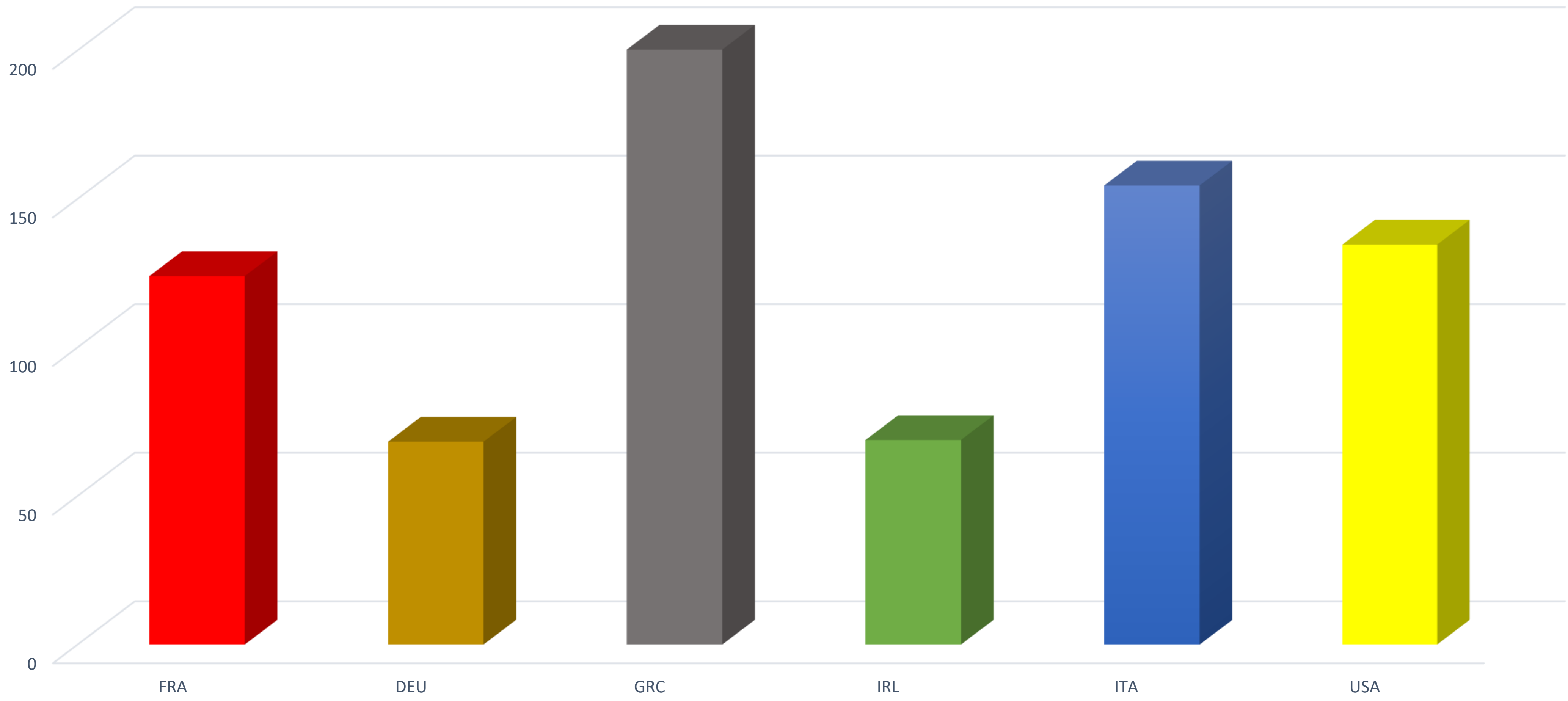
Lecture 5: Public Debt

Petros Varthalitis

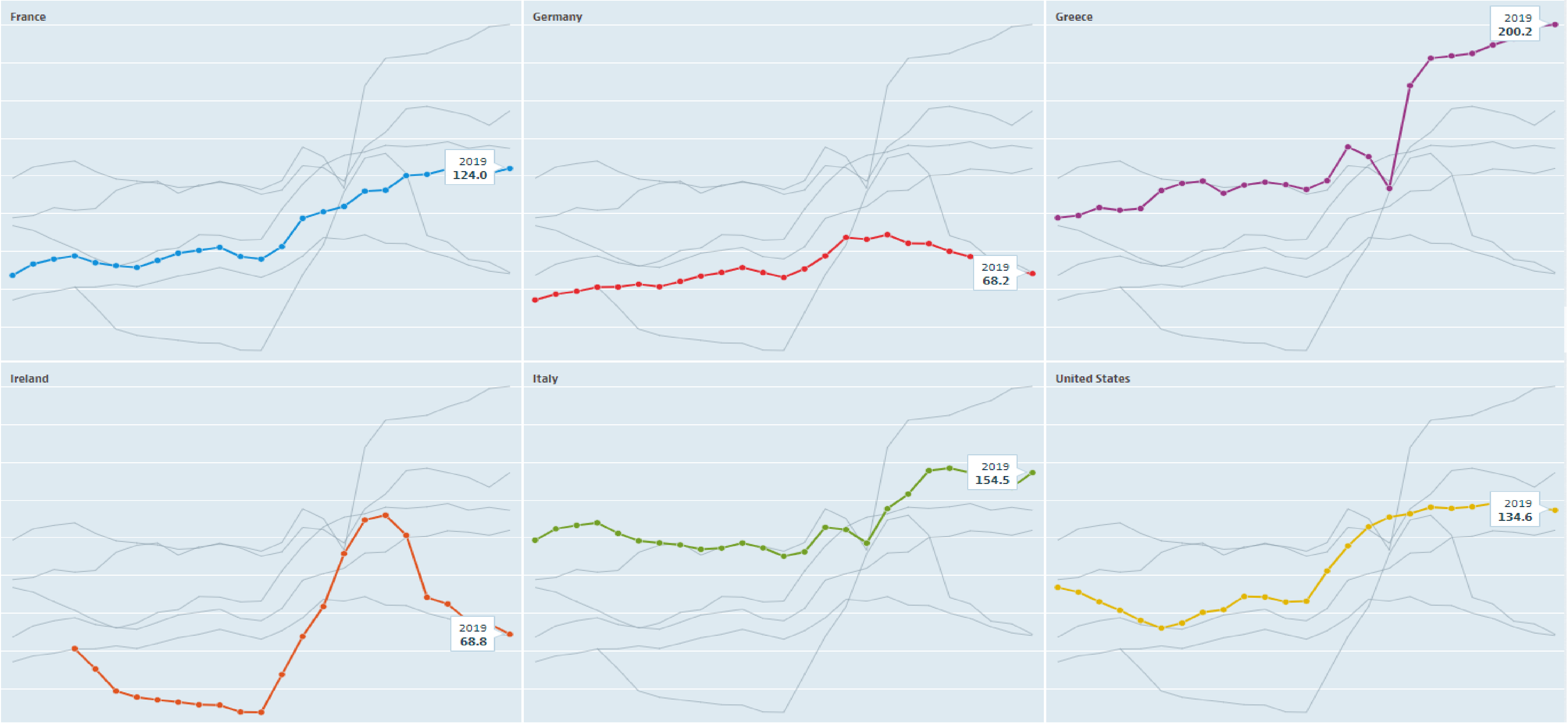
What is Public Debt?

- Governments face budget constraints like households.
- To finance their expenditures, they collect taxes.
- When tax revenues are lower than expenditures governments should borrow.
- Thus, they accumulate public debt.

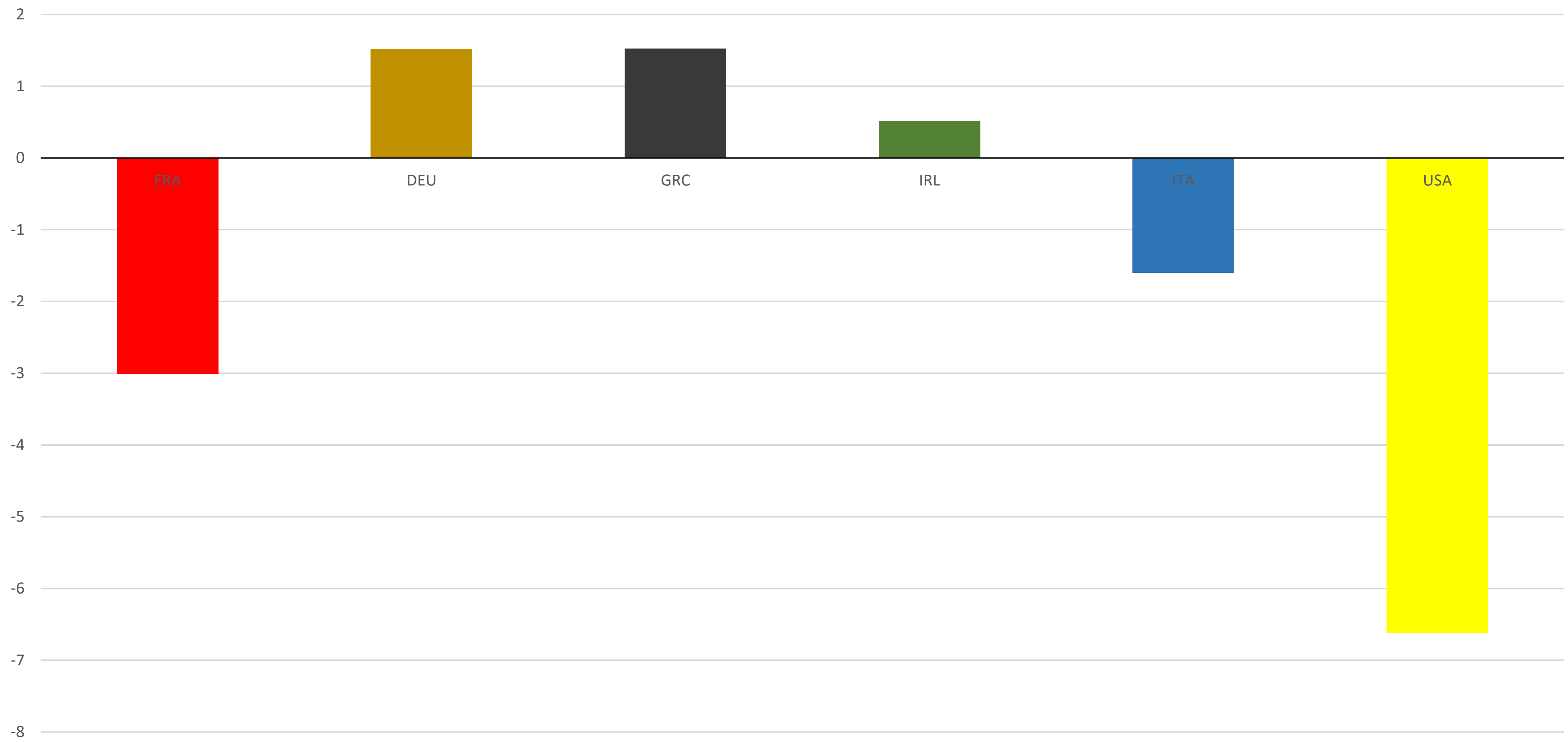
Public debt for a select of countries 2019 (% GDP)



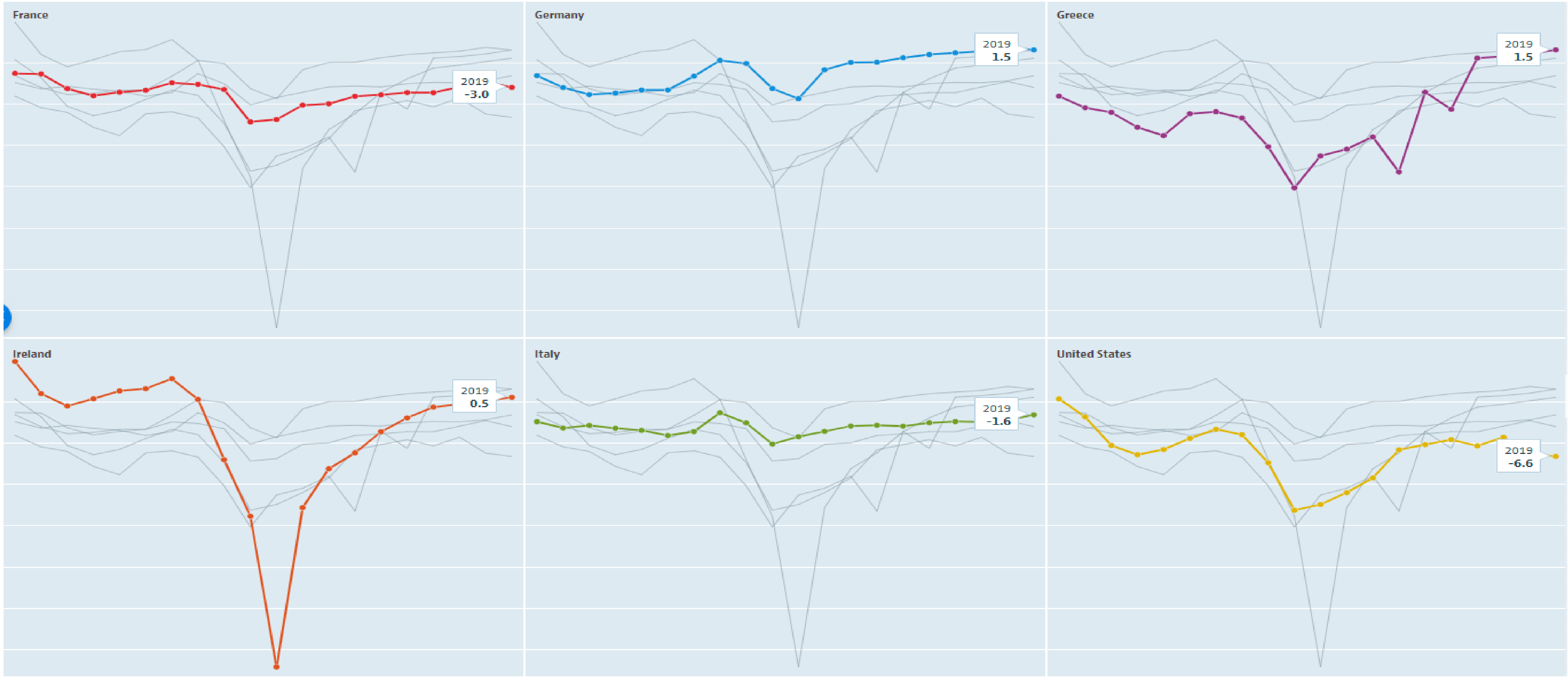
Evolution of public debt (% GDP) over 1995-2019



Public deficit for a select of countries 2019 (% GDP)



Evolution of deficits (% GDP) over 1995-2019



Government budget constraint

The government budget constraint in nominal terms is:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

- B_t denotes public debt in nominal terms at the end of period t.
- G_t denotes government spending in nominal terms.
- T_t denotes tax revenues in nominal terms.
- r_t denotes nominal interest rate, thus, $r_t B_{t-1}$, are interest payments on previous period public debt.

Deficit/Surplus

- The term, $G_t - T_t$, is government's primary deficit/surplus.
- When $G_t > T_t$ the government runs a deficit.
- When $G_t < T_t$ the government runs a surplus.
- When $G_t = T_t$ the government runs a balanced budget.

Example: Two periods

- In period 1 with initial debt equal to B_0 :

$$B_1 = (1 + r_1)B_0 + G_1 - T_1$$

- In Period 2:

$$B_2 = (1 + r_2)B_1 + G_2 - T_2$$

Let us assume that $B_2 = 0$, i.e., the government cannot be indebted at the end of the world.

- Solving period 2:

$$B_1 = -\frac{G_2 - T_2}{(1 + r_2)}$$

- And substitute B_1 in the government budget of period 1:

$$B_0 = -\frac{G_1 - T_1}{(1 + r_1)} - \frac{G_2 - T_2}{(1 + r_1)(1 + r_2)}$$

$$\underbrace{B_0}_{\text{Initial debt}} = \underbrace{\frac{T_1 - G_1}{(1+r_1)} + \frac{T_2 - G_2}{(1+r_1)(1+r_2)}}_{\text{Present value of future suspluses}}$$

- A debtor country ($B_0 > 0$) should run future surpluses to pay back its debt.

- A creditor country ($B_0 < 0$) could liquidate its assets and run future deficits (i.e., to sustain spending higher than tax revenues).

Government Budget Constraint in real terms

To express the GBC in real terms we divide both sides with the aggregate price level, P_t:

$$\frac{B_t}{P_t} = (1 + r_t)\frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{G_t}{P_t} - \frac{T_t}{P_t}$$

Government Budget Constraint as a share of GDP

To express the GBC as a share of real GDP we divide both sides with the real GDP level, Y_t:

$$\frac{B_t}{P_t Y_t} = (1 + r_t)\frac{B_{t-1}}{P_{t-1} Y_{t-1}} \frac{P_{t-1} Y_{t-1}}{P_t Y_t} + \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t}$$

Notice that $P_t Y_t$ is nominal GDP.

Notation

- We denote real variables as a share of GDP with small letters, e.g., $b_t \equiv \frac{B_t}{P_t Y_t}$ denotes real public debt as a share of GDP (see slides 3-4).
- $d_t \equiv \frac{G_t}{P_t Y_t} - \frac{T_t}{P_t Y_t} = g_t - \tau_t$ denotes deficit as a share of GDP (see slides 5-6).
- Gross inflation rate: $1 + \pi_t = \frac{P_t}{P_{t-1}}$.
- Gross GDP growth rate: $1 + \gamma_t = \frac{Y_t}{Y_{t-1}}$.

Using the above notation the evolution of real public debt is:

$$b_t = \frac{(1 + r_t)}{(1 + \pi_t)(1 + \gamma_t)} b_{t-1} + d_t$$

For convenience we define $1 + z_t = \frac{(1+r_t)}{(1+\pi_t)(1+\gamma_t)}$, where z_t is nominal interest rate scaled by inflation and GDP growth.

For algebraic simplicity in what follows we assume that $z_t = z$ is constant over time.

Sustainability of Public Finances

- Government issues debt to finance its deficits.
- For investors to be willing to hold the government debt of a country, they must be confident that the government can repay this debt.
- When investors anticipate that the Government will repay its debt in the long run, public debt is sustainable.
- When investors have concerns whether the Government will repay its debt in the long run, public debt might become unsustainable, (i.e., can rises indefinitely).

In what follows we use the GBC to study fiscal sustainability issues:

$$b_t = \frac{(1+r)}{(1+\pi)(1+\gamma)} b_{t-1} + d_t$$

The GBC is a first order difference equation that determines the evolution of public debt to GDP.

- b_{t-1} is **the current state** of public finances.
- d_t is primary deficit and can be thought as a **fiscal policy** variable.
- The parameter $1+z=\frac{(1+r)}{(1+\pi)(1+\gamma)}$ represents the equilibrium outcome of this economy.

Parameter z

In this case parameter z reflects the equilibrium outcome of the economy, i.e., the equilibrium interest rate, inflation rate and real growth. We could use a model to find these values.

We consider two cases:

Nominal income grows faster than interest payments

- Case 1:

$$1 + z < 1 \qquad r < \pi + \gamma$$

Interest payments grow faster than nominal income

- Case 2:

$$1 + z > 1 \qquad r > \pi + \gamma$$

Case 1: Stable

When $1 + z < 1$ thus, $r < \pi + \gamma$

$$b_t = (1 + z)b_{t-1} + d_t$$

This is a stable difference equation and can be solved backwards.

Solving a difference equation backwards

Iterate one period forward:

$$b_{t+1} = (1+z)b_t + d_{t+1}$$

Iterate two periods forward:

$$b_{t+2} = (1+z)b_{t+1} + d_{t+2}$$

….

Iterate T periods forward:

$$b_{t+T} = (1+z)b_{t+T-1} + d_{t+T}$$

By successive substitution backwards

Public debt after T periods:

$$b_{t+T} = (1+z)^{T+1} b_{t-1} + \sum_{i=0}^{T} (1+z)^{i} \; d_{t+T-i}$$

Taking the limit as $T \to \infty$:

$$\lim_{T\to\infty} b_{t+T} = \lim_{T\to\infty} \; (1+z)^{T+1} b_{t-1} + \lim_{T\to\infty} \sum_{i=0}^{T} (1+z)^{i} \; d_{t+T-i}$$

Sustainability analysis

Since $1+z<1$ i.e. $r<\pi+\gamma$:

$$\lim_{T\to\infty} \quad (1+z)^{T+1} b_{t-1} = 0$$

Can the Government sustain a constant level of debt to GDP b?

$$\lim_{T\to\infty} b_{t+T} = b$$

Assume $d_t = d$ then:

$$\lim_{T\to\infty} \sum_{i=0}^{T} (1+z)^i \; d_{t+T-i} = \frac{1}{1-z} d$$

Substituting these terms into the GBC we get the following condition for fiscal sustainability:

$$b = \frac{1}{1-z} d = \frac{(1+\pi)(1+\gamma)}{(1+r)} d$$

Policy Implications when $r < \pi + \gamma$

- When nominal income grows faster than interest payments, this erodes initial public debt stock.

$$\lim_{T\to\infty} (1+z)^{T+1} b_{t-1} = 0$$

- If $\frac{(1+\pi)(1+\gamma)}{(1+r)} > 1$, the debt to GDP ratio will remain sustainable at a constant level, b, even if the government runs a permanent primary deficit $d > 0$

Case 2: Unstable

- $1 + z > 1 \quad$ thus, $r > \pi + \gamma$

$$b_t = (1 + z)b_{t-1} + d_t$$

The GBC is an unstable difference equation backward solution shows that debt can rise indefinitely:

$$\lim_{T \to \infty} \quad (1 + z)^{T+1} b_{t-1} \to \infty$$

The GBC is solved forwards

We write the GBC as:

$$b_{t-1} = \frac{1}{(1+z)} b_t - \frac{1}{(1+z)} d_t$$

Iterate and substituting backwards the GBC is written:

$$b_{t-1} = \left(\frac{1}{1+z}\right)^{T+1} b_{t+T} - \frac{1}{1+z} \sum_{i=0}^{T} \left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

Taking the limit as $T \to \infty$:

$$b_{t-1} = \lim_{T\to\infty} \left(\frac{1}{(1+z)}\right)^{T+1} b_{t+T} - \lim_{T\to\infty} \frac{1}{1+z} \sum_{i=0}^{T} \left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

No-Ponzi condition

The no-Ponzi condition rules out funding debt interest payments by issuing more debt.

$$\lim_{T\to\infty}\left(\frac{1}{1+z}\right)^{T+1} b_{t+T} = 0$$

- If $\left(\frac{1}{1+z}\right)^{T+1} > 0$ then $b_{t+T} = 0.$
- If $b_{t+T} > 0$ then $\left(\frac{1}{1+z}\right)^{T+1} = 0.$

Forward solution

$$b_{t-1} = -\frac{1}{1+z}\sum_{i=0}^{T}\left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

The present value of future surpluses must be sufficient to meet current debt obligations.

Assume constant $d_t = d$ then:

$$b_{t-1} = \frac{(1+\pi)(1+\gamma)}{(1+r) - (1+\pi)(1+\gamma)}(-d)$$

Where $-d$ is a primary surplus.

Policy implications of $r > \pi + \gamma$

- For fiscal sustainability the *PV* of fiscal surpluses must be sufficient to repay existing public debt stock.

- In contrast to case 1, now the Government should run a permanent constant surplus to pay off its initial debt.

Fiscal and monetary policy

- We assume that monetary and fiscal policy follow simple rules.

- Monetary policy sets the nominal interest rate according to a Taylor rule:

$$(1 + r_t) = \varphi(1 + \pi_t)$$

- Fiscal policy sets deficit by reacting to public debt to GDP ratio:

$$(d_t) = -\vartheta b_{t-1}$$

The evolution of real public debt is:

$$b_t = \frac{(1 + r_t)}{(1 + \pi_t)(1 + \gamma_t)} b_{t-1} + d_t$$

Using policy rules:

$$b_t = \frac{\overbrace{\varphi(1 + \pi_t)}^{(1+r_t)}}{(1 + \pi_t)(1 + \gamma_t)} b_{t-1} \overbrace{-\vartheta b_{t-1}}^{d_t}$$

Monetary-fiscal policy nexus

$$b_t = \left[\frac{\varphi}{1+\gamma_t} - \vartheta\right] b_{t-1}$$

The evolution of public debt depends on monetary-fiscal policy nexus:

$$\left[\frac{\varphi}{1+\gamma_t} - \vartheta\right]$$

Additional Reading

- Michael Wickens (2008). Macroeconomic Theory: A Dynamic General Equilibrium Analysis. Princeton University Press. Chapter 5 (Advanced)

- Eichengreen B., El-Ganainy A. Esteves R. and Mitchner K. (2019): ***Public debt through ages***. voxeu.gr

https://voxeu.org/article/public-debt-through-ages

- Alogoskoufis G. (2021). ***The pandemic and Greece's debt: The day after***. Voxeu.gr (

https://voxeu.org/article/pandemic-and-greece-s-debt-day-after

- Krugman P. (2019). ***Melting snowballs and the winter of debt.*** The New York Times.

https://www.nytimes.com/2019/01/09/opinion/melting-snowballs-and-the-winter-of-debt.html

Additional Reading

- Wyplosz C. (2019). Olivier in Wonderland. Voxeu.gr (https://voxeu.org/content/olivier-wonderland)

- Blanchard O. (2019): *Reexaming the Economic Costs of Debt*. PIIE Prepared remarks submitted to the US House of Representatives Committee on the Budget hearing on "Reexamining the Economic Costs of Debt“

(https://www.piie.com/commentary/testimonies/reexamining-economic-costs-debt)

Technical Appendix

Forward solution for $T = 3$

$$b_{t-1} = \left(\frac{1}{1+z}\right)^{T+1} b_{t+T} - \frac{1}{1+z}\sum_{i=0}^{T}\left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

The above solution is written as:

$$b_{t-1} = \left(\frac{1}{1+z}\right)^{3+1} b_{t+3} - \frac{1}{1+z}\sum_{i=0}^{3}\left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

Develop the sum

$$-\frac{1}{1+z}\sum_{i=0}^{3}\left(\frac{1}{1+z}\right)^{i} d_{t+i}$$

For i=0

$$-\frac{1}{1+z}\left(\frac{1}{1+z}\right)^{0} d_{t+0}$$

For i=1

$$-\frac{1}{1+z}\left(\frac{1}{1+z}\right)^{1} d_{t+1}$$

For i=2

$$-\frac{1}{1+z}\left(\frac{1}{1+z}\right)^{2} d_{t+2}$$

For i=3

$$-\frac{1}{1+z}\left(\frac{1}{1+z}\right)^{3} d_{t+3}$$

Solution

$$b_{t-1} = \left(\frac{1}{1+z}\right)^4 b_{t+3} - \frac{1}{1+z}\left\{d_{t+0} + \left(\frac{1}{1+z}\right)^1 d_{t+1} + \left(\frac{1}{1+z}\right)^2 d_{t+2} + \left(\frac{1}{1+z}\right)^3 d_{t+3}\right\}$$