$$V(\hat{\beta}) = \sigma^2 \underbrace{(X'X)^{-1}}_{\substack{k\times n \; n\times k \\ k\times k}} \Rightarrow$$

$$V(\hat{\beta}) = E\left[\underbrace{\underset{k\times 1}{(\hat{\beta}-\beta)}\;\underset{1\times k}{(\hat{\beta}-\beta)'}}_{k\times k}\right]$$

$$= \begin{pmatrix} V(\hat{\beta}_1) & Cov(\hat{\beta}_1,\hat{\beta}_2) & \cdots & Cov(\hat{\beta}_1,\hat{\beta}_k) \\ Cov(\hat{\beta}_2,\hat{\beta}_1) & V(\hat{\beta}_2) & \cdots & Cov(\hat{\beta}_2,\hat{\beta}_k) \\ \vdots & & & \\ Cov(\hat{\beta}_k,\hat{\beta}_1) & \cdots & \cdots & V(\hat{\beta}_k) \end{pmatrix}$$

$$V(\hat{\beta}_1) = \left(\sigma^2 (X'X)^{-1}\right)_{(1,1)}$$

$$V(\hat{\beta}_2) = \left(\sigma^2 (X'X)^{-1}\right)_{(2,2)}$$

$$\vdots$$

$$V(\hat{\beta}_k) = \left(\sigma^2 (X'X)^{-1}\right)_{(k,k)}$$

$$S^2 = \frac{1}{n-k} \sum_{t=1}^{n} \hat{\varepsilon}_t^2 , \quad \hat{\varepsilon}_t = y_t - x_t \hat{\beta} = y_t - \hat{y}_t$$

$\Uparrow$

ESTIMATOR OF σ^2

$$\hat{b} = (X'X)^{-1}X'Y$$

$$\hat{Y} = X\hat{b}, \quad \hat{\varepsilon} = Y - \underbrace{\hat{Y}}_{} \Rightarrow$$

$$\Rightarrow \hat{\varepsilon} = \underbrace{Y}_{} - \underbrace{X\hat{b}}_{}$$

$$s^2 = \frac{1}{n-k}\sum_{t=1}^{n}\hat{\varepsilon}_t^2 = \frac{1}{n-k}\underbrace{\underset{1\times n}{\hat{\varepsilon}'}\;\underset{n\times 1}{\hat{\varepsilon}}}_{1\times 1}$$

$$\varepsilon \sim \underline{\underline{N}}(0, \sigma^2 I_n)$$

$$\hat{\beta} = \underline{\underline{\beta}} + (X'X)^{-1} X' \underline{\underline{\varepsilon}} \Rightarrow$$

$$\hat{\beta} \sim N$$

$$E(\hat{\beta}) = \beta$$

$$V(\hat{\beta}) = \sigma^2 (X'X)^{-1}$$

(ΔΙΟΤΙ ΣΥΝΑΡΤΗΣΗ ΓΡΑΜΜΙΚΗ ΤΟΥ $\varepsilon \sim N$)

$$\Rightarrow \hat{\beta} \sim N(\beta, \sigma^2 (X'X)^{-1})$$

ΕΣΤ$^{\underline{o}}$ $k=4$

$$b=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

και $b_2 = 0$

$b_3 + b_4 = 3$

$\nu = 2$

$$R b = q$$

$\nu \times k$ και $\nu \times 1$

ν ΓΡΑΜΜΙΚΟΥΣ ΠΕΡΙΟΡΙΣ.

$$R b = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$b_1 - b_2 = 1$$
$$b_3 - 2b_1 = 5$$
$$b_3 + 3b_4 = 0$$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix}$$

$$b_1 = b_2 \Rightarrow \left(b_1 - b_2 = 0 \right.$$
$$3b_1 = 7 - 2b_3 \Rightarrow 3b_1 + 2b_3 = 7 \quad \Bigg| \Rightarrow$$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 0 & 0 \\ 3 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$$

$v < k$ ← ΑΡΙΘΜΟΣ ΕΞΟΓΕΝΩΝ ΜΕΤΑΒ.

ΑΡΙΘΜΟΣ
ΓΡΑΜ. ΠΕΡΙΟΡ.

$$\hat{b} \sim N\left(b, \sigma^2 (X'X)^{-1}\right)$$

$$E(R\hat{b}) = R\,E(\hat{b}) = Rb$$

$$V(R\hat{b}) = R\,V(\hat{b})\,R' = \sigma^2 R (X'X)^{-1} R' \quad \Rightarrow$$

$$R\hat{b} \sim N$$

$$R\hat{b} \sim N\left[Rb,\ \sigma^2 R(X'X)^{-1}R'\right]$$

$$X \sim N(\mu, A) \Rightarrow (X-\mu)'A^{-1}(X-\mu) \sim \chi^2_{r(A)}$$

$r(A)$ = ΒΑΘΜΟΣ ΤΗΣ A

$$\Rightarrow (R\hat{b}-Rb)'\left[\sigma^2 R(X'X)^{-1}R'\right]^{-1}(R\hat{b}-Rb) \sim \chi^2_{r(R(X'X)^{-1}R')} = \chi^2_v$$

$$H_0: \underset{v \times k}{R} \underset{k \times 1}{\beta} = \underset{v \times 1}{q}$$

$$(R\hat{\beta} - q)' \left[\sigma^2 R (X'X)^{-1} R' \right]^{-1} (R\hat{\beta} - q) \overset{H_0}{\sim} \chi^2_v$$

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$\sum_{t=1}^{n} \hat{\varepsilon}_t^{2} = \hat{\varepsilon}'\hat{\varepsilon}$$

$$\hat{\varepsilon} = Y - X\hat{\beta} = Y - X(X'X)^{-1}X'Y =$$

$$= \underbrace{\left[I_n - X(X'X)^{-1}X' \right]}_{M} Y = MY$$

ΞΕΡΟΥΜΕ ΟΤΙ M ΣΥΜΜΕΤΡΙΚΗ + ΤΑΥΤΟΔΥΝΑΜΗ

και $r(M) = tr(M) = n - K$

ΕΠΙΣΗΣ

$$M'X = MX = \left(I_n - X(X'X)^{-1}X'\right)X =$$

$$= X - X\underbrace{(X'X)^{-1}X'X}_{I_k} = X - X = 0$$

M ΟΡΘΟΓΩΝΙΑ ΤΗΣ X.

$$\hat{\varepsilon} = My \Rightarrow \hat{\varepsilon} = M(X\beta + \varepsilon) \Rightarrow$$

$$\Rightarrow \hat{\varepsilon} = \underbrace{MX}_{=0}\beta + M\varepsilon \Rightarrow \boxed{\hat{\varepsilon} = M\varepsilon}$$

$$\varepsilon \sim N(0, \sigma^2 I_n)$$

$$\hat{\varepsilon} = M\varepsilon \quad \Rightarrow \quad \hat{\varepsilon} \sim N(0, \sigma^2 M)$$

$$E(\hat{\varepsilon}) = E(M\varepsilon) = M E(\varepsilon) = M \cdot 0 = 0$$

$$V(\hat{\varepsilon}) = V(M\varepsilon) = M V(\varepsilon) M' =$$

$$= M \sigma^2 I_n M' = \sigma^2 M M = \sigma^2 M$$

$$\hat{\varepsilon}'\hat{\varepsilon} = (M\varepsilon)'(M\varepsilon) = \varepsilon' M' M \varepsilon =$$

$$= \varepsilon' M M \varepsilon = \varepsilon' M \varepsilon$$

$$\varepsilon \sim N(0, \sigma^2 I_n) \Leftrightarrow$$

$$\frac{1}{\sigma}\varepsilon \sim N(0, I_n) \Rightarrow$$

$$\left(\frac{1}{\sigma}\varepsilon\right)' M \frac{1}{\sigma}\varepsilon \sim \chi^2_{r(M)} \Rightarrow$$

$$\frac{1}{\sigma^2}\varepsilon' M \varepsilon \sim \chi^2_{n-k} \Rightarrow \frac{1}{\sigma^2}\hat{\varepsilon}'\hat{\varepsilon} \sim \chi^2_{n-k}$$

$$\frac{1}{\sigma^2}\hat{\varepsilon}'\hat{\varepsilon} = \frac{1}{\sigma^2}\varepsilon' M \varepsilon$$

$$(R\hat{\beta} - q)'\left[R(X'X)^{-1}R'\right]^{-1}(R\hat{\beta} - q)\frac{1}{\sigma^2} \sim \chi^2_v$$

$$\frac{1}{\sigma^2}\hat{\varepsilon}'\hat{\varepsilon} \sim \chi^2_{n-K}$$

$$\Rightarrow$$

$$\frac{(R\hat{\beta} - q)'\left[R(X'X)^{-1}R'\right]'(R\hat{\beta} - q)\frac{1}{\cancel{\sigma^2}\, v}}{\frac{1}{\cancel{\sigma^2}}\frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-K}} \overset{H_0}{\sim} F(v, n-K)$$

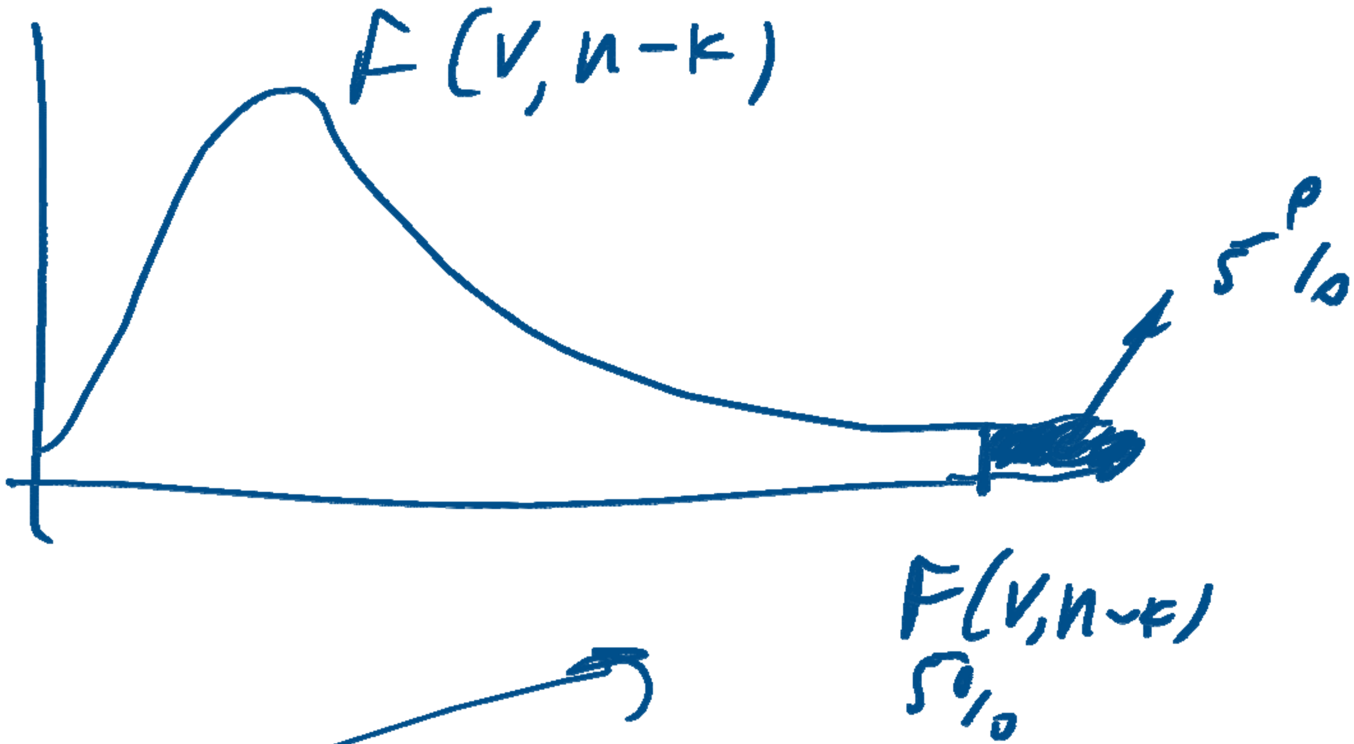
$$\Rightarrow F = \frac{\dfrac{(R\hat{\beta}-q)'[R(X'X)^{-1}R']^{-1}(R\hat{\beta}-q)}{v}}{\dfrac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k}} \sim F(v, n-k)$$

$$\frac{\hat{\varepsilon}'\hat{\varepsilon}}{n-k} = s^2$$

Reject H_0 iff

$$F > F_{5\%}(v, n-k)$$

(ΔΕΞΙΑ ΟΥΡΑ)



F > F(v, n-k): Reject H_0

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}$$

Έστω $K=4$

$$H_0: \beta_3 = 0 \qquad q=(0)$$

$$R = (0,0,1,0)$$

$$R\hat{\beta} = (0,0,1,0)\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \hat{\beta}_4 \end{pmatrix} = \hat{\beta}_3$$

$$R\hat{\beta} - q = \hat{\beta}_3 - 0 = \hat{\beta}_3$$

$$R\,(X'X)^{-1}R' =$$

$$(0, 0, 1, 0)\ (X'X)^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} =$$

$$= \left((X'X)^{-1}\right)_{(3,3)}$$

$$\frac{\hat{\beta}_3 \, (X'X)^{-1}_{(3,3)} \, \hat{\beta}_3 / 1}{s^2} \sim F(1, n-k)$$

тогда $V(\hat{\beta}_3) = s^2 (X'X)^{-1}_{(3,3)}$

$$\Rightarrow \left(\frac{\hat{\beta}_3}{V(\hat{\beta}_3)} \right)^2 \sim F(1, n-k) \Rightarrow$$

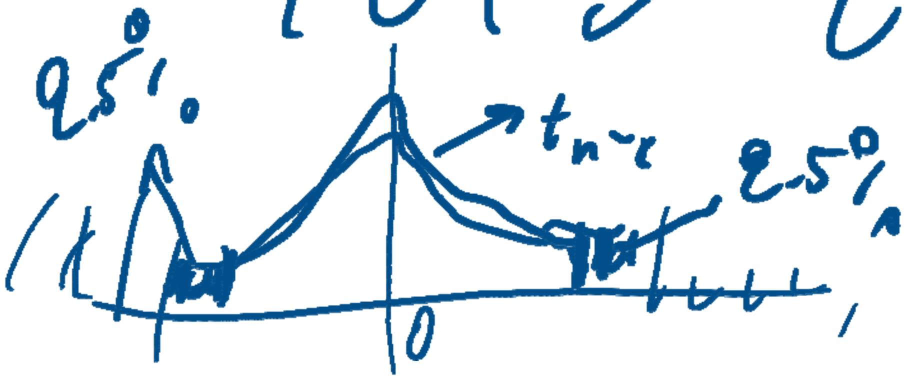
$$\frac{\hat{\beta}_3}{\sqrt{V(\hat{\beta}_3)}} \sim t_{n-k}$$

$H_0: \hat{\beta}_3 = 0 \qquad H_1: \beta_3 \neq 0$

$$t = \frac{\hat{\beta}_3}{s.e.(\hat{\beta}_3)} \sim t_{n-k}$$

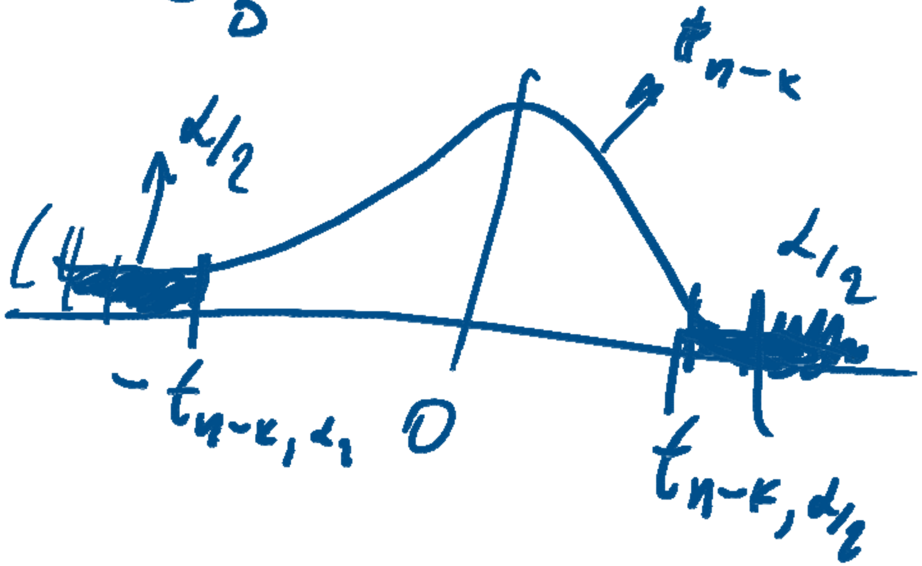
Reject H_0 iff

$$|t| > t_{n-k,\,2.5\%} \quad (5\% \text{ size test})$$



$H_0: \beta_3 = 0$ v.s. $H_1: \beta_3 \neq 0$ size test α

$$t = \frac{\hat{\beta}_3 - 0}{s.e.(\hat{\beta}_3)}$$



Reject H_0 iff $|t| > t_{n-k,\alpha/2}$

Reject H_0 iff $P.V.(t) < \alpha$

$$P.V.(t) = P(t_{n-k} < -|t|) + P(t_{n-k} > |t|)$$

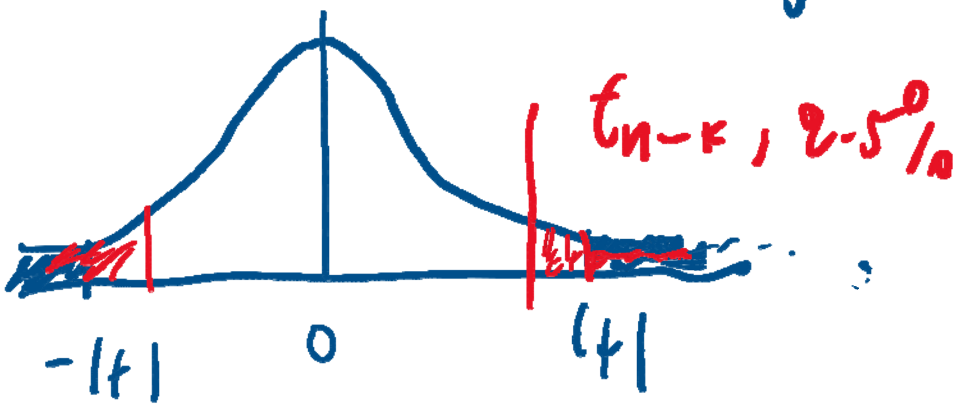
$$= 2P(t_{n-k} > |t|)$$

$H_0: b_2 = 0$ v.s. $H_1: b_2 \neq 0$

Reject H_0 iff $P.V.(t) < 5\%$

$$t = \frac{\hat{b}_2}{s.e.(\hat{b}_2)} \overset{H_0}{\sim} t_{n-k}$$

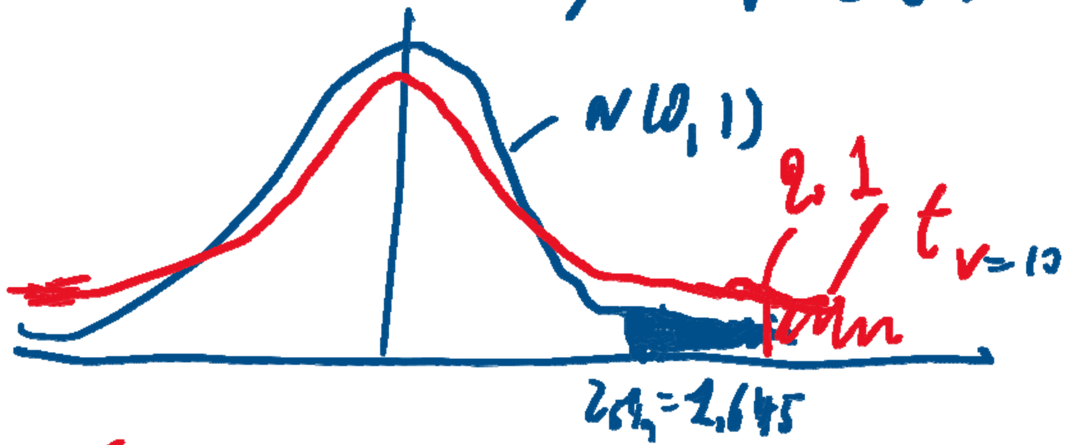
$$P.V.(t) = P(t_{n-k} < -|t|) + P(t_{n-k} > |t|) = 2P(t_{n-k} > |t|)$$



t_ν συμμετρική ως προς 0

$$t \sim t_\nu$$

$$E(t) = 0, \quad V(t) = \frac{\nu}{\nu-2} \quad \underline{\underline{\nu > 2}}$$



Για $\nu > 50$ (ΜΕΓΑΛΟ) $\Rightarrow$

$$\boxed{t_\nu \overset{A}{\sim} N(0,1)}$$

Dependent Variable: R1-RF
Method: Least Squares
Date: 10/24/19 Time: 10:23
Sample: 1960M01 2003M12
Included observations: 528

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.012192	0.082434	-0.147905	0.8825
RMRF	0.896798	0.019628	45.69048	0.0000
HML	0.272386	0.030257	9.002418	0.0000
SMB	1.252069	0.025830	48.47303	0.0000
UMD	0.004131	0.019786	0.208788	0.8347

R-squared	0.922954	Mean dependent var	0.791042
Adjusted R-squared	0.922365	S.D. dependent var	6.384388
S.E. of regression	1.778889	Akaike info criterion	3.999280
Sum squared resid	1655.006	Schwarz criterion	4.039707
Log likelihood	-1050.810	Hannan-Quinn criter.	4.015107
F-statistic	1566.287	Durbin-Watson stat	2.088172
Prob(F-statistic)	0.000000		

$t_{n-k} =$

DO NOT Reject

0 x 1

Reject H_0

$t_{528-5} = t_{523} \sim N(0,1)$

$\alpha = 5\%$

$Z_{\alpha/2} = Z_{2.5\%} = 1.96$

$H_0: \beta_i = 0$ v.s $H_1: \beta_i \neq 0$

$i = 1, 2, 3, 4, 5$

Reject iff $|t| > Z_{\alpha/2}$

$= \left| \frac{\hat{\beta}_i}{S.E.(\hat{\beta}_i)} \right| > 1.96$

$H_0: b_i = 0$ v.s. $H_1: b_i \neq 0$ $\quad i = 1, 2, \dots, k$

$$t = \frac{\hat{b}_i}{s.e.(\hat{b}_i)}$$

Reject H_0 iff $|t| > t_{n-k, \alpha/2}$ or $\equiv$

Reject H_0 iff $P.V.(t) < \alpha$

If H_0 is <u>NOT</u> Rejected $\Rightarrow$ η μεταβλητή στατιστικά ΜΗ-ΣΥΜΑΝΤΙΚΗ ΔΗΛΑΔΗ ο b_i συντελεστής είναι 0.

If H_0 is Rejected $\Rightarrow$

Η ι μεταβλητή είναι στατιστικά σημαντική Δηλαδή $b_i \neq 0$.