International Negotiations
Games, Strategies and Negotiations
Game Theory, solution concepts: Nash equilibrium

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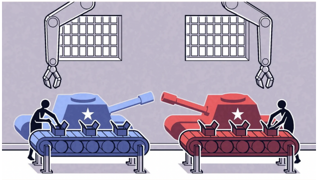

The battle of the sexes ♀♂

The battle of the sexes ♀♂

- Sometimes it is not obvious how we want to play the game, irrespective of what the other players do
- There are cases where our preferred move depends on what other parties play
- A couple wishes to meet out. They have two alternatives: either go to the fight (boxing) or go to the opera
- The man prefers football and the woman prefers the concert, but both prefer being together to being alone, even if that means agreeing to the less-preferred recreational pastime:

		Woman	
		F	O
Man	F	2, 1	0,0
	O	0,0	1,2

The battle of the sexes ♀♂

			Woman	
			F	O
Man	F		2, 1	0,0
	O		0,0	1,2

- It is no longer evident that a player always prefers to play (or not to play) one strategy. What each player wishes to play depends on what the other party will do
- If the man is certain that the woman will be in the Opera, he will choose opera because actually getting together is more important than the venue
- The prime target for the players are to coordinate. Choosing a venue is or secondary concern

The battle of the sexes ♀♂

		Woman	
		F	O
Man	F	2, 1	0,0
	O	0,0	1,2

- There is no longer an obvious way to play irrespective of what the other player does
- So, what could be a solution for such a game?
  Nash Equilibrium

The battle of the sexes ♀♂

		Woman F	Woman O
Man	F	2, 1	0,0
	O	0,0	1,2

- {Fight, Fight} is a Nash equilibrium. So is {Opera, Opera} (Two Nash equilibria in pure strategies)
- Nash equilibrium: No party has an incentive to deviate UNILATERALLY
- {Fight} is optimal for the man given that the woman chooses {fight}. And vice versa
- {Opera} is optimal for the woman given that the man chooses {Opera}. And vice versa

Strategy profile

Definition (Strategy profile)

In a game of n players, a strategy profile is a vector of n strategies, one for each player

- For example, a strategy profile for the file of the sexes could be (O, F) another could be (F, F) and so on...
- If player 1 has 2 strategies and player 2 has 3 strategies, we will have in total $2 \times 3 = 6$ strategy profiles in the game
- A (pure) strategies profile in a game of n players is denoted by $[s_1, s_2 \ldots, s_i \ldots, s_n]$

Nash equilibrium

Definition (Nash equilibrium)

Nash Equilibrium (NE) of a game of n players is a strategy profile of the game, such that each player wishes to play the strategy subscribed by the profile if all other $n-1$ players play the strategies subscribed to them by the profile

- In the battle of the sexes, both players want to play the profile (O,O) if the other player plays the profile
- The same is the case with the profile (F, F)

Another example: Stug hunt game (Rousseau: “Discourse on the origin of inequality”)

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- Stag hunt game: “Two hunters simultaneously decide whether to hunt for a deer or a hare. To catch a stag they need to coordinate: they both need to hunt in the same area. The stag gives a lot of meat. Each of the hunters can unilaterally hunt and catch a hare on their own for less meat”
- Players: the two hunters $\mathcal{N} = \{1, 2\}$.
- Strategies: the strategy set is $S_i = \{\text{S, H}\}$ for each player $i \in \{1, 2\}$.

Another example: Stug hunt game (Rousseau: "Discourse on the origin of inequality"

> *In this manner, men may have insensibly acquired some gross ideas of mutual undertakings, and of the advantages of fulfilling them: that is, just so far as their present and apparent interest was concerned: for they were perfect strangers to foresight, and were so far from troubling themselves about the distant future, that they hardly thought of the morrow. If a deer was to be taken, every one saw that, in order to succeed, he must abide faithfully by his post: but if a hare happened to come within the reach of any one of them, it is not to be doubted that he pursued it without scruple, and, having seized his prey, cared very little, if by so doing he caused his companions to miss theirs.*

J. J. Rousseau, "Discourse on the origin of inequality"

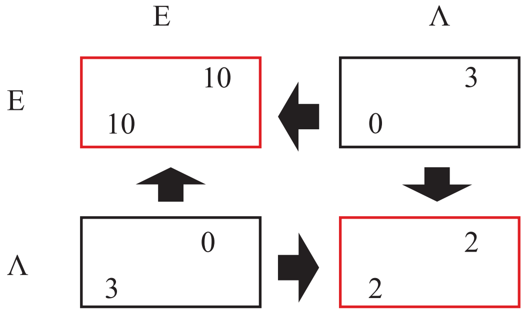
Stag-hunt game: best responses

- Let's examine the payoffs for the stag-hunt game in normal form

	S	H
S	10,10	0,3
H	3,0	2,2

Best responses

- This game has neither dominant nor dominated strategies
- Let's see the best responses for each player
- Payoffs: The players payoffs (left and slightly down) for row, and right and slightly up for column) are the players utility function $u : S \mapsto \mathbb{R}$.



Best responses

- Lets highlight row's best responses in green colour

	S	H
S	10,10	0,3
H	3,0	2,2

- and column's best responses in red colour:

	S	H
S	10,10	0,3
H	3,0	2,2

Best responses

The best response function $s_i(s_{-i})$ gives player's i optimal strategy s_i when the other players play a strategy s_{-i}.
In other words the best response function is a function that prescribes what is best for player i for each strategy of the other players

Best responses

Let's examine the best responses (BR) (green denotes row's BR and red colour denotes column's BR) in a game where player 1 has 2 strategies and player 2 has 3 strategies

	L	C	R
U	4,0	3,10	2,0
D	2,12	5,9	6,4

Best responses

And now BR for rock-paper-scissors

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Best responses and Nash equilibrium

Definition (Nash equilibrium in pure strategies)

A Nash Equilibrium of a game of N players is a strategy profile $s^* \in S$ such that for each player $i \in \mathcal{N}$:

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*)$$

for every strategy $s_i' \in S_i$ (in player's i strategy space).

- The Nash equilibrium is a strategy profile (a strategy for each player) such that the strategy prescribed by the profile for each player is a best response to the rest of the strategies in the profile
- That is, if all other players play the strategies in the profile, then player *i* also wants to play the strategy prescribed to him/her by the profile
- If all players play the profile, no player has an incentive to deviate from the profile UNILATERALLY

Examples of Nash equilibria

Let's examine the Nash equilibria in a number of games.
Remember the prisoner's dilemma:

	C	D
C	-1,-1	-9,0
D	0,-9	-6,-6

NOTICE: if IDSDS leads to a unique solution of the game, then this profile is necessarily a Nash equilibrium

Examples of Nash equilibria

The stug-hunt game has two N. E.

	S	H
S	10,10	0,3
H	3,0	2,2

Examples of Nash equilibria: the battle of the sexes with BR

	F	O
F	4,3	1,1
O	1,1	3,4

Examples of Nash equilibria: technology-adoption

Examples of Nash equilibria: technology-adoption

"Two researchers decide between Microsoft Windows or Apple Mac OS X as their operating system. Having the same OS is beneficial to both. Both researchers will be better off if they choose the Mac as their operating system (it is superior). However, a researcher will only be willing to do so if he is confident that his colleague will coordinate with him and they can work together."

- Players: Researcher 1 and 2. $\mathcal{N} = \{1, 2\}$.
- Strategies: each researcher chooses an OS: Windows or Mac: $S_i = \{\text{Windows}, \text{Mac}\}$, for $i = 1, 2$.

Examples of Nash equilibria: technology-adoption

- Payoffs: are given by the normal-form matrix

	Windows	Mac
Windows	5,5	3,2
Mac	2,3	9,9

As in the stag-hunt game, coordination is essential

A game of conflict: Hawk and dove (a.k.a. ”Chicken”)

A game of conflict: Hawk and dove

Two members of a gang contest for the leadership. They move face on towards each other inside two vehicles. Each combatant can either fight (play "Hawk") or chicken out and concede to his opponent (play "Dove"). A Hawk beats a Dove and wins the prize without conflict. Two Doves share the prize equally. Two Hawks destroy the prize and are damaged in the fight.

- Players: Combatants 1 (row) and 2 (column). $\mathcal{N} = \{1, 2\}$.
- Strategies: Each combatant chooses Hawk or Dove: $\mathcal{S}_i = \{\text{Hawk, Dove}\}$ for $i = 1, 2$.

A game of conflict: Hawk and dove

- Payoffs are given in the following normal-form matrix with Nash equilibria

	Hawk	Dove
Hawk	$-10, -10$	$10, 0$
Dove	$0, 10$	$5, 5$

A game of conflict: Hawk and dove

Hawk and dove with parametric payoffs

- Consider the Hawk and Dove with more general payoffs (NE are given)

	Hawk	Dove
Hawk	$-c, -c$	$v, 0$
Dove	$0, v$	$\frac{v}{2}, \frac{v}{2}$

A game of pig competition: When Strength Is Weakness

When Strength Is Weakness[1]

- An example from animal psychology: pigs establish dominance-subordinateness relations
- Psychologists put two pigs, one dominant, one subordinate, in a long pen. At one end of the pen was a lever that would release a portion of food to a trough located at the other end of the pen
- Which pig would push the lever? Which pig would eat first?
- Suprisingly, the dominant pig pressed the lever and the subordinate pig waited by the food and ate most of it

[1] Adapted from Hal Varian, Intermediate microeconomics

When Strength Is Weakness

- Payoffs are given in the following normal-form matrix with Nash equilibria

		Dominant pig	
		Don't press	Press
Subordinate pig	Don't press	0, 0	4, 1
	Press	0, 5	2, 3

When Strength Is Weakness

		Dominant pig: Don't press	Dominant pig: Press
Subordinate pig	Don't press	0, 0	4, 1
	Press	0, 5	2, 3

- For the subordinate pig, pressing the lever is a dominated strategy
- if the dominant pig could refrain from eating all the food and reward the subordinate pig for pressing the lever, it could gain more
- but by exercising power when it has it, it forces the subordinate pig to play hardball, as it has nothing to lose

Penalty shootout: when there is no N.E. (in pure strategies)

Penalty shootout

- Let's consider the case of a kicker and a goalie at a penalty-shoutout
- Suppose for simplicity that the kicker can either kick left or right
- And that if the goalie guesses the side right, he will catch the ball with probability 1. Otherwise the ball will go to the nets with probability 1
- Suppose the payoffs are the following:

		Goalie	
		Left	Right
Kicker	Left	0, 1	1, 0
	Right	1, 0	0, 1

Penalty shootout

- Let's consider examine the 2 players' Best Responses in colour (red for kicker, green for goalie):

		Goalie	
		Left	Right
Kicker	Left	0, 1	1, 0
	Right	1, 0	0, 1

- We see that no 2 BR for the two players exist in the same cell, meaning that the game has no NE in pure strategies
- There is of course a way to kick a penalty (randomising), but this is something we won't examine in our lectures
- Those of you who must learn how to kick a penalty are urged obtain a copy of the course material by Lampros Pechlivanos...!