrial Organization: Markets and Strategies
Paul Belleflamme and Martin Peitz, 2d Edition

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5. PRODUCT DIFFERENTIATION



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Contents

5.1 Horizontal Product Differentiation (Hoteling Model)

5.2 Vertical Product Differentiation

Differentiated products are "similar", but not identical (they are close, but not perfect, substitutes).

There are two major types of product differentiation:

1. Horizontal differentiation: If all products were the same price, consumers disagree on which product is most preferred.

Consumers have different tastes over them.

E.g. films, cars, clothes, books, cereals, ice-cream flavours, Starbucks (by geographic location), . . .

2. Vertical differentiation: If all products were the same price, all consumers agree on the preference ranking of products, but differ in their willingness to pay for the top ranked versus lesser ranked products.

If the products are all sold at the same price, consumers all choose the product with the "highest" quality.

E.g. computer memory/processors, airline tickets, ...

5.1 Horizontal Product Differentiation

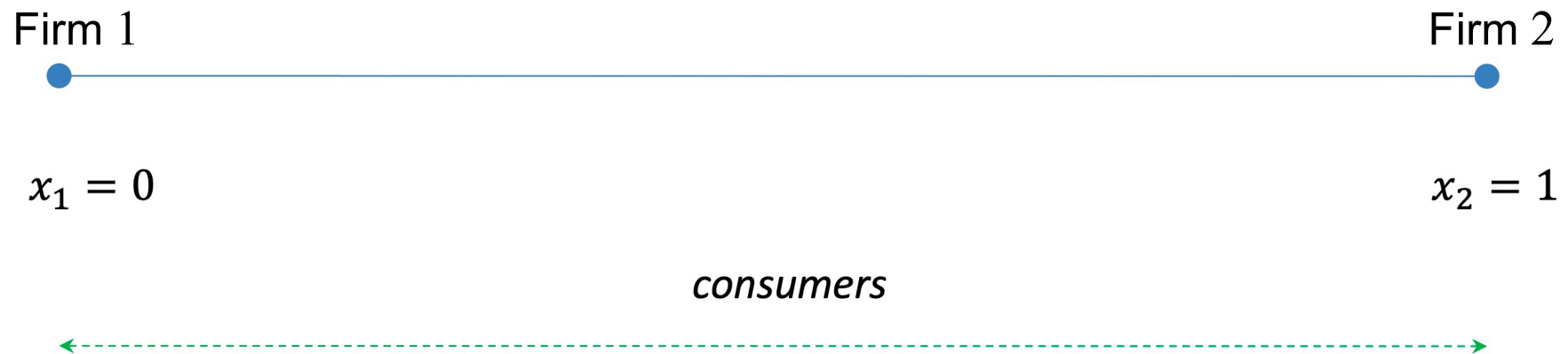
Hoteling (Linear City) Model

5.1 Horizontal Product Differentiation

Hoteling model with Exogenous Locations

- **Model Assumptions:**

- A "street" or a "space of tastes" represented by the interval $[0, 1]$
- $n = 2$ firms sell the same product on this street; Firm 1 is located at the beginning of the street $x_1 = 0$; Firm 2 is at the end of the street $x_2 = 1$.
- Each firm faces a constant *MC*: c
- A mass 1 of consumers are distributed uniformly along the interval.



5.1 Horizontal Product Differentiation

Hoteling model with Exogenous Locations

- **Model Assumptions** (cont.):

- Each consumer buys 0 or 1 unit of the product (i.e., unitary demand).

Utility of the product for a consumer: v

But there is a transportation cost the consumer pays to get the good: t per unit of distance (quadratic function).

→ Net utility of consumer is: $v - p - \text{transportation cost}$

- Static game:

Both firms choose simultaneously and separately the prices of their products, p_1 and p_2 .

5.1 Horizontal Product Differentiation

Hoteling model with Exogenous Locations

- **Model Solution – Equilibrium:**

Before solving for the Nash equilibrium prices, we need to find the demand faced by each firm and to do so, we first need to determine the ‘indifferent (or marginal) indifferent consumer’.

In the presence of consumers with heterogeneous tastes, the indifferent consumer is the consumer who is indifferent between two possible choices.

In this model, the indifferent consumer is the one which is indifferent between buying from firm 1 and buying from firm 2.

- *Where is the indifferent consumer approximately located?*

The indifferent consumer is defined by:

$$v - \left(p_1 + (\tilde{x} - 0)^2 t \right) = v - \left(p_2 + (1 - \tilde{x})^2 t \right)$$

Thus, his/her location is:

$$\tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

5.1 Horizontal Product Differentiation

Hoteling model with Exogenous Locations

- **Model Solution – Equilibrium (cont.):**

From the location of the indifferent consumer, it follows that demand of firm 1 is given by:

$$D_1(p_1, p_2) = \tilde{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

And the demand of firm 2:

$$D_2 = 1 - \tilde{x} = 1 - D_1$$

- *We move now to the choice of prices by each firm i :*

Each firm i chooses its price p_i (taking as given the price of its rival p_j) in order to maximize its profit which is given by:

$$\pi_i = (p_i - c) \left(\frac{1}{2} + \frac{p_j - p_i}{2t} \right)$$

FOC:

$$\rightarrow p_i = R_i(p_j) = \frac{c + t + p_j}{2}$$

Best-response
functions

5.1 Horizontal Product Differentiation

Hoteling model with Exogenous Locations

- **Model Solution – Equilibrium (cont.):**

Solving the system of the two BRs we find the Nash equilibrium prices:

$$p_1^* = p_2^* = c + t$$

Note: The equilibrium price increases with t . If $t = 0$, then there is no product differentiation and we have again the ‘Bertrand paradox’ in equilibrium.

In the Hoteling model with exogenous (fixed) locations, an increase in the degree of product differentiation (measured by t) increases firms’ market power.

5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Assumptions:**

Same assumptions as before (as in the Hoteling model with Exogenous Locations) with the only difference that now:

- Dynamic game:

Stage 1: Both firms choose their locations.

Stage 2: Both firms choose simultaneously and separately the prices of their products, p_1 and p_2 .

5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Solution – Equilibrium:**

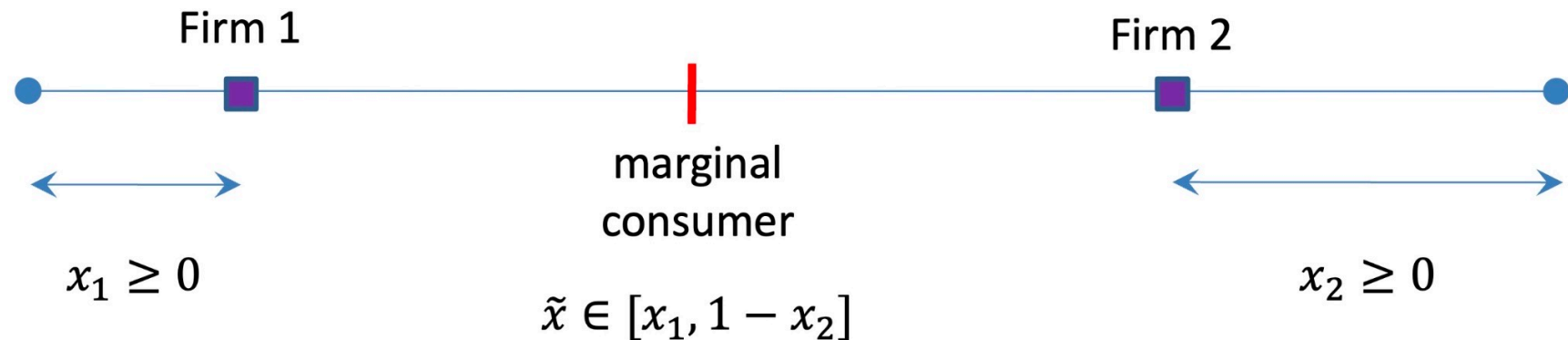
Subgame Perfect Nash equilibrium - Solution by Backward Induction:

First, we look for the equilibrium of the price competition (last) Stage 2.

Next, we solve for the equilibrium locations in Stage 1 assuming that firms expect equilibrium prices to prevail in Stage 2 (*subgame perfection*).

- Stage 2: Choice of prices

- We need first to determine the demand of each firm after determining the indifferent consumer.



5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Solution – Equilibrium (cont.):**

The indifferent consumer is defined by:

$$v - \left(p_1 + (\tilde{x} - x_1)^2 t \right) = v - \left(p_2 + (1 - x_2 - \tilde{x})^2 t \right)$$

Thus, his/her location is:

$$\tilde{x} = x_1 + \frac{1 - x_1 - x_2}{2} + \frac{p_2 - p_1}{2t(1 - x_1 - x_2)}$$

From the location of the indifferent consumer, it follows that if $0 \leq \tilde{x} \leq 1$ then, the demand of firm 1 and of firm 2 are given by:

$$D_1 = \tilde{x} \text{ and } D_2 = 1 - \tilde{x}$$

5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Solution – Equilibrium (cont.):**

- *We move now to the choice of prices by each firm i :*

Each firm i chooses its price p_i (taking as given the price of its rival p_j) in order to maximize its profit which is given by:

$$\pi_i = (p_i - c) \left(x_i + \frac{1 - x_1 - x_2}{2} + \frac{p_j - p_i}{2t(1 - x_1 - x_2)} \right)$$

FOCs: → Two Best-response functions.

The solution of the system of the BRs gives the equilibrium prices in terms of firms' locations:

$$p_1^* = c + t(1 - x_1 - x_2) \left(1 + \frac{x_1 - x_2}{3} \right)$$

$$p_2^* = c + t(1 - x_1 - x_2) \left(1 + \frac{x_2 - x_1}{3} \right)$$

5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Solution – Equilibrium (cont.):**

Stage 1: Choice of Locations

Firm 1 chooses its location taking the location of firm 2 as given. It anticipates the equilibrium price of Stage 2. Therefore, its profit maximization problem is:

$$\max_{x_1} \left(p_1^*(x_1, x_2) - c \right) D_1 \left(x_1, x_2, p_1^*(x_1, x_2), p_2^*(x_1, x_2) \right)$$

Firm 2 faces a similar problem.

FOCS: → Two Best-response functions.

The solution of the system of the BRs gives the equilibrium firms' locations:

Firm 1 locates at $x_1 = 0$ and Firm 2 locates at $x_2 = 1$.

'maximum differentiation'

Why? (See next page)

5.1 Horizontal Product Differentiation

Hoteling model with Endogenous Locations

- **Model Solution – Equilibrium (cont.):**

An increase in a has a positive and a negative effect on the profit of firm A :

- Demand effect (positive): Moving towards the center, firm A increases its market share (given prices).
- Competition effect (negative): Moving towards the center, firm A generates more competition, i.e. prices decrease.

In equilibrium (with quadratic transportation costs) the competition dominates the demand effect.

In the Hoteling model with endogenous locations, firms choose maximum differentiation.

Hoteling model with Endogenous Locations

- **Comparison with Social Optimum (Welfare maximization)**

How do firms' location choices compare to the social optimum? Do firms differentiate themselves *too much* or *too little*?

The socially optimal locations are those that minimize production and transportation costs. These costs are minimized when the two firms are located at $1/4$ and at $3/4$.

→ Therefore, there is too much differentiation.

- **Variation: Exogenous (fixed) prices**

What are the firms' locations in equilibrium in this case?

In equilibrium, the two firms produce the same (average) variety → there is minimal differentiation.

5.2 Vertical Product Differentiation

Vertical Differentiation model

- **Model Assumptions:**

- A "street" or a "space of quality" represented by the interval $[0, 1]$
- $n = 2$ firms sell a (potentially) differentiated product on this street
Firm A is located at a and Firm B located at b .
- Each firm faces $MC = 0$.
- A mass 1 of consumers are distributed uniformly along the interval.

Consumers have unitary demand and prefer the product closest to 1 (highest quality).

The utility of consumer located at x buying from firm i , with $V > 1/3$, is:

$$U(x) = \begin{cases} V + ax - p_A & \text{if } i = A \\ V + bx - p_B & \text{if } i = B \end{cases}$$

Vertical Differentiation model

- **Model Assumptions (cont.):**

- Dynamic game:

Stage 1: Both firms choose their locations.

Stage 2: Both firms choose simultaneously and separately the prices of their products, p_1 and p_2 .

Vertical Differentiation model

- **Model Solution – Equilibrium:**

Subgame Perfect Nash equilibrium - Solution by Backward Induction:

- Stage 2: Choice of prices

We suppose $b > a$.

- We need first to determine the demand of each firm after determining the indifferent consumer.

If $p_B \leq p_A$ all consumers buy in B (higher quality at lower price).

If $p_B > p_A$ we have $D_A = \bar{x}$ and $D_B = 1 - \bar{x}$ where \bar{x} satisfies:

$$\begin{aligned} a\bar{x} - p_A &= b\bar{x} - p_B \\ \Leftrightarrow \bar{x} &= \frac{p_B - p_A}{b - a} \end{aligned}$$

5.2 Vertical Product Differentiation

Vertical Differentiation model

- **Model Solution – Equilibrium (cont.):**

- *We move now to the choice of prices by each firm i :*

Each firm i chooses its price p_i (taking as given the price of its rival p_j) in order to maximize its profit which is given by:

$$\max_{p_A} p_A \frac{p_B - p_A}{b - a}$$
$$\max_{p_B} p_B \left(1 - \frac{p_B - p_A}{b - a}\right)$$

FOCs:

$$0 = \frac{p_B - 2p_A}{b - a}$$
$$0 = 1 - \frac{2p_B - p_A}{b - a}$$

Vertical Differentiation model

- **Model Solution – Equilibrium (cont.):**

The solution of the system of the BRs gives the equilibrium prices in terms of firms' locations:

$$p_A(a, b) = \frac{1}{3}(b - a)$$
$$p_B(a, b) = \frac{2}{3}(b - a)$$

Notes:

1. $p_B > p_A$, the firm that produces higher quality charges a higher price.
2. $p_A > MC = 0$, both firms charge a price larger than marginal cost.

Vertical Differentiation model

- **Model Solution – Equilibrium (cont.):**

Stage 1: Choice of Locations (Qualities)

Substituting the prices (as functions of the locations) into the demand functions, we obtain:

$$D_A = \frac{p_B(a, b) - p_A(a, b)}{b - a} = \frac{1}{3}$$
$$D_B = 1 - \frac{p_B(a, b) - p_A(a, b)}{b - a} = \frac{2}{3}$$

Thus, profits are:

$$\pi_A = p_A(a, b)D_A = \frac{b - a}{3} \frac{1}{3} = \frac{b - a}{9}$$
$$\pi_B = p_B(a, b)D_B = \frac{2(b - a)}{3} \frac{2}{3} = \frac{4(b - a)}{9}$$

5.2 Vertical Product Differentiation

Vertical Differentiation model

- **Model Solution – Equilibrium (cont.):**

From the FOCs we see that:

$$\begin{aligned}\frac{\partial \pi_A}{\partial a} &= -\frac{1}{9} < 0 \\ \frac{\partial \pi_B}{\partial b} &= \frac{4}{9} > 0\end{aligned}$$

Thus, the equilibrium quality choices are:

Firm *A* locates at $a = 0$ and Firm *B* locates at $b = 1$.

‘maximum differentiation’

I.e., firms specialize in producing quality for different groups of consumers.

Why? Although quality is produced at zero cost and preferred by the consumers, firm *A* prefers to produce low quality in order to differentiate from its rival.

5.2 Vertical Product Differentiation

Vertical Differentiation model

- **Model Solution – Equilibrium (cont.):**

The resulting equilibrium process and profits are:

$$p_A = \frac{1}{3} \quad \pi_A = \frac{1}{9}$$
$$p_B = \frac{2}{3} \quad \pi_B = \frac{4}{9}$$

Notes:

- Firm with higher quality charges a higher price.
- The price difference increases with the degree of differentiation.

In the vertical differentiation model, firms choose maximum differentiation.