International Negotiations
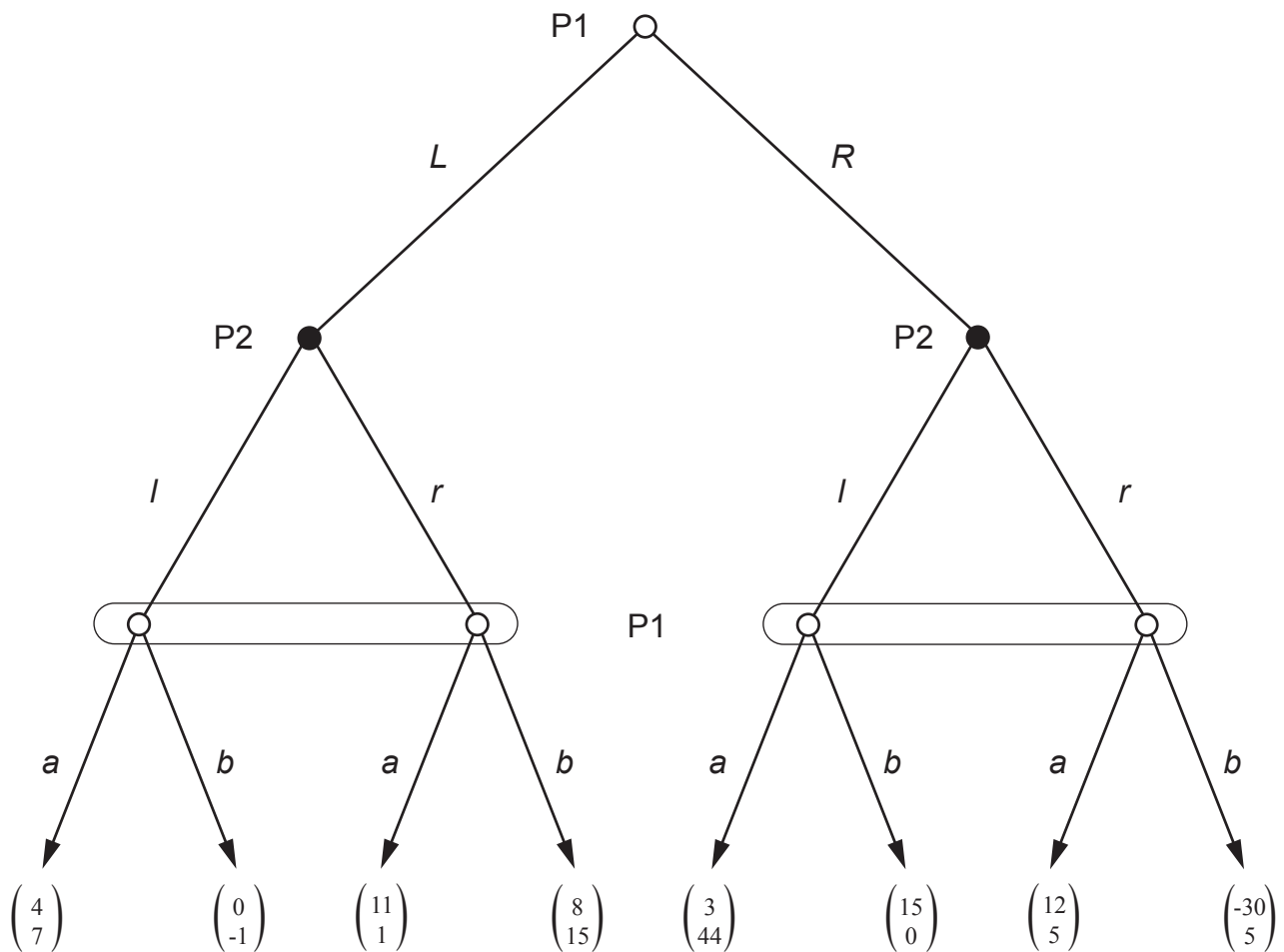# Games, Strategies and Negotiations
exercises

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Please start by attempting as many exercises as you can. This document will be updated with more exercises in coming 2 weeks.
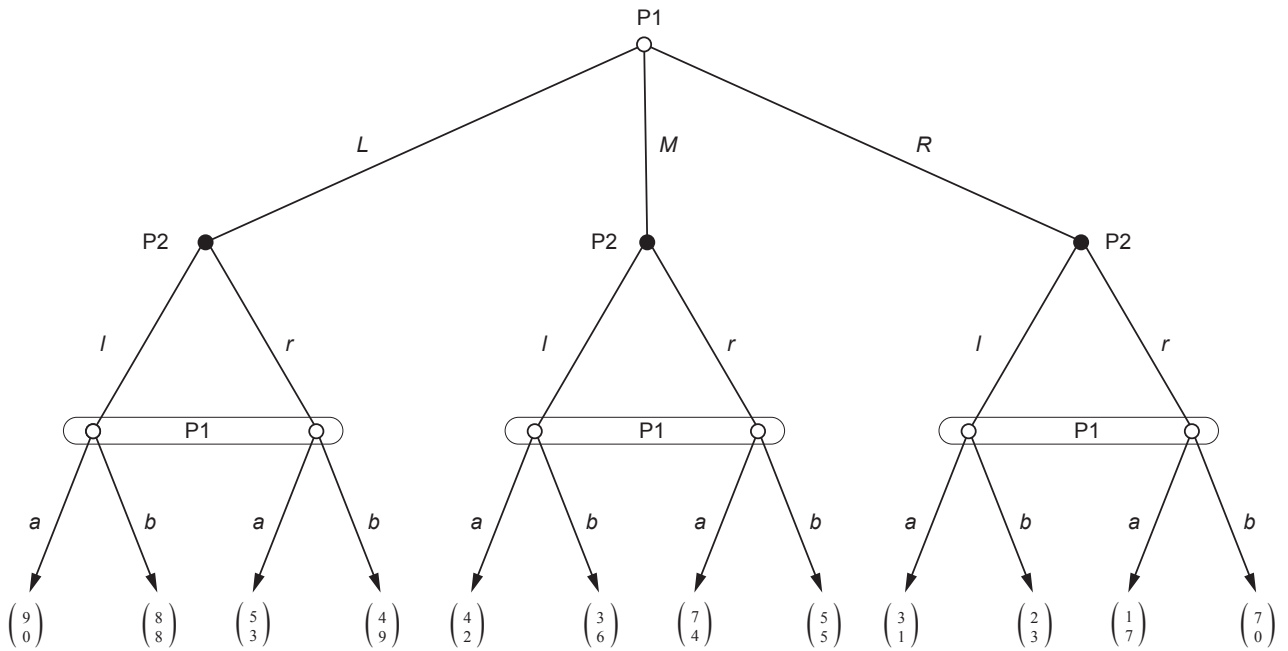
1. For the extensive form game in figure 1. give the normal (strategic) form representation of the game. Find all Nash equilibria of the game

Figure 1: Game 1

2. (Stolen from Lambros Pechlivanos and the other class in games for negotiations!). For the extensive form game in figure 2. give the normal (strategic) form representation of the game. Find all Nash equilibria of the game

Figure 2: Game 2

P1

L   M   R

P2   P2   P2

l   r   l   r   l   r

P1   P1   P1

a   b   a   b   a   b   a   b   a   b   a   b

$\binom{9}{0}$ $\binom{8}{8}$ $\binom{5}{3}$ $\binom{4}{9}$ $\binom{4}{2}$ $\binom{3}{6}$ $\binom{7}{4}$ $\binom{5}{5}$ $\binom{3}{1}$ $\binom{2}{3}$ $\binom{1}{7}$ $\binom{7}{0}$

3. Consider the extensive form game in figure 3.

   (a) give the normal (strategic) form representation of the game.

   (b) Find all Nash equilibria of the game

   (c) Describe if any of these N.E. consists a threat by one of the players. Why?

   (d) Does the threat survive backwards induction? What are the SPNE of the game?

Figure 3: Game 3

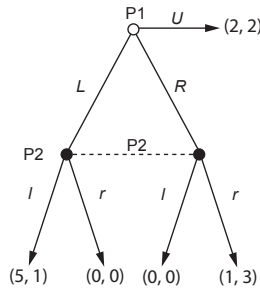P1
U $\binom{2}{2}$

D

P2

l   r

$\binom{3}{1}$ $\binom{0}{0}$
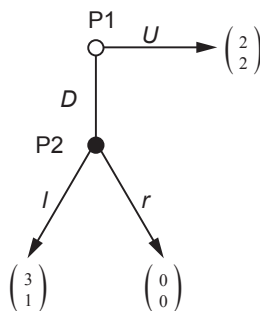
4. Consider the extensive form game in figure 4.

   (a) give the normal (strategic) form representation of the game (note the information set when P1 playes L or R).

   (b) Find all Nash equilibria of the game

   (c) Can you think whether we can solve for SPNE and why or why not?

   (d) What are the SPNE of the game?

Figure 4: Game 4

5. Consider the following strategic form game:

		Player 2	
		t1	t2
Player 2	s1	10, 0	5, 2
	s2	10, 11	2, 0

   (a) are there any strictly dominated strategies?

   (b) find the NE of the game

   (c) if you proceed with Iterated Deletion of Weakly Dominated Strategies, what is/are the solutions(s) of the game?

   (d) Can you see how erasing a weakly dominated strategy can get rid of possible Nash Equilibria of the Game?

   (e) Can you argue that the Nash equilibrium you deleted with IDWDS, was possibly the most probable way to play the game?

6. Consider the following strategic form game:

		Player 2		
		t1	t2	t3
	s1	100, 0	5, 1	4, -200
Player 2	s2	10, 100	5, 0	0. -100

   (a) are there any strictly dominated strategies?

   (b) find the NE of the game

   (c) if you proceed with Iterated Deletion of Weakly Dominated Strategies, what is/are the solutions(s) of the game?

   (d) Can you see how erasing a weakly dominated strategy can get rid of possible Nash Equilibria of the Game?

   (e) Can you argue that the Nash equilibrium you deleted with IDWDS, was possibly the most probable way to play the game?

7. Consider the following strategic form game:

		Player 2			
		t1	t2	t3	t4
	s1	200, 6	3, 5	4, 3	0, -1000
Player 2	s2	0, -10000	5, -1000	6, 3	3, 20

   (a) What are the NE of the game?

(b) Put yourselves in the shoes of player 2 (column). If you played this game without pre-play negoti-ation, what do you think your most probable play would be? What do you think the most likely outcome of this game might be?

(c) What does this make you think about NE?

8. Consider the following strategic form game:

<table>
<tr><td></td><td></td><td colspan="3">Player 2</td></tr>
<tr><td></td><td></td><td>t1</td><td>t2</td><td>t3</td></tr>
<tr><td></td><td>s1</td><td>12, -1</td><td>4, 5</td><td>8, 2</td></tr>
<tr><td></td><td>s2</td><td>11, 11</td><td>2, 9</td><td>-5, 12</td></tr>
<tr><td>Player 2</td><td>s3</td><td>5, -3</td><td>3, 6</td><td>9, 7</td></tr>
</table>

(a) What are the NE of the game?

(b) Suppose the game is played twice. Can you give 4 SPNE of the game involving NE of the static game?

(c) In the twice repeated game with no discount, can you think of another SPNE of the game in which both players do better than in any of the previous SPNE of the game?

(d) (Difficult, don't worry if you can't solve it) Suppose that the players discount the future with discount factors δ_1, δ_2. What are the minimum values of δ_1, δ_2 for which the SPNE of question 8c continues to be a SPNE of the twice repeated game?

(e) (More difficult, only for those of you who are ok with maths and infinite series, otherwise don't worry, I included this just for fun!) Suppose that the players discount the future with discount factors δ_1, δ_2. What are the minimum values of δ_1, δ_2 for which the SPNE of question 8c continues to be a SPNE of the infinitely repeated game?

9. Consider the following extensive-form games in figure 5:

(a) Convert them to strategic form representation

(b) Find the Nash equilibria of the game (whenever possible)
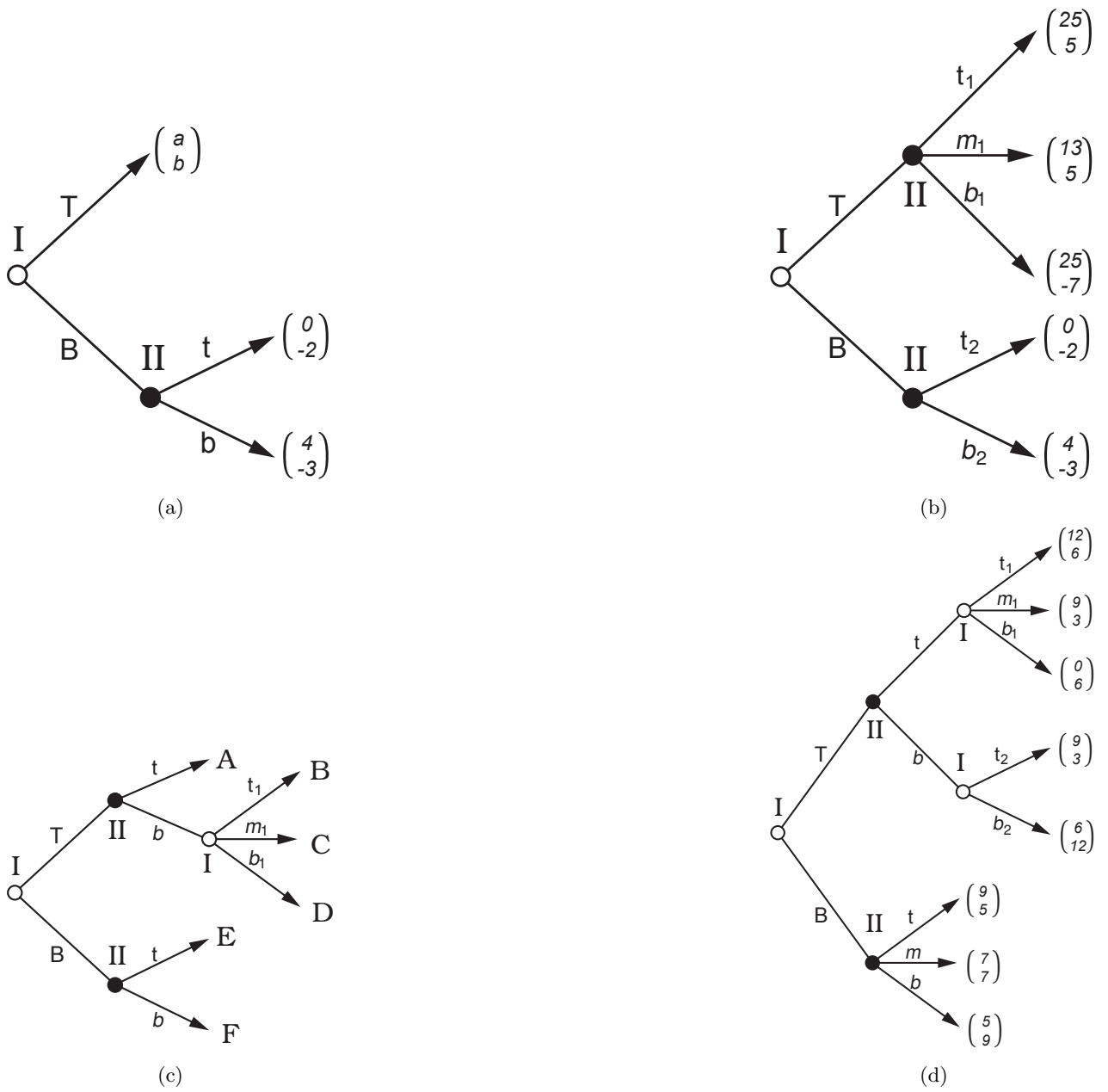
(c) Find the SPNE of the game (whenever possible)

(a)

(b)

(c)

(d)

Figure 5

10. Find all of the rationalizable strategies in the following games

Player 2

	c1	c2	c3	c4
r1	6, 2	6, 3	7, 6	2, 8
r2	8, 5	6, 9	4, 6	4, 7

Player 1

(a)

Player 2

	c1	c2
r1	9, 5	5, 3
r2	8, 6	8, 4

Player 1

(b)

Player 2

	c1	c2	c3	c4
r1	-1, 20	-7, -7	-1, 2	-5, 8
r2	27, 20	13, -1	21, 2	13, -1
r3	-5, 20	-3, 5	7, -1	3, -4

Player 1

(c)

Player 2

	c1	c2	c3	c4
r1	3, 7	0, 13	4, 5	5, 3
r2	5, 3	4, 5	4, 5	3, 7
r3	4, 5	3, 7	4, 5	5, 3
r4	4, 5	4, 5	4, 5	4, 5

Player 1

(d)

11. For each of the following normal form games,

    (a) indicate the set of rationalizable strategies for each player

    (b) give the Nash equilibrium/equilibria of the game

    (c) explain which strategy profile you think is the most likely to be played. Is it always the NE?

	s1	s2	s3	s4
t1	1, 6	2,2	5,4	9,1
t2	-1, 12	3, -1	2, -3	7, -2
t3	0, 9	6,5	4, 6	2, 4
t4	3, 0	8, 0	2, 1	11,-1

(a)

	s1	s2
t1	2,3	4,3
t2	0,0	4,6

(b)

	s1	s2	s3	s4	s5	s6
t1	0, 1	4, 6	-7, 11	1, 21	5, 2	0, 12
t2	22 , -2	-4, 25	7, 30	0, -9	3, -1	7, 22
t3	12, 4	-3, 9	4, -4	0, -2	1, 5	-1, 11
t4	-1, 0	-2, -2	9, 0	1, 12	4, 1	8, 3
t5	-9 ,2	6, 15	11, 12	0, 14	30, 3	8, 18
t6	10, 11	-5, 40	0, -3	-1,-7	0, 0	-2, 30

(c)

	a	b	c	d
ϵ	-1, 1	-2, -1	-1, 2	0, 0
δ	2, 1	1, 0	0, 1	0, 0
γ	0, 3	-1, -1	1, 0	1, -1
β	7, 1	0, 0	7, 4	0, 0
α	0, 0	-1, 1	1, 3	1, 4

(d)

|	c1	c2	c3	c4
r1	1000, -5000	2000, -500	7, 5	2000, -1000
r2	7, 300	2000, 15	3, 15	7, -2000
r3	20, 13	20, -1000	-2, 11	20, 12
r4	1500, -20000	8, 17	7, 15	6, -3000

(e)

	c1	c2
r1	0, 11	14, 11
r2	0, 25	14, 25

(f)

12. For the game depicted in figure 6 below player 1 (I) moves at white nodes, player 2 (II) at black nodes and player 3 at blue nodes as indicated with the relevant latin numerals next to each node.

    (a) find the Nash equilibrium/a using backwards induction

    (b) find (at least) one additional Nash equilibrium that is not subgame perfect

Figure 6
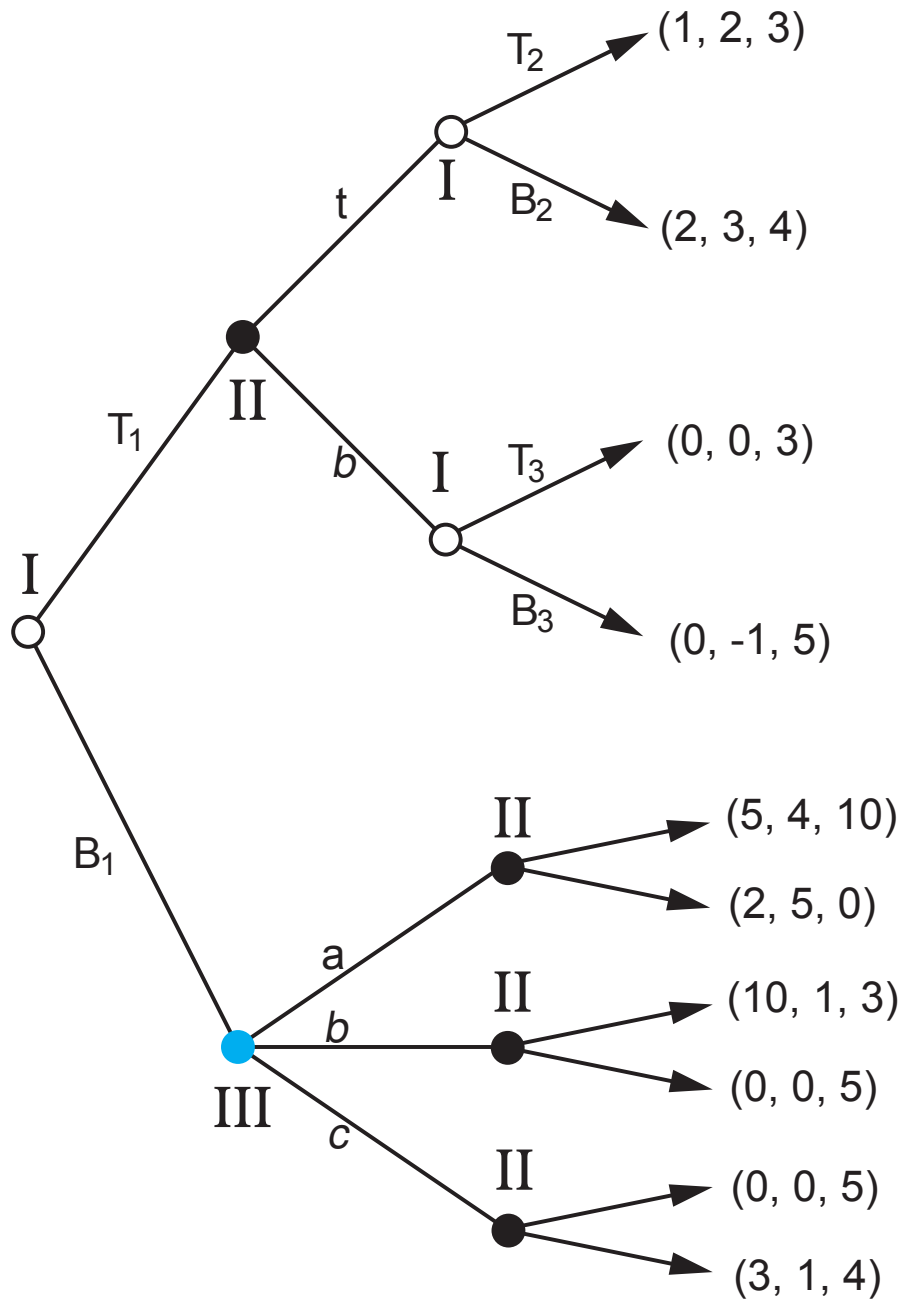
13. *** For the game depicted in figure 6 above
    (a) How many strategies does each player have?
    (b) how many strategy profiles are there?
    (c) what are the strategies of each player?
    (d) can you give a strategic form representation, in which, Player 1 chooses row, player 2 chooses column and player 3 chooses table (prepare that is one table for each of Player's III strategy)?

14. For the each of the following strategic form games:
    (a) find the strictly dominated strategies and delete iteratively
    (b) find the rationalizable strategies

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(c) find the Nash equilibria

(d) discuss if you think some Nash equilibria are more likely to be played than others (when appropriate)

(e) discuss incentives the different players might have to stir into a particular equilibrium (when appropriate)

	L	C	R
U	2,0	1, 1	4, 2
M	3, 4	1, 2	2, 3
D	1, 3	0, 2	3, 0

(a)

	L	C	R
U	1, 0	3, 0	2, 1
M	3, 1	0, 1	1, 2
D	2, 1	1, 6	0, 2

(b)

	t1	t4
s1	5, 3	3, -2
s2	-1, 2	5, -5
s3	2, 5	1, 12
s4	4, 1	2, 7

(c)

	L	R
U	1, 3	4, 3
D	1, 7	4, 7

(d)

	a	b
U	100, 100	0, 0
D	0, 0	1, 1

(e)

	L	C	R
T	11, 22	0, 7	11, 24
M	44, 2	-4, -8	0, 0
B	37, 9	41, 9	41, 9

(f)

	t3	t5
s1	0, 5	7, 6
s2	9, 4	5, -5
s4	3, 21	6, 7

(g)

	t1	t2	t3	t4
s1	5, 3	-1, 2	2, 5	4, 1
s2	4, 0	6, -1	3, 1	0, -1
s3	0, 5	9, 4	0, 9	3, 21
s4	3, -2	5, -5	1, 12	-1, 7

(h)

	t1	t2	t3	t4	t5
s1	5, 3	4, 4	0, 5	3, -2	7, 6
s2	-1, 2	6, 3	9, 4	5, -5	5, -5
s3	2, 5	3, 6	0, 9	1, 12	1, 12
s4	4, -2	11, -1	3, 21	2, 7	6, 7
s5	15, 4	3, 21	3, 5	4, 2	4, 0

(i)

	t1	t2	t3	t4
s1	5, 3	4, 4	0, 5	3, -2
s2	-1, 2	6, 3	9, 4	5, -5
s3	2, 5	3, 6	0, 9	1, 12
s4	4, -2	0, -1	3, 21	2, 7

(j)

	t1	t2	t3	t4
s1	5, 3	4, 0	0, 5	3, -2
s2	-1, 2	6, -1	9, 4	5, -5
s3	2, 5	3, 1	0, 9	1, 12
s4	4, 1	0, -1	3, 21	2, 7

(k)

	s1	s2
t1	8, 8	15, 0
t2	0, 16	11, 14

(l)

	L	R
U	2, 7	4, 7
D	2, 3	4, 7

(m)

|	s1	s2	s3	s4	s5
t1	2, 3	0, 7	4, 5	9, 9	1, 4
t2	2, 7	4, -2	-1, 0	0, -3	4, -4
t3	5, 0	6, 3	0, 2	1, 3	- 5, 1
t4	3, 23	0, 4	14, 3	0, 0	12, 2
t5	11, 10	4, 11	-5, 10	9, 10	9, 9

(n)

15. Consider the extensive form game in figure 7.

(a) give the normal (strategic) form representation of the game (note the information set when P1 playes L or R).

(b) Find all Nash equilibria of the game

(c) Is there any NE that contains what you could consider to be a threat? Which and why?

(d) What are the SPNE of the game?

(e) Are all threats removed when considering SPNE?

Figure 7
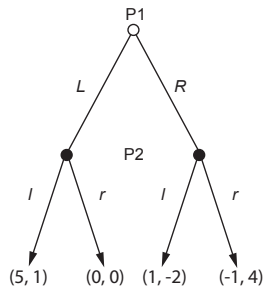
16. Consider the extensive form game in figure 8.

    (a) give the normal (strategic) form representation of the game (note the information set when P1 playes L or R).

    (b) Find all Nash equilibria of the game

    (c) Is there any NE that contains what you could consider to be a threat? Which and why?

    (d) What are the SPNE of the game?

    (e) Are all threats removed when considering SPNE?

Figure 8
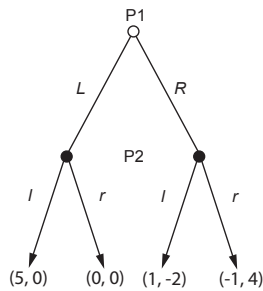
17. Consider the following game in strategic form representation

	L	C	R
T	4, 6	11, -9	4, 7
M	11, 7	9, 0,	1, 12
B	5, 8	12, 13	0, 14

(a) What are the NE of the game?

(b) If the game is repeated infinitely, can there be other strategy profiles (other than the NE that is) that can emerge in equilibrium as a SPNE?

(c) Do both players benefit from switching to strategy profiles that are not NE of the stage game? Explain briefly how this is achievable. What could it mean for long term negotiations?