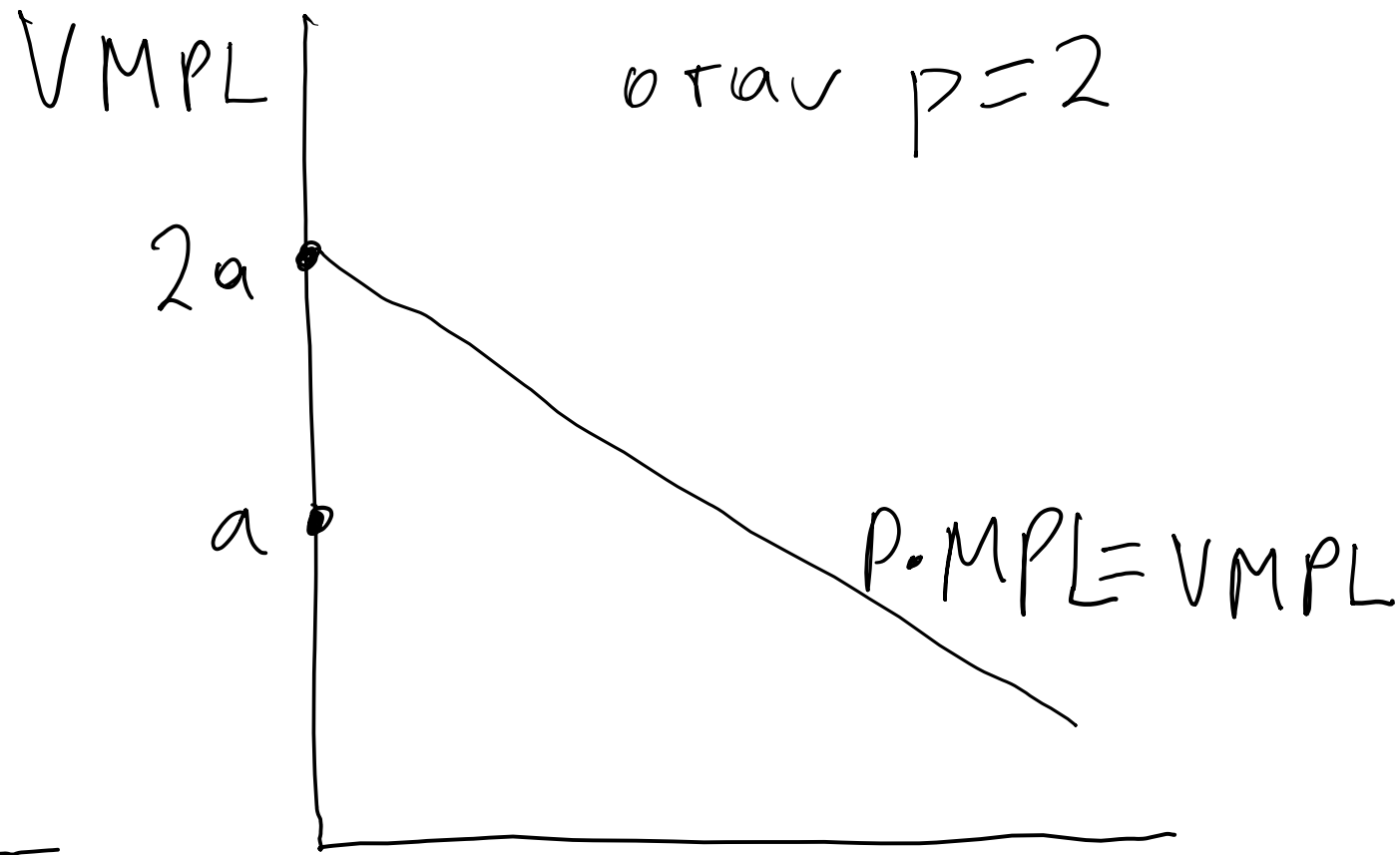


This diagram shows the (physical) marginal product of labour (MPL)



This diagram shows the value of the MPL (VMPL), which is equal to $P \cdot MPL$.

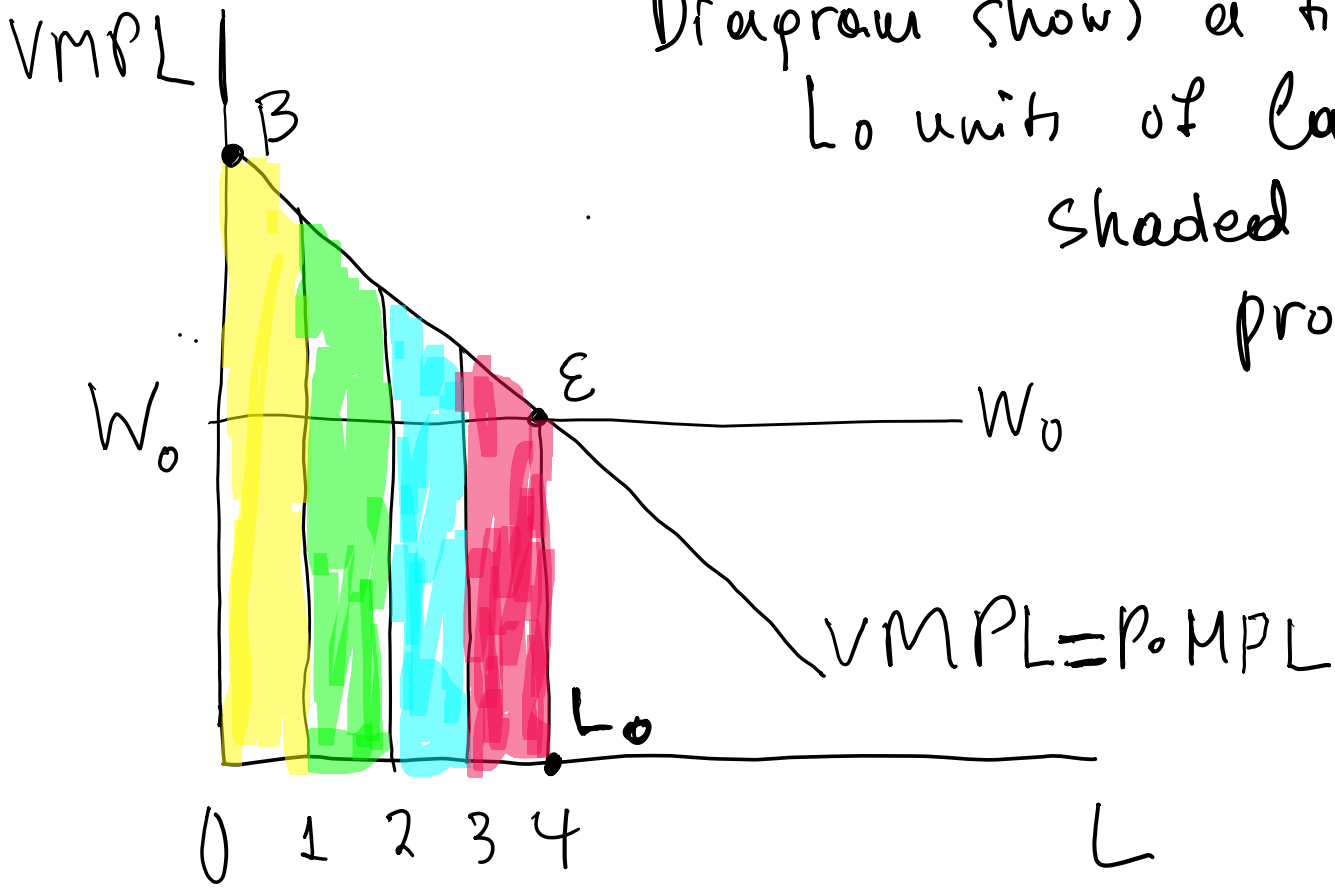


Diagram shows a firm which at wage W_0 employs L_0 units of labour (say 4). The yellow shaded area shows the value of output produced by the first unit of

labour, the green area shows the value of output produced by the second unit of labour, and so on. Adding the four shaded areas (yellow,

green, blue, red) we get the total value of output produced by the four units of labour. This is equivalent to the total revenue of the firm, i.e. the area $OBE L_0$.

Production Functions

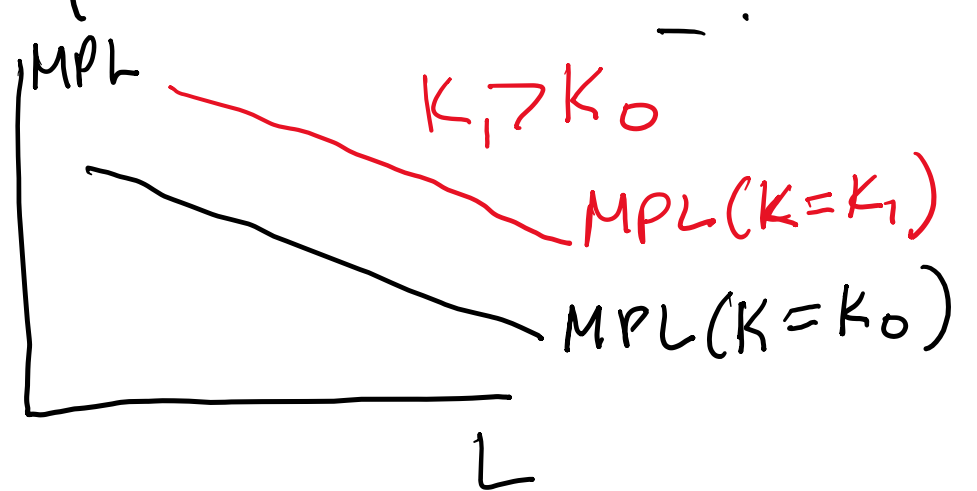
Assume that output is produced using labour (L) and capital (K). We write this as $Q = F(L, K)$, which is a production function. A widely used production function takes the form $Q = L^b K^{1-b}$, where b is a parameter, $0 < b < 1$.

$$MPL \equiv \frac{dQ}{dL} = b L^{b-1} K^{1-b} = b \left(\frac{K}{L}\right)^{1-b} \quad (1)$$

$$MPK \equiv \frac{dQ}{dK} = (1-b) L^b K^{-b} = (1-b) \left(\frac{L}{K}\right)^b \quad (2)$$

Από τις συναρτήσεις (1), (2) συνεπάγεται ότι \rightarrow

αν το $\frac{K}{L}$ (δηλ. ο λόγος κεφαλαίου προς, εργασία) είναι
 σταθερό, τότε το MPL και το MPK είναι σταθερά. Δηλαδή,
 αν, π.χ. διπλασιασθεί το L και το K, τότε το MPL και
 το MPK θα είναι τα ίδια όπως και πριν τον διπλασιασμό
 του K και του L. Αν όμως το $\frac{K}{L}$ αυξηθεί (δηλ. υπάρχει
 περισσότερο κεφαλαίο ανά εργαζόμενο) τότε το MPL γίνεται
 μεγαλύτερο (δηλ. ολόκληρη η καμπύλη του MPL μετατοπίζεται
 προς τα πάνω και δεξιά)



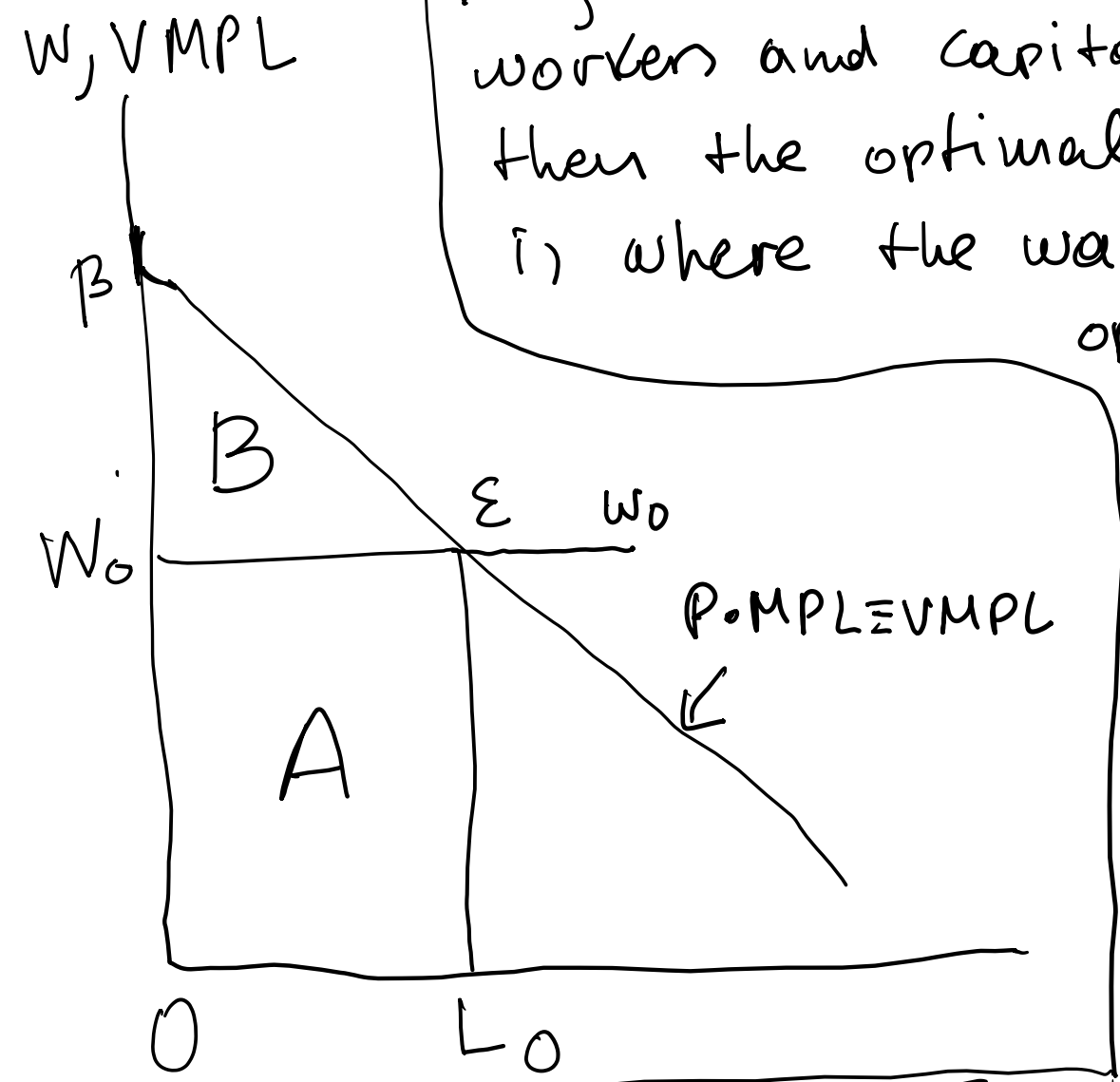
Αυριότροιχα, αν το L/K αυξηθεί (δηλ. υπάρχουν περισσότεροι εργαζόμενοι ανά μονάδα κεφαλαίου) τότε το ΜΡΚ θ' αυξηθεί (δηλ. ολόκληρη η καμπύλη του ΜΡΚ θα μετακινηθεί από τα πάνω και δεξιά). Επομένως, συνοψίζοντας, αν μετά από οποιαδήποτε μεταβολή έχουμε



ότι $\left(\frac{K}{L}\right) \uparrow$; τότε $MP_L \uparrow$ και $MP_K \downarrow$.

If output Y produced by L and K , then the value of output produced will be shared (not necessarily equally) between the owners of the two factors of production, i.e. the owners of labor (workers), and the owners of capital (capitalists). The following diagram shows how we can depict the distribution of income between labour (workers) and capital (capitalists).

Diagram shows the distribution of income between workers and capitalists. If w_0 is the (nominal) wage, then the optimal employment level for the firm is where the wage line cuts the VMPL curve, i.e. optimal employment is determined when $w = VMPL = P \cdot MPL \rightarrow$



where $\frac{w}{p} = MPL$. This means that at the optimal employment level, the real wage is equal to the MPL. Total value of output is area $OBEL_0 \equiv A + B$. Total wage payments made to the L_0 units of labour employed is area $OW_0EL_0 \equiv A$. The remaining is the total profits ($w_0BE \equiv B$), which is the income of capitalists.

$w, P \cdot MPL$

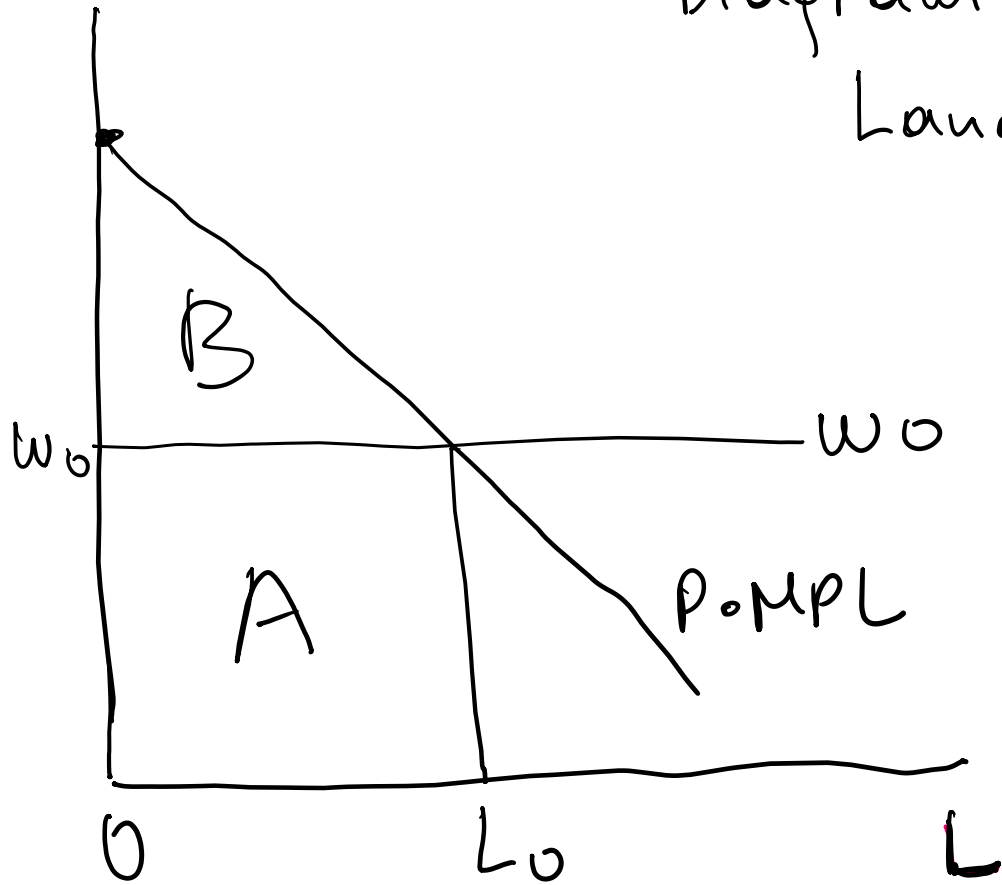


Diagram shows the case that Labour (L) and Land (T) are used to produce output.

Total value (i.e. total revenue) are equal to areas A and B, total wage payments are equal to A, thus the profits of landowners are equal to B (income of landowners).

Note: If the amount of land which is used for producing the good increases, the $P \cdot MPL$ curve will shift to the right.

Under perfect competition, the same way that at the optimal employment level $MPL = W/P$ (the marginal product of labour is equal to the real wage), we have also $MPK = \frac{r_K}{P}$ (the marginal product of capital is equal to the real reward $(\frac{r_K}{P})$ per unit of capital), and $MPT = \frac{r_T}{P}$ (the marginal product of land is equal to the real reward $(\frac{r_T}{P})$ per unit of land).

Here, r_K and r_T are the nominal rental rates per unit of capital and land (respectively). Thus, if MPK or MPT change, so will be their real rewards.

- The above imply that if the MPL is higher, then the MPK (or MPT) will be lower, which implies that a higher real wage will be associated with a lower real income per unit of capital (or land).
- Thus a rise in K/L implies a higher MPL, a higher real wage (w/p), a lower MPK and a lower income for capitalists per unit of capital.
- Similarly, a rise in T/L implies a higher MPL, a higher real wage (w/p), a lower MPT and a lower income for landowners per unit of land.
- All of the above hold if there is perfect competition, and the production function is like the one we used in slide 3.

