Introduction Data A Framework for Analysis Solow Growth Model Numerical Example and Predictions Conclusion

Ευρωπαϊκή Οικονομική Πολιτική

Δρ Ευάγγελος Διοικητόπουλος

Τμήμα Οικονομικής Επιστήμης (ΟΠΑ)

Ιανουάριος 2020



Outline of the Presentation

- Main Topic: Integration through Growth
- Questions
- Definitions
- Data
- Introduction of main mechanisms of growth theory A Parable

Methodology of Answering

- Define economic variables
- Study the empirical facts about the living standards and economic growth of different countries
- Build a model: Find ways to explain the evolution of world's income per capita and growth performance
- Replicate the facts using the model and if it works well!!! then

- policy prescriptions



GDP per capita

- A country's standard of living depends on its ability to produce goods and services (GDP).
- GDP measures a society's well being
- But, most important, GDP per capita (GDP/Population) measures how much income the average person enjoys.
 - Example: In economy A, GDP=200 and there are 2 people, then, the income per capita is 100 (200/2). In economy B, GDP=300 and there are 20 people, then, the income per capita is 15 (300/20).
- GDP per capita is an important measure to compare countries.
 We choose to work in a country that on average our income will be higher. i.e. in the previous example country A is rich while country B is relatively poor.

GDP per capita

	Highest GDP per Capita		Largest Economies		Most Populous Countries	
Rank	Country	GDP per Capita (\$)	Country	Total GDP (\$ trillions)	Country	Population (millions)
1	Qatar	159,469	United States	12.62	China	1,320
2	Luxembourg	84,525	China	10.08	India	1,160
3	United Arab Emirates	52,946	Japan	3,81	United States	307
4	Bermuda	52,090	India	3.76	Indonesia	240
5	Macao	51,057	Germany	2.66	Brazil	199
6	Norway	49,945	United Kingdom	2.07	Pakistan	181
7	Singapore	47,373	Russia	2.05	Bangladesh	154
8	Kuwait	46,639	France	1.98	Nigeria	149
9	Brunei	46,229	Italy	1.68	Russia	140
10	Australia	41,304	Brazil	1.62	Japan	127
11	United States	41,099	Mexico	1.29	Mexico	111

• GDP per capita can account better for differences in living standards.



GDP growth

- GDP growth rate (annual) measures how much income has changed as a percentage relative to the previous year $\frac{Y_2 Y_1}{Y_1}$.
- Example 1. If in 2015 GDP is 100 and in 2016 is 105, then, the growth rate of GDP is 105-100/100=0.05 or 5%.
- Example 2. If in 1950 GDP is 100 and in 2016 is 150, then, the trend growth rate of GDP is 150-100/100=0.5 or 50%.

Some Insights

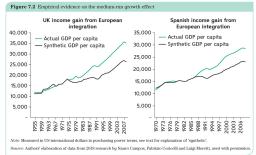
- Economic growth means producing more and more every year.
- European leaders have long emphasized the pro-growth aspects of European integration: it affects growth mainly via its effect on investment in human capital, physical capital, and knowledge capital.
- Growth effects fall naturally into two categories: medium term, like 'induced physical capital formation'; long term, involving a permanent change in the rate of accumulation, and thus, a permanent change in the rate of growth.

Some Insights

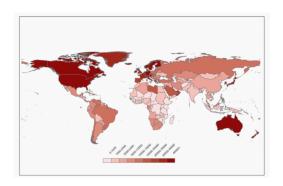
- Schematically: European integration (or any other policy) →
 allocation effect → improved efficiency → better investment
 climate → more investment in machines, skills and/or
 technology → higher output per person. Also, institutional
 and cultural changes/incentives.
- Under medium-run growth effects, the rise in output per person eventually stops at a new, higher level.
- Under long-run growth effects, the rate of growth is forever higher under an endogenous growth perspective.

Intergration and Growth

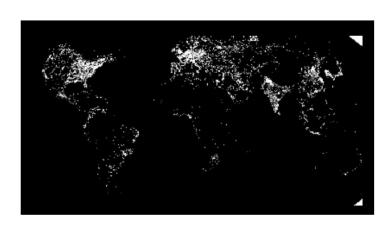
 Are growth and European integration related? → Statistical evidence shows sizeable medium-run effect of integration:



Το κατά κεφαλήν εισόδημα σήμερα



Το κατά κεφαλήν εισόδημα σήμερα απο ψηλά



Η Εξέλιξη της Οικονομική Ανισότητα στο Κόσμο



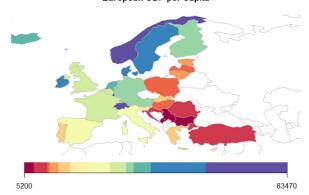
Η Εξέλιξη της Οικονομική Ανισότητα σε Ευρώπη και Ιαπωνία

Top 1% share of total income – Europe and Japan (L-shaped), 1900–2012 – Max Roser⁶



Κατά Κεφαλήν Εισόδημα στην Ευρώπη

European GDP per Capita



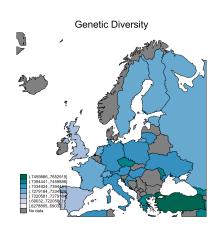
Βαθμός υπομονής στην Ευρώπη



Βαθμός Εμπιστοσύνης στην Ευρώπη

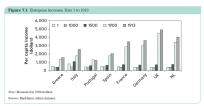


Γενετική Ποικιλομορφία



Η Ιστορική Πορεία του Κατά Κεφαλήν Εισοδήματος

 By historical standards, continuous economic growth is a relatively recent phenomenon. Before the Industrial Revolution, which started in Great Britain in the late 1700s, European incomes had stagnated for a millennium and a half.

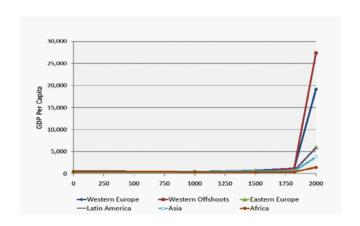


Η Ιστορική Πορεία του Κατά Κεφαλήν Εισοδήματος

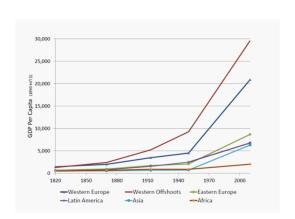
 With industrialization, incomes began to rise at a respectable rate of around 2 per cent per year. Growth rates, however, were hardly constant from this date:

Period	Real GDP	Real GDP per capita	Real GDP per hour	
1890-1913	2.6	1.7	1.6	
1913-1950	1.4	1.0	1.9	
1950-1973	4.6	3.8	4.7	
1973-1992	2.0	1.7	2.7	
Whole period 1890-1992	2.5	1.9	2.6	
orway, Sweden, Switzerland and the Note that the 1950-73 period is the scholing the unusual 1913-50 period errelation that first started economis	UK, all adjusted for bo aberration. Both befo). The Golden Age was a thinking about the g	(Austria, Belgium, Denmark, Finland unskurg changes). re and after this period, growth rates also the most intensive period of Eu- rowth effects of European integration icty Research 1908, published by Carr	were just under 2 per cent per an repean integration and it was this a.	

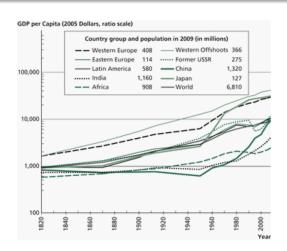
Η Ιστορική Πορεία του Κατά Κεφαλήν Εισοδήματος



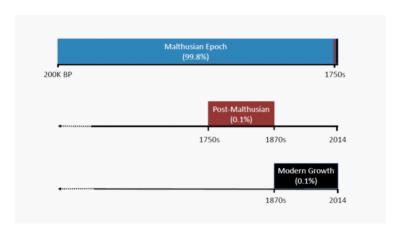
Κατά κεφαλήν εισόδημα απο το 1820 εως σήμερα



Κατά κεφαλήν εισόδημα απο το 1820 εως σήμερα



Οι τρείς φάσεις εξέλιξης του ανθρωπίνου εισοδήματος



These differences pose a mystery:

- Why some countries are poor while others are rich? Does have to be this way?
- Are there factors that we can change that lead to these gaps?
- Is enjoyment of the rich dependent on suffering of the poor?
- More population means more production and higher income per person?
- What can explain the divergence in income per capita between similar countries (South vs North Europe?)
- What policies should poor countries pursue to promote growth in order to join the developed world?

Questions about growth

- What differences between countries that have led to these divergent experiences?
- Will the richest continue to grow richer? Will the poor continue to trail behind?
- Poor or Advanced economies grow faster?
- Will limitations on resources prohibit the poor countries to catch up?
- What role will new technologies play in this process?

UGT - Comparative Economic Development

- What accounts for the differences in GDP/capita in the malthusian equilibrium?
 - First divergence leads to the great divergence.
- What accounts for the take-off from stagnation to growth in some countries and the persistent stagnation in others?
- What governs the differential timing of the demographic transition across the globe?
- Reversal of Fortunes.

What this lecture does?

- This course address these questions of why countries differ in their standards of living
- ② Why countries grow, or fail to grow, richer over time.
- What are the determinants of income and growth?
- What can we do?

Sum Up

- Why countries differ in their levels of income?
- What about their growth prospects?
 - We will introduce simple model cannot explain all of the phenomena we observe, but it is instructive to see how far the model can take us.

Observations from the data

- 1. Poor countries have lower income per capita than rich.
- 2. Many Poor countries seems to grow faster than advanced (convergence catch up reversal?)
- 3. Some **poor countries stagnate** and grow less than the advanced (e.g. Africa)
- 4. There are differences in income per capita among neighbour or advanced countries (e.g. south vs north Europe, US vs Europe)

Aim

- Goal: understand why some countries are rich and others poor
- Why countries become richer or poorer over time?
- What is the set of answers?

Factors of Production, Factor Accumulation and Preferences

- Source of differences in income between countries?
- Different quantities of labour and capital.
- Differences in other "factors of production".
- What determines the quantities of these factors that countries accumulate?
- Example: what determines the quantity of capital in a country: saving, foreign investment,
- Number of workers and how fast they grow.
- Other forms of capital: skills and knowledge of people



Consider A Parable

- Two Neighbour Countries, Sylvania and Fredonia
- Sylvania has GDP per capita=16000, Fredonia has GDP per capita=2000
- You are the president of a prestigious economic consulting firm in Sylvania
- The King of Sylvania employs you to explain why this happens?

Work on that (Production and Factors)

- Step 1. You set up a team that calculates the gap=14000, 8 times lower than Freedonia.
- Step 2. You measure the level population/labour in both countries. But, you find that are the same.
- Step 3. Then, you think about other factors of production.
 You find that both countries produce the goods and services with labour and capital (equipments, machines, tools)
- Step 4. You measure them. You find that Sylvania has less capital. Great, But why?

Working on that (Preference and Accumulation of Factors)

- Step 5. You dig more. Why more capital in Fredonia?
- Step 6. You measure that in Sylvania people consume more than what they save from what they produce.
- Step 7. Thus, in Freedonia the save more every year and they generate more capital (tools) that help them with the same labour to produce more
- You think you found the smoking gun!!!

Working on that (Productivity)

- Step 8. To test your theory you ask your staff to see that if you add the same amount of machines how much Sylvania income will be?
 - Unfortunately, Sylvania income will go from 2000 to 8000. Still another 8000 left to explain:(
- Step 9. You understand that something happens and the guys in Freedonia with the same numbers they produce more. This mean they more productive!!! You scratch your head!!
 Productivity!!! Yeah!!! This is it!!
- Step 10. Nice, lets see why they are more productive!!?

Working on that (Productivity=Technology and Efficiency)

- Step 11. Your staff is working hard and they find that
 - a) People in Freedonia combine better the existing tools and labour, in other word they have better ideas how to use them, or in other words they have better **Technology**!!
 - b) People in Fredonia have better way to combine Technology, Capital and Labour together (sometimes the ideas exist but we cannot materialize them). This means that have better Efficiency
- Step 12. You test if everything works. You combine everything and you find that a) higher savings (investment) b) more capital, c) better technology and efficiency explains the whole difference in income!!! You are extremely happy
- Step 13. You write the report to the King!!!

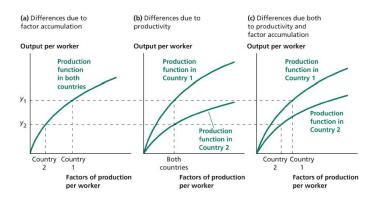
Almost Ready (Fundamentals)

- Step 14. The King reads the reports and ask you, What is the fundamental factors that can explain all?
- Step 15. You work more, you search in cultures, geography but those closed neighbour countries are the same
- Step 16. Yeah!!!Institutions!!!!You think more and you understand that in Sylvania there exists monarchy and in Freedonia, Democrasy!!!
 - In Sylvania guys want to save more but they know that if they save the King may expropriate their wealth
 - In Fredonia, clever investors think ideas about how to produce more output but in Sylvania more weapons. Thus, technology is less
 - In Sylvania to obtain Wealth and Status if you make happy the King's will. But in Freedonia you are appreciated if you are doing your job well. So, you are more efficient.

The End!!!

- Step 17. You complete your report. Your write to the King that the root is Monarchy.
- Racing toward the borders, pursued by the palace guard, you understand that your effort would be better rewarded in Freedonia.

Parable in Figures



Introduction

Production, Factors, Accumulation

- Production. $Y = AK^aL^{1-a}$, where $a \in (0,1)$
- Factors of Production, K, L
- Factor Accumulation, $K_{t+1} = I(S) + K_t \delta K_t$
- How Factors Accumulate: Preferences, Markets and others mechanisms we will deeply explain.

Productivity

- Productivity, A: a measure of how much output can be produced with a given quantity of factors of production.
- Productivity: technology (inventions) and efficiency.

Technology

- Why technology differs between countries?
- How new technologies are created and disseminated?
- How technological progress has contributed to growth historically?
- Extent differences in technology explain differences in income?

Technology

- Why technology differs between countries?
- How new technologies are created and disseminated?
- How technological progress has contributed to growth historically?
- Extent differences in technology explain differences in income?

Fundamentals

- Searching for "fundamental" determinants of income and development: form of government, geography and climate, institutions.
- The degree to which something is "fundamental" will vary from factor to factor, and will often be open to debate.
- The problem of the "endogeneity" of fundamental factors

What about growth?

- Turning the model of the determinants of income levels into a model of the determinants of income growth:
 - If two countries are the same in their fundamentals (or more generally, if we would expect them to have equal levels of income based on their fundamentals),
 - Then the country with a lower level of income will be expected to grow faster.
 - This could either be because the poorer country has income below the level that we would expect, given its fundamentals
 - Or because the richer country has income above the level that we expect from its fundamentals.

A Numerical Example

- $Y = K^{0.3}L^{0.7}$, where s=0.3 in Freedonia and s=0.2 in Sylvania. For both $\delta=0.02$
- ullet $K_1=K_0+I_0-\delta K_0$ and I=sY
- Step 1. Each economy starts with $K_0 = 40$ and population every year is L = 20.
- Step 2. Plug K_0 into Y and obtain $Y_0 = 40^{0.3}20^{0.7} = 24.76$
- Step 3. Calculate savings and investment in Freedonia: $S = sY_0 = 0.3 * 24.76 = 7.42$
- Step 4. Using the formula above calculate $K_1 = 46.58$. Then, repeate this process 5 times (in Step 3 for Sylvania use s = 0.2).

Numerical Example

The growth process: The effect of savings on growth in the short run

Numerical Example: $Y = K^{0.3}L^{0.7}$, d=0.02

 $K_0 = 40$ with high savings s=0.3

Time	Capital	Labour		Production	Income per capita	Growth rate
	2000	40	20	24.6228883	1.231144413	-
	2001	46.58686648	20	25.7750815	1.288754074	0.046793585
	2002	53.3876536	20	26.8505538	1.342527689	0.041725272
	2003	60.37506666	20	27.8598186	1.392990929	0.03758823
	2004	67.5255109	20	28.8112009	1.440560044	0.034148905
	2005	74.81836094	20	29.7114231	1.485571155	0.031245564

 $K_0 = 40$ with high savings s=0.2

Time	Capital	Labour		Production	Income per capita	Growth rate
	2000	40	20	24.6228883	1.231144413	
	2001	44.12457765	20	25,3585937	1.267929685	0.029878925
	2002	48.31380484	20	26.0580774	1.302903868	0.027583693
	2003	52.55914422	20	26.7248627	1.336243135	0.025588431
	2004	56.85293387	20	27.3619384	1.368096922	0.023838317
	2005	61.18826288	20	27.9718657	1.398593284	0.022291083

Higher savings, higher income per person and higher growth in the short-run (2000-2005)!!!

Summary 1

- Economic prosperity, as measured by real GDP per person, varies substantially around the world.
- The average income of the world's richest countries is more than ten times that in the world's poorest countries.
- The standard of living in an economy depends on the economy's ability to produce goods and services.

Summary 2

- Income per capita positively depends on how productive we are.
- Productivity depends on the amounts of physical capital, human capital, natural resources, and technological knowledge available to workers.
- Government policies and Institutions can influence the economy's growth rate in many different ways.

Summary 3

• Higher savings lead to higher income per person in the short-run and in the long-run.

Factors of Production

- Factors of production are the those inputs used in the production function to produce goods, e.g.
 - Physical capital (K)
 - Labour (L)
 - Technological knowledge (A)
 - Natural Resources/Land (X)
 - Human Capital (H)

Agricultural Land as a Fraction of Total Wealth in the United Kingdom

1688	64%
1798	55%
1885	18%
1927	4%
1958	3%

Five key characteristics of capital

- Capital is productive: raises the amount of output that a worker can produce.
- Capital is something that has itself been produced; it has been build or created. The process of producing capital is called investment. Needs sacrifice of consumption, thus, saving!!! Different from Land.
- **3** Capital is rival in its use. e.g. hammer. Opposite to ideas that are non-rival.

Five key characteristics of capital

- 4. It can earn a return and this is the incentive of its creation. Because capital is productive and its use is limited, it is often able to earn a return. If using a certain piece of capital will make a worker more productive, then the worker will be willing to pay to use it.
- 5. **Capital wears out**. The economic term for this wearing-out process is depreciation.
- 6. A large fraction of the investment that takes place in the economy serves only to replace capital that has depreciated.

The Production Function - Assumption 1

$$Y_t = AK_t^a L_t^{1-a}, where \ a \in (0,1)$$
 (1)

where $L_t > 0$ is labour/workers/hours and $K_t > 0$ is capital, and a > 0.

- Assumption 1
 - a) L labour increases production, $MP_L \equiv \frac{\partial Y_t}{\partial L_t} = A(1-a)K_t^aL_t^{-a} = A(1-a)(\frac{K_t}{L_t})^a > 0.$
 - b) L labour increases production with decreasing rate, $\frac{\partial MP_L}{\partial L_t} < 0$.

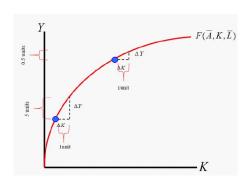
The Production Function - Assumption 2

$$Y_t = AK_t^a L_t^{1-a}$$
, where $a \in (0,1)$

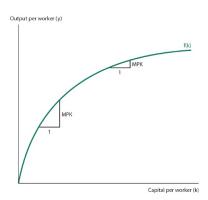
where $L_t > 0$ is labour/workers/hours and $K_t > 0$ is capital.

- Assumption 2.
 - a) K labour increases production, $MP_K \equiv \frac{\partial Y_t}{\partial K_t} = AaK_t^{a-1}L_t^{1-a} = Aa(\frac{K_t}{L_t})^{a-1} > 0.$
 - b) K capital increases production with decreasing rate, $\frac{\partial MP_K}{\partial K_t} < 0$. (Diminishing Marginal Product of Capital)

The Production Function - The MPk and K



The Production Function - The MPk and K/L



The Production Function - Assumption

- Assumption 3. Constant returns to scale.
 - a) Increasing both factors each by z increase output by z. $A(zK_t)^a(zL_t)^{1-a} = Az^aK_t^az^{1-a}L_t^{1-a} = AK_t^aL_t^{1-a}z^az^{1-a} = AK_t^aL_t^{1-a}z^{a+1-a} = AK_t^aL_t^{1-a}z = Y_tz$
 - b) Implication 1: Income per capita is a function of capital per worker: Can be written as

$$\frac{Y_t}{L_t} = \frac{AK_t^a L_t^{1-a}}{L_t} = \frac{AK_t^a}{L_t L_t^{-1+a}} = \frac{AK_t^a}{L_t^{1-1+a}} = \frac{AK_t^a}{L_t^a} = A\left(\frac{K_t}{L_t}\right)^a$$

$$y_t = A(k_t)^a$$

where $y_t \equiv A\left(\frac{K_t}{L_t}\right)^a$ is income per capita and $k_t \equiv \frac{K_t}{L_t}$ capital per worker

• c) Implication 2: $MP_LL + MP_KK = Y$. Implies zero profits as MP_L shall equal wage rate and MP_K the return on capital.



Factor Payments and Shares

- Firms maximize profits: P = Y wL rK
- Wage per hour for worker should be in a competitive economy equal to the marginal product: $w = MP_L$
- The rental cost of capital in a competitive economy will equal the marginal product of capital, $r = MP_K$.
- The share of capital holders in the production (return * amount of capital / output):

$$r*K = MP_KK = Aa(\frac{K_t}{L_t})^{a-1}K = aAK_t^aL_t^{1-a} = aY$$

$$\frac{MP_KK}{Y} = a$$

thus a is the share of total output that goes to capital holders.

Constant Labour and Technology

- Lets assume for the moment the Labour is constant over time.
- Also, assume that there is no technological progress over time.
- All the action in the economy comes from capital accumulation

Consumption, Saving and Investment

• Production (Y) is either consumed (C) or saved (S). (We are going to analyze the determinants of this decision later on).

$$Y = C + I$$

• Saving rate, γ , is the fraction of income that is saved. So $\frac{S}{V} = \gamma$ or

$$S = \gamma Y$$

• What it is saved, γY , becomes investment, $\gamma Y = I$.

$$I = S = \gamma Y$$

- Investment is used to buy/build new capital stock
- As we told before Capital wears out.



Capital Accumulation

Capital Evolves over time as follows:

$$K_{t+1} = I + K_t - \delta K_t$$

- The capital stock next period, K_{t+1} , is equal to
 - 1) The new that have been purchased through investment, I
 - 2) plus what remained from the previous year i.e what we had from the previous year K_t minus the part that died, δK_t

Capital Accumulation

• Using $I = \gamma Y$ we get

$$K_{t+1} = \gamma Y + K_t - \delta K_t \tag{2}$$

and under constant labour over time $(L_{t+1} = L_t)$ the capital stock per worker is given by

$$k_{t+1} = \gamma A k_t^a + k_t - \delta k_t \tag{3}$$

• The change of the capital stock over time, $\Delta k = k_{t+1} - k_t$ using (3) is given by:

$$\Delta k = \gamma A k_t^a - \delta k_t \tag{4}$$

Analysis of the growth of capital stock

$$k_{t+1} = \gamma A k_t^a + k_t - \delta k_t \Rightarrow \Delta k = \gamma A k_t^a - \delta k_t$$

• Consider the following example: Suppose that in the year 2010, the quantity of capital per worker in a certain country was equal to $k_0=100$, A=1, the quantity of output per worker $y_t=50$. The fraction of output saved/invested was 20%, and the depreciation rate was 5%.

Step 1. As $y_t = 50$ then investment is I = 50 * 20% = 10Step 2. The number of machines that will not survice are $\delta k_0 = 100 * 5\% = 5$.

Analysis of the growth of capital stock

$$k_{t+1} = \gamma A k_t^a + k_t - \delta k_t \Rightarrow \Delta k = \gamma A k_t^a - \delta k_t$$

Step 3. In the next period we will have $k_1 = 100 + 10 - 5 = 105$.

Step 4. The change in the capital stock will then be

$$k_1 - k_0 = 105 - 100 = 5.$$

Result: If investment (10) is higher than depreciation (5) then capital stock will increase, otherwise will decrease.

Analysis of the growth of capital stock

$$\Delta k = \gamma A k_t^a - \delta k_t$$

- $\Delta k > 0$ if $\gamma A k_t^a > \delta k_t$ and $\Delta k < 0$ if $\gamma A k_t^a < \delta k_t$.
- $\Delta k = 0$ if $\gamma A k_t^a = \delta k_t \Rightarrow A \gamma \frac{k_t^a}{k_t} = \delta \Rightarrow k_t^{a-1} = \frac{\delta}{A \gamma} \Rightarrow$

$$k_t^* = \left(rac{\delta}{A\gamma}
ight)^{rac{1}{a-1}}$$

where k_t^* is what we call the **steady-state** of the capital perworker.

• In other words, Δk depends on the level of k. If, $k < k^*$ then $\Delta k > 0$ and when $k > k^*$ then $\Delta k < 0$.

Why a steady-state exists?

- Assume that the economy starts at a k lower than the steady-state, thus $\Delta k > 0$
- As capital increases, output increases $(\gamma A k_t^a)$ but at a diminishing rate due to diminishing returns. So investment increases by a diminishing rate.
- ullet While, the capital depreciates with the same constant rate δk_t .
- Inevitably, at some point the rate of increase in output and, in turn, investment will be the same with the depreciation.
- Thus, from one hand capital will increase by the rate of investment but will decrease by the rate of depreciation. In other words, if the rates are the same, the capital will remain the same for every period. Thus, $\Delta k = 0$.

The Steady State of the Solow Model

- 1. Convergence to the steady-state (Stable)
- 2. Poor countries exhibit higher growth in output per capita

Determinants of the Long-run Position of the Economy

• Following the previous analysis, the steady-state denotes the long-run position of the economy. Once there, we stay there.

$$k_t^* = \left(\frac{\delta}{A\gamma}\right)^{\frac{1}{a-1}}$$

- What determines the long-run position
 - γ . The higher the savings/investment rate the higher the steady-state capital per worker, k_t^* and, in turn, the higher the output per worker $y^* = A(k^*)^a$.
 - A. Better technology, higher k_t^* and $y^* = A(k^*)^a$
 - δ . The higher the rate of depreciation, the lower k_t^* and $y^* = A(k^*)^a$.
- Those are some testable predictions of the Solow-Model.

The Process of Growth

$$\begin{array}{ccc} K_0 & \rightarrow & Y_0 \rightarrow \gamma Y_0 \rightarrow I \rightarrow \\ K_0 + I - \delta K_o & \rightarrow & K_1 \rightarrow Y_1... \end{array}$$

Numerical Example 1. Differences in Ko

- Assume two countries that have the same rate of investment but different levels of income. Country 1, $K_0 = 40$ and Country 2, $K_0 = 20$.
- $Y = K^{0.3}L^{0.7}$, where L = 20, $\gamma = 0.3$ and $\delta = 0.02$.
- $K_1 = K_0 + I_0 \delta K_0$ and $I = \gamma Y$
- Question 1. Which country has higher income per capita today?
- Question 2. Which economy will be richer in the long-run.
- Question 3. Will capital per worker and income per worker be increasing or decreasing?
- Question 4. Which county is going to growth faster until the long-run position?
- Question 5. What will the growth rate of output be in the long-run.

Numerical Example 1. Differences in Ko

- For Country 1:
- Step 1. The economy starts with $K_0 = 40$.
- Step 2. Plug K_0 into Y and obtain $Y_0 = 40^{0.3} 20^{0.7} = 24.76$
- Step 3. Calculate savings/investment in Country 1: $S = I = \gamma Y_0 = 0.3 * 24.76 = 7.42$
- Step 4. Using the formula before calculate $K_1 = 46.58$. Then, repeate this process 5 times
- For Country 2: We just do the same process but for $K_0 = 20$.

Numerical Example 1. Differences in Ko

Time	K	L	Υ	k=K/Y	y=Y/L	Growth rate
2000	40	20	24.6228883	2	1.23114441	-
2001	46.5868665	20	25.7750815	2.32934332	1.28875407	0.04679359
2002	53.3876536	20	26.8505538	2.66938268	1.34252769	0.04172527
2003	60.3750667	20	27.8598186	3.01875333	1.39299093	0.03758823
2004	67.5255109	20	28.8112009	3.37627554	1.44056004	0.03414891
2005	74.8183609	20	29.7114231	3.74091805	1.48557116	0.03124556
Time	K	L	Υ	k=K/Y	y=Y/L	Growth rate
2000	20	20	20	1	1	-
2001	25.6	20	21.5373857	1.28	1.07686929	0.07686929
2002	31.5492157	20	22.9307134	1.57746079	1.14653567	0.06469345
2003	37.7974454	20	24.2080458	1.88987227	1.21040229	0.05570399
2004	44.3039103	20	25.3894688	2.21519551	1.26947344	0.04880291
2005	51.0346727	20	26,489917	2.55173363	1.32449585	0.0433427

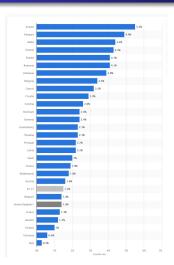
Numerical Example 1. Differences in Ko

- What the model predicts:
- Question 1. Which country has higher income per capita today? Country 1 because for higher capital per worker.
- Question 2. Which economy will be richer in the long-run?
 None. They will arrive at the same steady-state.
- Question 3. Will capital per worker and income per worker be increasing or decreasing? Increasing because we are below the ss.
- Question 4. Which county is going to growth faster until the long-run position? Country 2 grows faster because of diminishing returns to capital that affect mostly Country 1.
- Question 5. What will the growth rate of output be in the long-run. Zero growth as capital investment will be equal to depreciation in the long-run.

What the model predicts

- If two countries have the same rate of investment but different levels of income, the country with lower income will have higher growth.
- This is generally true and is called the convergence hypothesis.
- But not always.

European Example



Numerical Example 2 - Differences in savings

- Assume two countries that have the same rate $K_0=40$ but different savings rate. Country 1, $\gamma=0.3$ and Country 2, $\gamma=0.2$.
- Question 1. Which country has higher income per capita today?
- Question 2. Which economy will be richer in the long-run.
- Question 3. Will capital per worker and income per worker be increasing or decreasing?
- Question 4. Which county is going to growth faster until the long-run position?
- Question 5. What will the growth rate of output be in the long-run.

Numerical Example 2 - Differences in savings

Higher savings, higher income per worker and growth in the short-run

The growth process: The effect of savings on growth in the short run

Numerical Example: $Y = K^{0.3}L^{0.7}$, d=0.02

 $K_0 = 40$ with high savings s=0.3

Time	Capital	Labour	Production	Income per capita	Growth rate
2000	40	20	24.6228883	1.231144413	
2001	46.58686648	20	25.7750815	1.288754074	0.046793585
2002	53.3876536	20	26.8505538	1.342527689	0.041725272
2003	60.37506666	20	27.8598186	1.392990929	0.03758823
2004	67.5255109	20	28.8112009	1.440560044	0.034148905
2005	74.81836094	20	29.7114231	1.485571155	0.031245564

 $K_0 = 40$ with high savings s=0.2

Time	Capital		Labour	Production	Income per capita	Growth rate
21	000	40	20	24.6228883	1.231144413	
21	001	44.12457765	20	25.3585937	1.267929685	0.029878925
21	002	48.31380484	20	26.0580774	1.302903868	0.027583693
21	003	52.55914422	20	26.7248627	1.336243135	0.025588431
21	004	56.85293387	20	27.3619384	1.368096922	0.023888317
21	005	61.18826288	20	27.9718657	1.398593284	0.022291083

Higher savings, higher income per person and higher growth in the short-run (2000-2005)!!!



Numerical Example 2 - Differences in savings

Higher savings, higher income per worker in the long-run but not in the short-run.

		$K_0 = 40$ and	high savings s	-0.3				$K_0 = 40$	and low	savings	s=0.2	
Time	Capital	Labour		come per capita (Growth rate	Time	Cs	pital Labour	Proc	luction I	ncomo per capita G	rowth rate
	2000	60	20 24.62288827	1.751168613			2000	40	20	24.62288827	1.231144413	
	2001	40.50036040	20 25.77500140	1.200754074	0.046793585		2001	44.12457700	20	25.3585937	1.207929083	0.029878925
	2002	53.3874534	20 26.65055377	1.342527689	0.041725272		2002	48.51580484	20	26.05897757	1.502905868	0.027585095
	2003	60.37906666	20 27.85091857	1.302000029	0.03798823		2003	52.50014422	20	26.72496209	1.336243133	0.023300431
	2004	67.5255100	20 28.81110089	1.440560044	0.034148905		2004	00.80290087	20	27,30193843	1,368090922	0.023838317
	2005	74.81886004	20 29.7114231	1.485571155	0.081245564		2005	61.18625260	20	27.97100500	1.398393284	0.022291003
	2006	82.23542064	20 30.56600648	1.529300224	0.028762721		2000	CD 30687070	20	28.00000004	1.427542947	0.02091303
	2007	E9.76051559	20 31.57954164	1.568977082	0.026615751		2007	69.95906513	20	29.11834845	1.455942423	0.019679668
	2009	07.37016581	20 32.15991055	1.607709928	0.024741244		2000	74.30365352	20	29.65032091	1.402976445	0.013563053
	2000	105.0783557	20 32.8984152	1.54492076	0.023090767		2009	78.82788623	20	30.1833975	1.509019895	0.017561607
	2010	112.8463181	20 38.60089015	1.680404057	0.021626604		2010	60.28740609	20	39.66276671	1.534129333	0.015645195
	2011	120.6721566	20 14 20213068	1714641534	0.020319397		2011	87.75821727	20	31.16788513	1.558354257	0.015810116
	2012	128 5457587	20 34.94236875	1.747468458	0.014145083		2012	92.23662996	20	31.62676133	1.501020091	0.015043594
	2013	136.6606341	20 35.58141470	1.7790707339	0.019084610		2013	96.71924971	20	32.05036094	1.604519047	0.014328355
	2014	144,4058459	20 36.19065384	1.809552692	0.017122395		2014	301.2029409	20	32.52961602	1.626480801	0.013687437
	2015	152.3749251	20 36.77858772	1.858929385	0.01.6245452		2015	209.6942093	20	32.05526153	1.647763076	0.013094161
	2016	160.3610029	20 37.34556181	1.86732809	0.015443054		2016	210.1621625	20	33.36834252	1.668402131	0.012525400
	2017	168.3577514	20 37.89578711	1.894789355	0.014706181		2017	214.6325268	20	33.76862330	1.68843117	0.012004028
	2018	176.3593325	20 38.4273584	1.92136792	0.014027187		2018	219.0036009	20	34.15761339	1.707930594	0.011519288
	2019	184.3603554	20 38.94216937	1.947111468	0.013599593		2010	123.5432517	20	34,53557612	1.726778806	0.011065241
	2030	192.3558271	20 39.44142534	1.972071267	0.012817845		2020	127.9795019	20	34.90302928	1.745151464	0.010620244
	2021	200.3411382	20 39.92565393	1.996282696	0.012277157		2021	131.4005177	20	35.26045417	1.763022709	0.010240512
	2022	208.3120116	20 40.59571419	2.019785709	O.OL1773389		2022	136.8045082	20	35.60810715	1.780114857	0.03486196
	2023	216.2644856	20 40.85230437	2.042615218	0.011302986		2025	341.1901655	20	35.94697348	1.797148574	0.009511163
	2024	224.1948872	20 41.29606856	2.054803428	0.010852648		2024	145.555757	20	36.27687087	1.813843510	0.00417732
	2025	232.09981	20 41.72760249	2.086380124	0.010449758		2025	119.900016	20	36.5081/054	1.820017177	0.009961922
	2026	239.9793946	20 42.14745540	Z 2075719Z5	0.010001825		2026	254.2215855	20	36.91174956	1.845587478	0.008565228
	2017	767 8776707	22 41 5941 0274	2 127807489	0.009		2027	252.5196012	20	37.21718788	1.860869394	0.008

Stage of Development Savings rate Tolenological Progress Government Policy and Human Capital Culture

Numerical Example 2 - Differences in savings

- What the model predicts:
- Question 1. Which county has higher income per capita today? The both have the same income today because of the same capital per worker.
- Question 2. Which county will be richer in the long-run? The one that saves more will arrive in a higher steady-state.
- Question 3. Will capital per worker and income per worker be increasing or decreasing? Increasing because we are below the ss.
- Question 4. Which county is going to growth faster until the long-run position? Country 1 as it has higher savings rate.
- Question 5. What will the growth rate of output be in the long-run. Zero growth as capital investment will be equal to depreciation in the long-run because of diminishing returns.

What the model predicts

- If two countries have the same level of income but different rates of investment, then the country with a higher rate of investment will have higher growth at least in the short run.
- A country that raises its level of investment will experience an increase in its rate of income growth.

What we learn about the initial questions

- Why some countries are poor?
 - Low capital per worker, low savings, low technology
- Why some poor countries seem to grow faster than advanced
 - At low capital stock, the marginal productivity of capital is high. Thus, increases in capital boost production more than in a country where the marginal productivity is low
- Why some poor countries stagnate?
 - Very low savings rate, high consumption. But why? This is not explained by the Solow model.

We still need to explain more.

- The Solow model predicts zero growth in the long-run. But this is not true. How this happens?
- What determines the savings rate?
- Only a part of income differences can be explained.

Technological Progress

- The only way an economic to grow over the long-run is only through continuing technological progress.
- This can be exogenous, $A_{t+1} = (1+g)A_t$
- Or endogenous, $A_{t+1} = (1 + g(H, L, K, G, \tau))A_t$.

Numerical Example 1. Differences in Rate of TP

- Assume two countries that have the same rate of investment, same capital depreciation, same initial capital, 40, and initial size of population, 20, same initial technology $A_0=1$ and same population growth
- They different only in the rate of technological progress
- Country A has g = 0, Country B, g = 0.06.
- $Y = K^{0.3}(AL)^{0.7}$, where $\gamma = 0.3$ and $\delta = 0.02$.
- $K_1 = K_0 + I_0 \delta K_0$ and $I = \gamma Y = 0.3$

Numerical Example. No TP

- For Country 1 No technological Progress:
- ullet Step 1. The economy starts with, $A=1,\ \mathcal{K}_0=40$ and $\mathcal{L}_0=20$.
- Step 2. Plug K_0 into Y and obtain $Y_0 = 40^{0.3} 20^{0.7} = 24.76$
- Step 3. Calculate savings/investment in Country 1: $S = I = \gamma Y_0 = 0.3 * 24.76 = 7.42$
- Step 4. Using the formula before calculate $K_1=46.58$. Then, $K_1=46.58$ and $L_1=L_0+(0.02)L_0=20.4$ gives $Y_1=26.13$ and $\frac{Y_1}{L_1}=1.281$. We repeate this process 30 times

Numerical Example. TP 0.06

- For Country 2 Technological Progress g = 0.06.:
- ullet Step 1. The economy starts with $A_0=1,\ K_0=40$ and $L_0=20$.
- Step 2. Plug K_0 into Y and obtain $Y_0 = 40^{0.3} 20^{0.7} = 24.76$
- Step 3. Calculate savings/investment in Country 1: $S = I = \gamma Y_0 = 0.3 * 24.76 = 7.42$
- Step 4. Using the formula before calculate $K_1=46.58$. Then, $K_1=46.58$ and $L_1=L_0+(0.02)L_0=20.4$, $A_1=1*1.06$ gives $Y_1=27.22$ and $\frac{Y_1}{L_1}=1.231$ and growth=0.084. We repeate this process 30 times

Technological Progress zero

```
40.00
                    20.00
                              1.00
                                         24.62
                                                        2.00 1.251 -
2001
          46.59
                    20.40
                              1.00
                                         26.13
                                                        2.28 1.281 0.041
          60.71
                              1.00
                                         29.09
                                                        2.86 1.371
          68.23
                              1.00
                                                        3.15 1.411
                              1.00
                                         32.00
                                                        3.44 1.449
                                                        3.73 1.485 0.025
                              1.00
                                         34.59
                                                        4.02 1.518
         101.07
                                                        4.31 1.550 0.021
                              1.00
         109.95
                                                        4.60 1.581 0.020
         119.08
                              1.00
                                                        4.88 1.609 0.018
         128,47
                              1.00
                                                        5.17 1.637 0.017
         135.11
                                         42.17
                                                        5.45 1.663 0.016
         148.00
                                         43.66
                                                        5.72 1.687 0.015
         158.14
                    26.19
                                         45.16
                                                        5.99 1.711 0.014
         168.52
                              1.00
1.00
1.00
         179.15
                                                        6.53 1.755 0.012
         190.03
                                                        6.79 1.776 0.012
         201.15
                                                        7.04 1.796 0.011
                              1.00
                                         52.88
                                                        7.29 1.815 0.011
         224.13
                                                        7.54 1.833
         236.00
                                                        7.79 1.851 0.010
                              1.00
         245.11
                                         57.75
                                                        8.02 1.868 0.009
         260.47
                                                        8.26 1.884 0.009
2024
         273.09
                                         61.11
                                                        8.49 1.900 0.008
         285.96
                                                        8.71 1.915 0.008
         299.09
                                                        8.94 1.929 0.008
                                                        9.15 1.945 0.007
         326.12
         340.04
                    35.52
                                                        9.57 1.969 0.007
                                                        9.78 1.982 0.006
```

Technological Progress positive

```
2001
                     20.40
                              1.05
                                                       2.28 1.334
                     20.81
                                                       2.59 1.443
                                                      2.91 1.553
                              1.26
2006
           90.27
                              1.42
                                        43.63
                                                       4.01 1.937
2007
          101.55
                                                      4.42 2.078
                                                       4.86 2.227
                                                       5.32 2.384
          141.76
                     24.38
                                        62.16
                                                       5.81 2.550
2013
          193.41
                     25.87
                                        80.19
                                                       7.48 3.107
                                                                      0.067
                     26.39
                              2.26
          235.63
                                        95.15
                                                                      0.066
          259.47
                                       103.44
                                                                      0.066
          285.31
                     28.00
                                       122.11
                                                      10.97 4.275
          343.69
                     30.31
                                       156.20
                                                      13.60 5.153
          450.86
                                                      14.58 5.481
          492.68
                              3.82
                                                      15.62 5.828
2024
          537.97
                     32.17
                                       199.33
                                                      16.72 6.196
2025
          587.01
                     32.81
                              4.29
                                       216.11
                                                      17.89 6.586
                                                                      0.063
          640.10
                     33.47
                                       234.25
                                                      19.13 6.999
2027
          697.58
                                       253.88
                                                      20.41 7.417
2028
          759.79
                     34.82
                                       275.10
                                                      21.82 7.900
                                                                      0.062
          827.12
                     35.52
                              5.42
                                       298.05
                                                     23.29 8.392
                                                                      0.063
                                                      24.84 8.912
```

Technology Creation

- Creating new technologies requires investment.
- Nature of investment in technology varied across different places and times.
- In a modern economy, vast resources are devoted to (R&D), to create new products or processes.

Research and Development

- A fairly recent phenomenon.
- Before the mid-nineteenth century technological advance was primarily the product of "tinkerers" rather than formally trained scientists.
- Even today, formal R&D of big corporations is often overshadowedby hackers working in their spare time out of a garage.
- Most R&D conducted by private firms seeking to maximize profits.
- Unique nature of technology led governments to play a role in research



Transfer of Technology

- Transfer of Technology is non-rival: one person's use of a piece of technology in no way prevents others from using it just as effectively.
- Non-rivalry of technology means we should focus on transfers between firms or countries.
- Easy transferability means that the incentives for creating technology are diminished.
- European integration has been proven to promote technology transfer.

Patents and Trade Secrets

- Transferability reduces incentive to create new technologies.
- Venice patent law of 1474
- Patenting an invention requires a detailed public description. Makes
 it easier to come up with a close substitute –and once the patent
 has expired others will be able to copy it exactly.
- Coca-Cola was never patented, and has remained secret for more than a century.

Deep routes. The Origins of Innovation and Trust

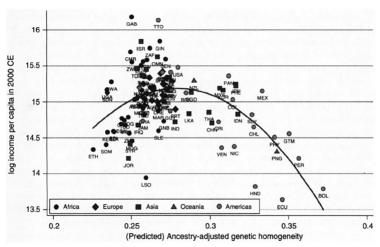
- The role of ancestry genetic diversity on economic development
- Reading: Ashraf and Galor, 2013, "The "Out of Africa" Hypothesis, Human Genetic Diversity, and Comparative Economic Development", American Economic Review.
- 1. (Channels) Heterogeneity raises the likelihood of disarray and mistrust, reducing cooperation and disrupting the socioeconomic order.
- 2. (Channels) Aggregate productivity is enhanced by an increased capacity for technological advancement and innovation
- The closer the migratory distance from africa, the higher the genetic diversity. (Africa very diverse, Latin America very homogeneous)

The Origins of Innovation and Trust

TABLE 9... COSTS AND RENEETS OF DIVERSITY

	int	Degree of	ist	Scientific articles per capita per year 1981–2000			
	Full sample (1)	Full sample (2)	Common sample (3)	Full sample (4)	Full sample (5)	Common sample (6)	
Predicted diversity	-1.880**	-2.226**	-1.920**	2.484***	1.860***	3.023**	
(ancestry adjusted)	(0.829)	(0.862)	(0.940)	(0.566)	(0.550)	(1.222)	
log Neolithic transition	0.069	0.089	0.091	-0.085*	-0.080*	-0.189**	
timing (ancestry adjusted)	(0.062)	(0.063)	(0.060)	(0.047)	(0.046)	(0.085)	
log percentage of arable land	0.004 (0.019)	-0.002 (0.018)	-0.014 (0.019)	0.008 (0.016)	0.005 (0.015)	-0.005 (0.037)	
log absolute latitude	-0.003	0.003	-0.008	0.046*	0.055**	0.079	
	(0.027)	(0.028)	(0.031)	(0.024)	(0.023)	(0.073)	
Social infrastructure	0.200***	0.146*	0.149*	0.702***	0.547***	0.627***	
	(0.069)	(0.079)	(0.081)	(0.109)	(0.125)	(0.197)	
Ethnic fractionalization	0.060	0.048	0.054	0.091	0.073	0.088	
	(0.065)	(0.064)	(0.067)	(0.094)	(0.093)	(0.155)	
Percentage of population at	-0.063	-0.033 (0.092)	-0.048	0.093*	0.135***	0.026	
risk of contracting malaria	(0.090)		(0.100)	(0.054)	(0.048)	(0.120)	
Percentage of population	-0.074	-0.040	-0.046	0.020	0.051	0.113	
living in tropical zones	(0.053)	(0.059)	(0.062)	(0.057)	(0.055)	(0.181)	
Mean distance to nearest	0.094	0.098	0.077	0.104***	0.115***	0.159**	
waterway	(0.064)	(0.060)	(0.058)	(0.038)	(0.035)	(0.067)	
Years of schooling		0.013 (0.010)	0.017* (0.010)		0.031*** (0.008)	0.021 (0.023)	
Observations	58	58	56	93	93	56	

Genetic Diversity and Economic Development



- Assume Population L=1 and no population growth
- Assume that the production function is now given by:

$$Y_t = K_t^a (H_t)^{1-a} \tag{1}$$

where H_t denotes the level of human capital

 Production is used for consumption, investment and government spending:

$$Y_t = C_t + I_t + G_t \tag{2}$$

where G denotes government spending on education.

ullet Government Finances G by levying a tax on income:

$$G_t = \tau Y_t \tag{3}$$

• After paying taxes, a fraction γ of the disposable (after taxes) income $(1-\tau)Y$ goes to consumption and investment

$$I = \gamma (1- au) \, Y_t$$
 and $C = (1-\gamma) (1- au) \, Y_t$

- Capital accumulation follows Solow.
- Capital tomorrow is determined by investment in new capital, $I=\gamma(1-\tau)\,Y$, plus what capital left from the previous period $(K_t-\delta K_t)$

$$K_{t+1} = \gamma(1-\tau)Y_t + K_t - \delta K_t \tag{4}$$

 Human capital accumulation will be driven by the provision of public schooling from the government. Denoting public investment in education by G, we can write the dynamics of Human capital as:

$$H_{t+1} = G_t + H_t - \delta^H H_t \tag{5}$$

where $\delta^{H}~(0<\delta^{H}<1)$ is the depreciation rate of human capital.

• Substructing $\frac{Y_t}{Y_t}$ in both sides of (9) we get $\frac{Y_{t+1}}{V} - \frac{Y_t}{V} = A\gamma^a(1-\tau)^a\tau^{1-a} - \frac{Y_t}{V}$:

$$Y_{t+1} - Y_t \qquad \qquad Y_{t+1} - Y_t \qquad \qquad Y_{t+1} - Y_t \qquad \qquad Y_{t+1} - Y_t = Y_t - Y_t -$$

$$g \equiv \frac{Y_{t+1} - Y_t}{Y_t} = A\gamma^a (1 - \tau)^a \tau^{1-a} - 1$$

which is the growth rate of income per capita end depends on:

- Productivity, A (positively)
- Savings/Investment rate, γ (positively)
- and the level of tax rate (inverse U-shaped)

What drives differences in savings rates?

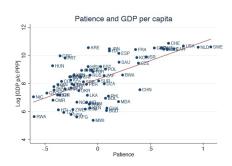
- The motive for savings depends on
- a) Market: the return on savings (productivity of the capital stock)
- b) Culture: the subjective discount rate (rate of time preference)
- Aristotle said "Patience is bitter, but its fruit is sweet"
- This is confirmed by the empirical data.

Distribution of Time Preference



Figure 2: Distribution of Patience Across Countries

Time Preference and GDP per capita



Conclusions - Exogenous Growth

- Income per worker depends positively on capital per worker.
 More machines per worker, more production per worker, higher income per worker.
- The long-run level of capital per worker depends on savings, technology and depreciation.
- Savings/investment rates positively affect capital per worker in the short-run and in the long-run.
- Growth inevitably stops in the long-run if capital experiences diminishing returns in production.
- An one time increase in saving or technology has only a positive short-run effect on growth.



Conclusions - Endogenous Growth

- Technological progress is the key to explain persistent long-run growth
- But, where technological progress comes from? Answer, the decision of agents.
- That is, savings, years of education, taxation, spending on research and development, trust, cooperation.
- This in turn explains differences in income per capita around Europe.
- Answers European policys and growth pacts targeting those factors.



Conclusions - Example Greece

