International Negotiations
Games, Strategies and Negotiations
Dynamic Games

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 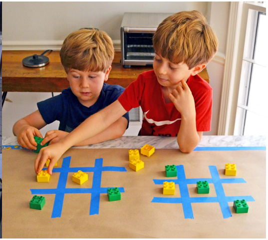

Why do we need dynamic analysis?

- Often economic transactions take place in stages. E.g. negotiations: offer, acceptance, or counter-offer, followed by acceptance, rejeciton or counter-offer etc.
- Threats/promises. By their nature need stages: (rejection$\longrightarrow$make good on threat/promise)
- Many negotiations with long-term partners are repeated in time
- A producer enters a market (moves first). Incumbents react to entrant's move (e.g. with a price war). Following that the entrant might have to decide his/her course of action

Example: a game of threat

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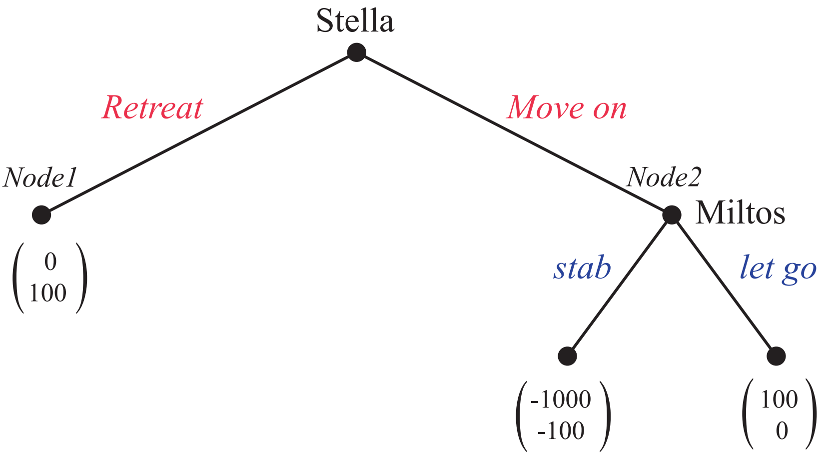
- Miltos, a working-class guy is left at the alter
- He sees his bride-to-be, Stella next morning in the street
- He threats to kill her if she doesn't repent or leave
- What is her best course of action? Leave or stay?
- Assume that getting killed is the worst outcome (u(getting killed) $= -1000$), followed by killing someone (u(imprisonment) $= -100$), followed retreating (u(retreating) $= 0$), followed by saving face (not backing off) (u(saving face) $= 100$)

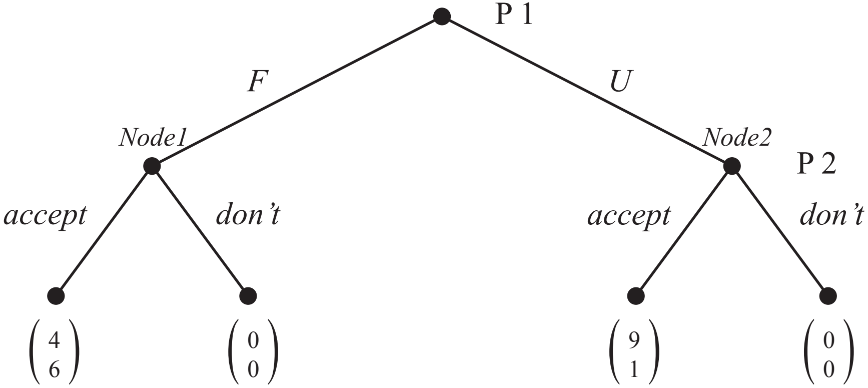
Example: a game of threat



Example: a(nother) game of threat

- Player 1 decides whether to offer player 2 favourable terms on a deal or not
- Player 2 decides whether to accept player's 1 conditions (favourable or unfavourable)
- If the terms are accepted, the deal goes through. If not the players walk away with zero payoffs
- Let's represent this game in extensive form:

A game of threat



Strategies in dynamic games

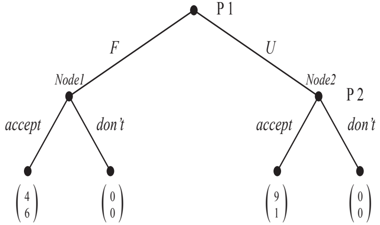
- To analyse how this game might be played out, we need to define the notion of strategy for each player in a dynamic game
- For player 1 a strategy would be to play either F(avourable) or U(nfavourable). He has 2 strategies
- For player 2 however, a strategy is a set of instructions about how to play the game IN EVERY POSSIBLE SITUATION SHE MIGHT FIND HERSELF IN
- A strategy is a full manual that player 2 can give to a third party and have them (the 3rd party) play the game in her place as her proxie. $\longrightarrow$ A strategy should contain instructions about how the game should be played in node 1 *as well as* in node 2

Players' strategies

Let's write down the two players' strategies:

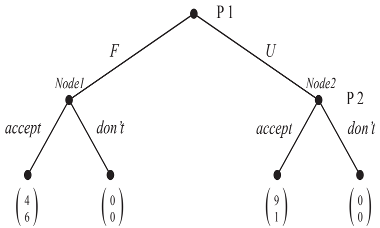
- Player 1 has two strategies: F and U
- Player 2 on the.other hand has 4 strategies: all the possible combinations of action in the two nodes he might be called upon to play:
  1. Strategy 1: {accept, accept} [Play "accept" if you find yourself playing at node 1 and "accept" if you find yourself playing at node 2]
  2. Strategy 2: {accept, don't accept} [Play "accept" if you find yourself playing at node 1 and "don't accept" if you find yourself playing at node 2]
  3. Strategy 3: {don't accept, accept} [Play "don't accept" if you find yourself playing at node 1 and "accept" if you find yourself playing at node 2]
  4. Strategy 1: {don't accept, don't accept} [Play "don't accept" if you find yourself playing at node 1 and "don't accept" if you find yourself playing at node 2]

Extensive and formal (or strategic) form representation of this game of threat



	a-a	a-d	d-a	d-d
F	4,6	4,6	0,0	0,0
U	9,1	0,0	9,1	0,0

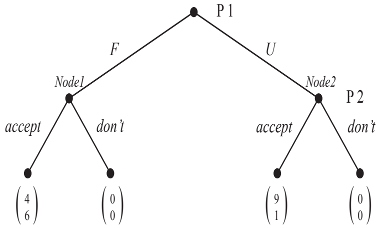
Nash equilibria of the game



	a-a	a-d	d-a	d-d
F	4,6	4,6	0,0	0,0
U	9,1	0,0	9,1	0,0

- We note that the game has 3 Nash equilibria
- If P1 plays F, P2 wants to play [a-d] and the other way around
- If P1 plays U, P2 achieves her most wanted outcome either by [a-a] or by playing [d-a] and reversely: if P2 plays either [a-a] or [d-a], P1 prefers to play U

A special equilibrium



	a-a	a-d	d-a	d-d
F	4,6	4,6	0,0	0,0
U	9,1	0,0	9,1	0,0

- The equilibrium {F and [a-d]} is of particular interest: P2 forces P1 to give P2 favourable conditions by threatening P1 that he will cause a collapse of the negotiations
- How credible is such a threat?
- Suppose we deem this threat not credible. Is there a way to get rid of Nash equilibria that contain non-credible threats?

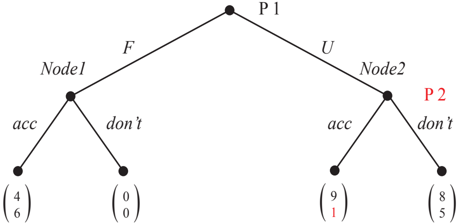
Subgame Perfect Nash Equilibrium (SPNE)

- The equilibrium in which P2 fell for player's 1 threat, might not be an acceptable way to play the game, particularly if P2 is sophisticated or experienced
- The reason why such an outcome bothers us is because it entails falling for a bluff: P1 threatens to hurt himself as well and P2 believes him
- To avoid predictions that the game will be played in such a way that some players fall for non-credible threats, SPNE solves the game by backwards induction: starting from the end and solving for optimal strategies backwards towards the beginning
- That is we go to the final nodes of the game and examine what the player playing at the penultimate node would play. Whatever is a “wrong” move is deleted. Next we proceed to the previous node doing the same thing until we have erased all strategies that could never be played

Subgame Perfect Nash Equilibrium (SPNE)

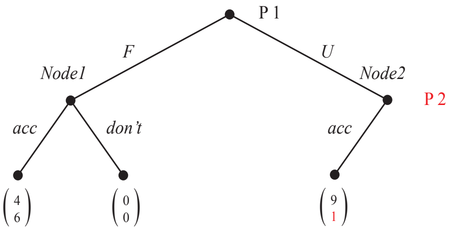
- Before we give a proper definition of a subgame, let's see how we can proceed to search for SPNE

Subgame Perfect Nash Equilibrium (SPNE)



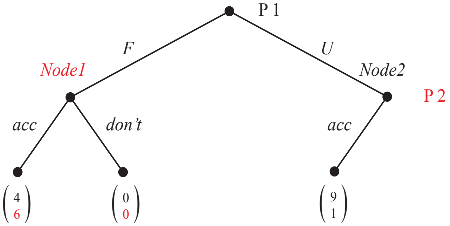
Let's compare P2's payoffs when she chooses node 2 [accept] with her payoffs when she chooses [don't] (red colour)

Subgame Perfect Nash Equilibrium (SPNE)



- If P2 ever reached node 2, she would definitely choose [accept]

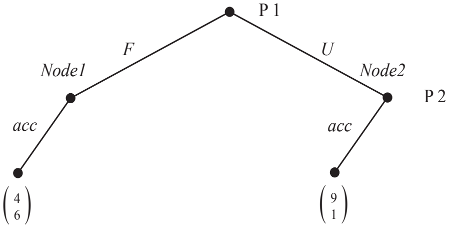
Subgame Perfect Nash Equilibrium (SPNE)



Let's compare P2's payoffs when she chooses node 1 [accept] with her payoffs when she chooses [don't] (red colour)

- If P2 ever reached node 1, she would definitely choose [accept]

Subgame Perfect Nash Equilibrium (SPNE)



- If P2 ever reached node 1, she would definitely choose [accept]
- If P2 ever reached node 2, she would definitely choose [accept]

Subgames

Now that you have seen how we solve for a SPNE, we can give a more proper definition of a subgame:

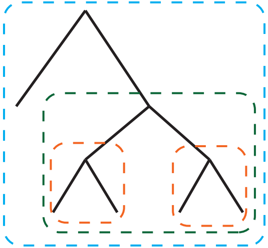
Definition (Subgame)

A subgame of an extensive-form game is a subset of the game with the following properties:

1. *It starts from an information set that contains only one node*
2. *It contains all the subsequent nodes*
3. *If the subgame contains a node x, then every other node that belongs in the same information set as x must also belong to the subgame*

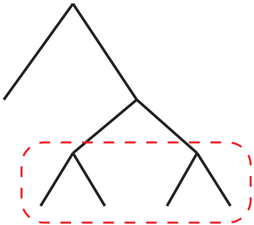
Subgames

Let's see some examples of subgames:



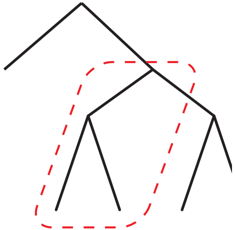
Subgames

It might help to understand better what a subgame is if we see counter examples to the three conditions given in the definition. For example let's see what it would mean if the first condition was violated



Subgames

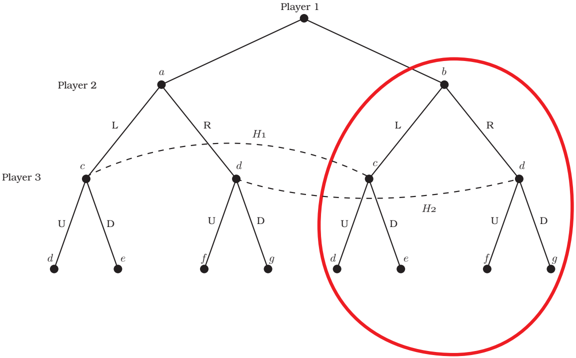
Or what it would mean if the second condition was violated



This is not a subgame

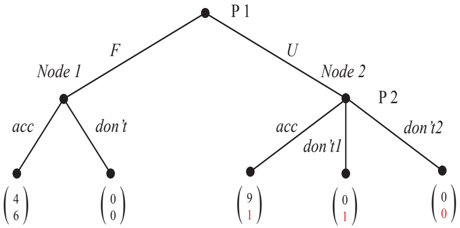
Subgames

And this is what a violation of the third condition might look like:



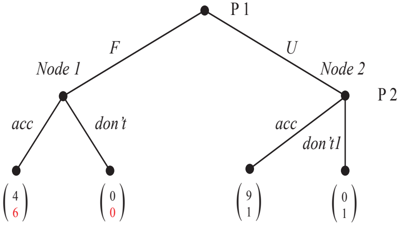
The subset encircled by red is not a subgame. It abides by conditions 1 and 2 but not by condition 3: the subset breaks the information set H2!

A credible threat



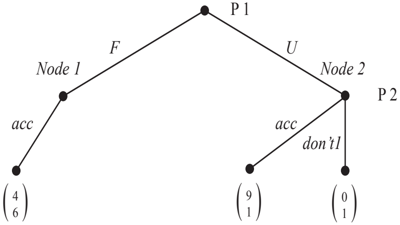
- Let's revisit the favourable-unfavourable terms negotiation game. What will happen if P2 except from accepting P1's offer has an outside alternative offer by another shadow party, which is at least as good as P1's unfavourable terms? So that he can refuse all offers, accept the third party's offer (don't 1) or accept P1's offer
- If that is the case, the threat to refuse P1's offer is now credible. Let's examine P2's play in the final nodes
- If he is brought to play at node 2, he will choose either [accept], or [don't 1], both of which are equally good for P2

A credible threat



- P2 would never play [don't2]
- What will she do if she is brought to play at node1?
- There she can choose only [accept]

A credible threat



- P2 would never play [don't2]
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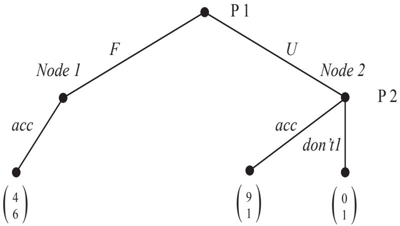
- There she can choose only [accept]

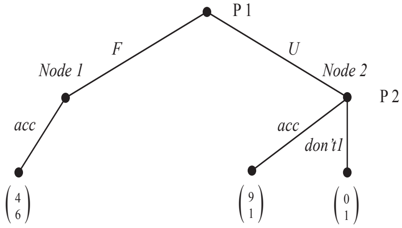
A credible threat



- Hence P1 would only play [accept] at node1 and at node2, she would play either [accept] or [don't1]

- P2 has 2 possible strategies in any SPNE (remember STRATEGY = COMPLETE COURSE OF ACTION):
- Strategy 1: [accept, accept]
  Strategy 2: [accept, don't1]
- both are credible because in neither of the two does P2 inflict harm upon herself

A credible threat



- Against P2's strategy 1, P1 would choose U (which would guarantee him 9)
- Against P2's strategy 2, P1 would choose F and end up with 6

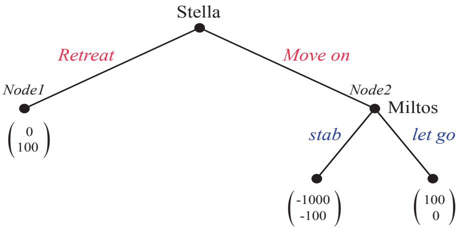
- This game has 2 SPNE
- SPNE 1: { P1: U, P2: [accept, accept]}
  SPNE 2: { P1: F, P2: [accept, don't1]}
- By making a credible threat of playing [don't1] if he is brought to play at node2, P2 can lead the game to the more favourable equilibrium for her, that is SPNE 2 and end up with a payoff of 6

Dynamic analysis

- Dynamic analysis focuses on strategic interactions in environments in which players move serially
- The notion of SPNE takes into account that some threats are not credible and focuses on equilibria that contain strategies which do not fall for such bluffs
- Dynamic analysis allows us to analyse the rational component of strategic behaviour (and can of course often be mistaken as people are very often ... less than rational...)
- Let's now examine some more Dynamic games and their equilibria

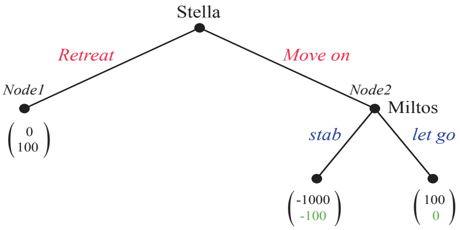
Remember the game from the film Stella (1955)?

Stella (1955)



- What would Miltos play if Stella keeps moving towards him?
- He should clearly let her go if she "called his bluff"

Stella (1955)



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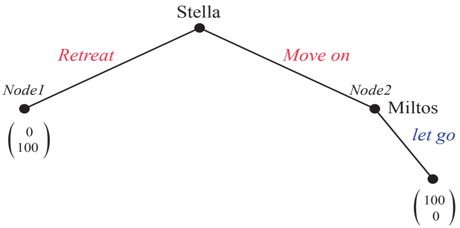

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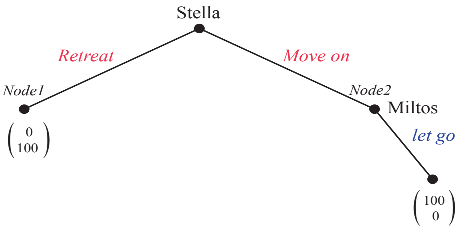


- What would Miltos play if Stella keeps moving towards him?
- He should clearly let her go if she “called his bluff”

- Hence the only SPNE-compatible strategy for Miltos would be: [let go if brought to node2]
- and hence, solving backwards, given that in node2 Miltos would let her go, Stella’s best course of action is to actually move on. . .

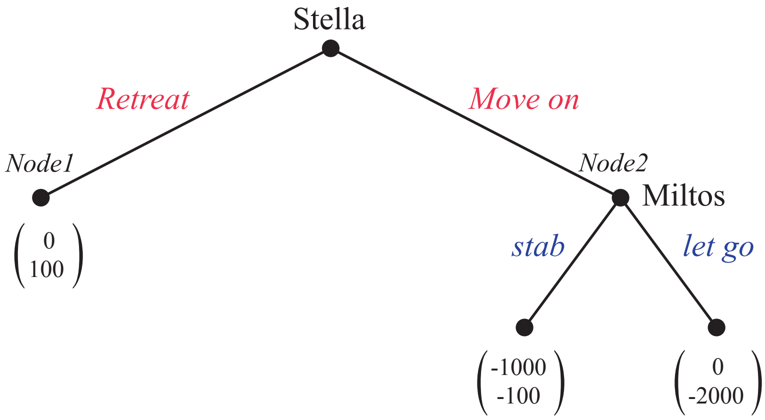
Stella (1955)

- Actually Stella moves on according to SPNE prediction, however ...
- When she reaches Miltos, he actually ... stabs her ...
- where did our analysis go astray?
- we can make some conjectures

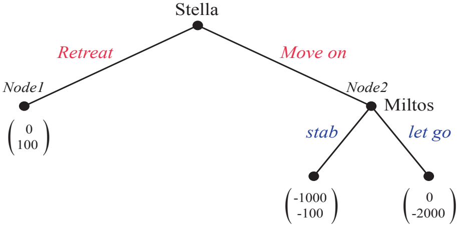
Stella (1955) and rationality

- One possible explanation is that although Stella is rational and plays according to game theoretic predictions, Miltos is not (and Stella doesn't know it to incorporate Miltos' irrationality in her decision process)
- Another possible explanation is that we got Miltos' payoffs wrong. Suppose that for Miltos, losing face is far worse (in an almost pre-modern society) than actually ending up in jail. Then Miltos' payoffs could look something like this:

Stella (1955) and correct payoffs

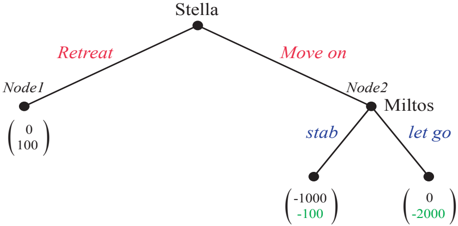


Stella (1955) and correct payoffs



- Would such payoffs suffice to explain the observed outcome?
- well, not quite. Let's solve by backwards induction:

Stella (1955) and correct payoffs



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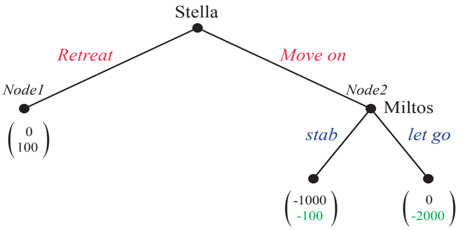
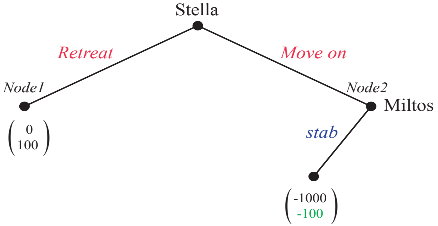


- At node2, Miltos would choose stab ($-100 > -2000$)
- Miltos' only rational strategy would be to [stab if at node2]

Stella (1955) and correct payoffs



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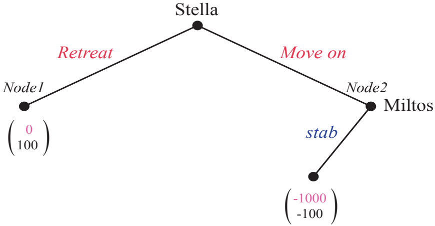
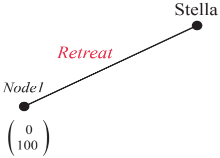


- But then it would never be rational for Stella to move on
- Only SPNE: [Retreat, stab at node2]

Stella (1955) and correct payoffs

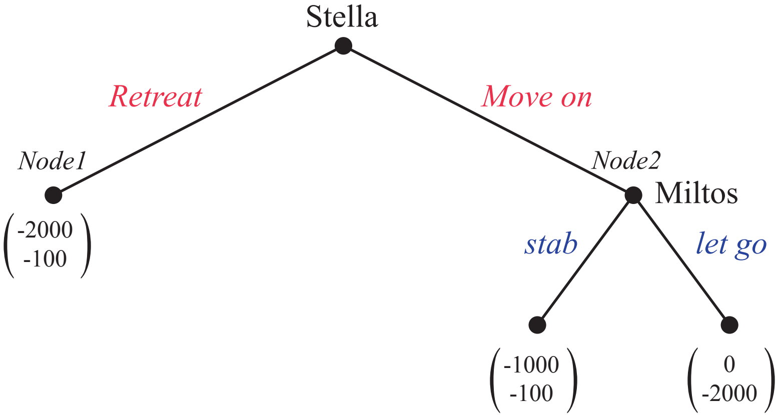


- But then it would never be rational for Stella to move on
- Only SPNE: [Retreat, stab at node2]

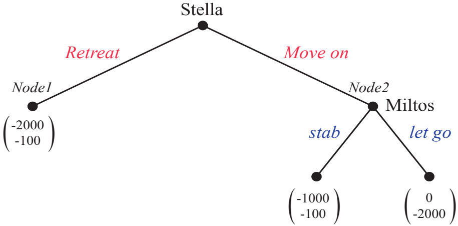
Stella (1955) and correct payoffs

- So, under these payoffs, whereas it is rational for Miltos to stab, it is not rational for Stella to move on. These payoffs cannot explain our heroes' behaviour
- We need a slight recalibration of Stella's payoffs to explain what actually happened in the film:

Stella (1955) and correct payoffs

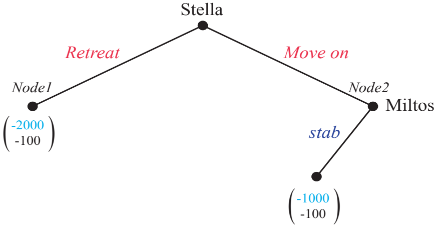


Stella (1955) final payoffs



- The subgame starting with Miltos' move is unchanged:
- Miltos' only rational strategy would be to [stab if at node2]

Stella (1955) final payoffs



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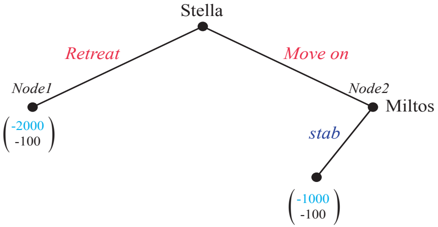
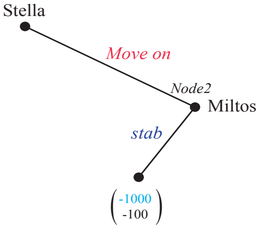
Stella (1955) final payoffs



- But now Stella does not wish to retreat
- under new payoffs, losing face is worse than dying

Stella (1955) final payoffs



- Hence only SPNE of the re-calibrated game is:
- [Stella: move on, Miltos stab if Stella moves on]

Stella (1955) final payoffs

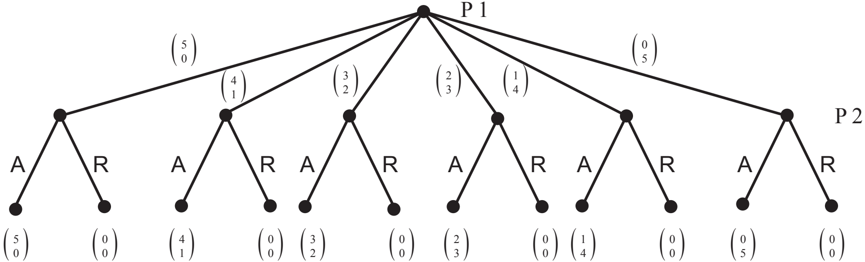
- Such payoffs are likely more compatible with our characters' personalities and this sets up the film's tragic ending!
- Lesson to be taken away: in cases of strategic interaction, it is very important to GET THE PAYOFFS RIGHT
- It is also equally important to correctly assess players' rationality/irrationality, as these can tilt the outcomes to whole new directions

The ultimatum game

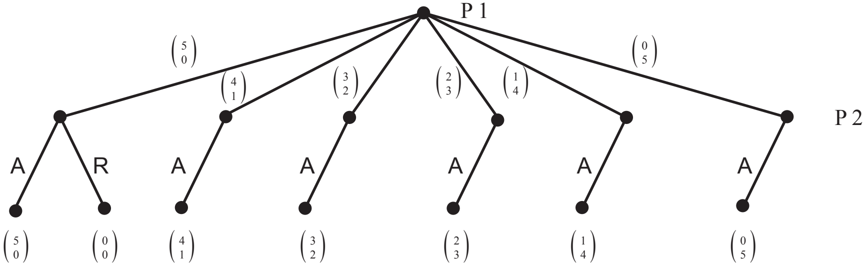
Divide €5, by €1 increments

- The negotiation takes the form of an ultimatum (often parties try to do this in negotiations. Why?):
- P1 suggests a split to P2
- P2 can either accept P1's proposal in which case each gets the amount proposed by P1, or reject the proposal. If P2 rejects both players end up with nothing
- SPNE: [P1 suggest keeping the whole amount (except for a very small amount - the smallest possible subdivision of the amount- which he offers P2. P2 accepts]

The utimatum game with discrete amounts

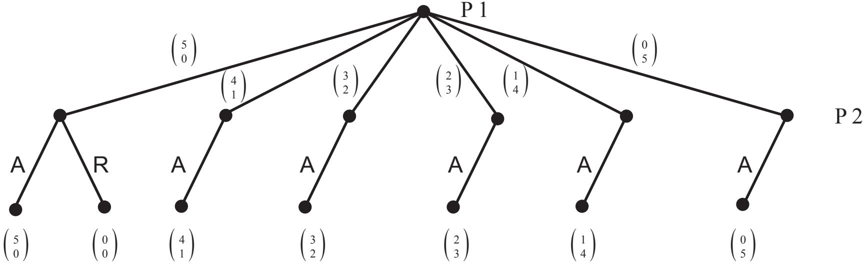


The utimatum game with discrete amounts



- P1 Might choose Refuse only in node 1. In all other nodes she would pick accept. Hence there can be only two possible strategies for P2 in a SPNE: Strategy 1: [RAAAAA]
  Strategy 2: [AAAAAA]

The utimatum game with discrete amounts



- 2 SPNE:

  SPNE 1: {P1: $\begin{pmatrix}4\\1\end{pmatrix}$, P2: [RAAAAA] }

  SPNE 2: {P1: $\begin{pmatrix}5\\0\end{pmatrix}$, P2: [AAAAAA] }

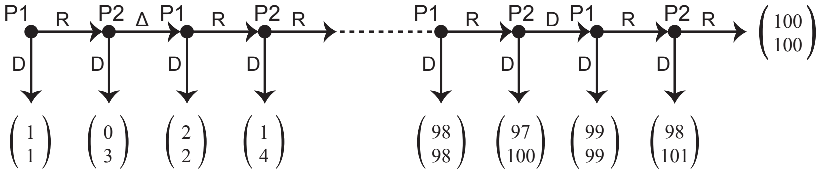
The ultimatum game with continuous payoffs

- The two players split €S. P1 suggests any real division and P2 either accepts or refuses
- unique SPNE: [P1: suggests she keeps $S - \epsilon$ and hands P2 ϵ. P2: accepts]

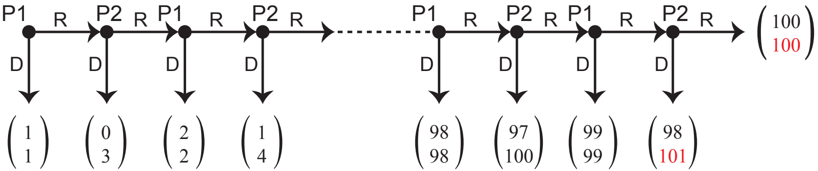
The ultimatum game

- The ultimatum game predicts that the party that gives the ultimatum will seize most of the prize. P2 will go away with something close to zero
- What would we expect in such a case in reality?

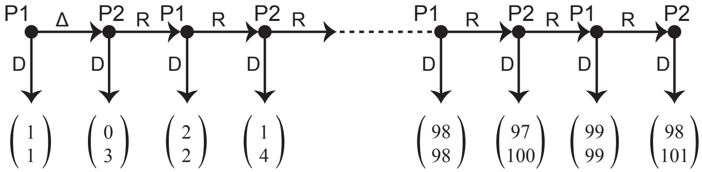
The centipede



The centipede



The centipede



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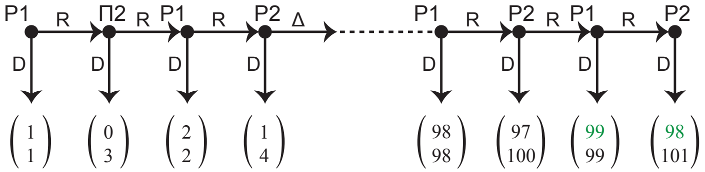

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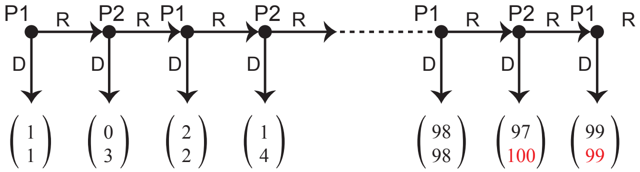

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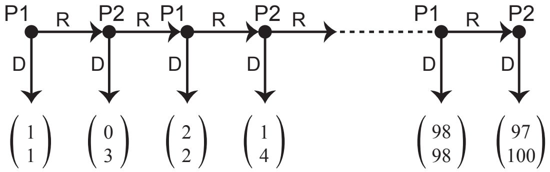

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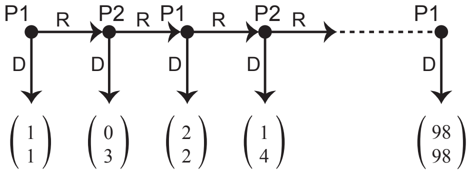

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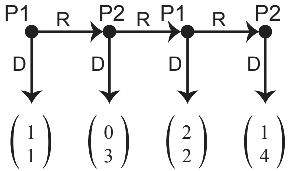

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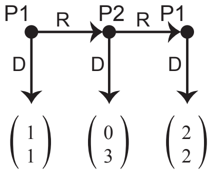
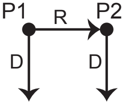


The centipede



$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

The centipede

Repeated games

- In many negotiations, the parties come together more than once to repeat their strategic interaction (think of a negotiation each year with a long term supplier to fix prices and quantities supplied)
- A repeated game is a strategic (normal) form game that is repeated either finitely or infinitely many times t, where $t \in \{1, 2, \ldots T\}$ (T may be equal to ∞)
- What are the equilibria of repeated games?
- If S_t is the set of actions in each stage t
- A strategy in a repeated game specifies an action at any given point t for any history of actions $H(t) = S_1 \times S_2 \times \ldots \times S_{t-1}$

Repeated games

- In simple words, a strategy specifies an action for any kind of play by the players so far
- The strategy space of repeated games grows super-exponentially with T
- Repeated games have much much richer strategy spaces than the stage game
- And the Nash equilibria of repeated games are much richer than the N.E. of the stage game
- What can we say about the NE of repeated games?

Repeated games

Theorem

If $S_1, S_2, \ldots S_n$ are NE of the stage game, then any strategy profile that specifies any $S_i \in \{S_1, S_2, \ldots S_n\}$ for each stage of the repeated game is a N. E. of the repeated game

- Hence playing a NE in every stage is a NE of the repeated game
- However, as we shall see, there might also be other NE of the repeated game