Χρηματοοικονομική Οικονομετρία.
Παρακαλώ απαντήστε σε όλες τις ερωτήσεις. Χρόνος 75 λεπτά

Consider the following MA(1) process:

$$r_t = z_t - \theta z_{t-1}$$

where z_t is an iid process with 0 mean and unit variance

1. **State the stationarity (second order) conditions for the above model. State the conditions for invertibility for the above model.**
2. **Derive the autocorrelation function for the above model.**

Consider the Dow-Jones weekly excess returns over the period from 31/08/1979 to 23/05/2008, a sample size of 1500 observations.
Table 1 presents the correlogram and the Q- tests (up to ten lags) for Dow Jones excess returns and Table 2 the correlogram and the Q- tests for their squares.

Table 1 (DJ excess returns - Correlogram)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
*| |	*| |	1	-0.063	-0.063	5.9647	0.015
| |	| |	2	0.052	0.049	10.081	0.006
| |	| |	3	-0.022	-0.016	10.814	0.013
| |	| |	4	-0.017	-0.022	11.272	0.024
| |	| |	5	-0.049	-0.049	14.834	0.011
| |	| |	6	0.063	0.059	20.819	0.002
| |	| |	7	-0.034	-0.023	22.527	0.002
| |	| |	8	-0.020	-0.032	23.153	0.003
| |	| |	9	0.013	0.013	23.394	0.005
| |	| |	10	-0.035	-0.032	25.276	0.005

Table 2 (Squared DJ excess returns - Correlogram)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
|** |	|** |	1	0.232	0.232	81.118	0.000
|* |	| |	2	0.094	0.043	94.474	0.000
| |	| |	3	0.063	0.034	100.36	0.000
| |	| |	4	0.040	0.016	102.82	0.000
| |	| |	5	0.031	0.015	104.32	0.000
|* |	|* |	6	0.079	0.068	113.77	0.000
|* |	|* |	7	0.133	0.103	140.51	0.000
|* |	| |	8	0.070	0.011	147.97	0.000
| |	| |	9	0.047	0.012	151.31	0.000
| |	| |	10	0.045	0.019	154.39	0.000

3. **According to the results presented in Table 1, is there any autocorrelation in the data (5% level)?**
4. **According to the results presented in Table 2, are the squared excess returns autocorrelated of order 1 at 5% level**
5. **What is the value of the statistic for testing the autocorrelation of the squared returns (Table 2) of up to 6^{th} order (including the 6^{th} order)? What is the value of the statistic for testing the 6^{th} order autocorrelation of the squared returns?**

Consider the GARCH (1,1) Model

$$r_t=\gamma+\varepsilon_t, \quad \sigma^2_t=C+\alpha\,\varepsilon^2_{t-1}+\beta\,\sigma^2_{t-1}$$

Table 3 (GARCH (1,1) Estimation for the Dow Jones data)

Dependent Variable: DJ
Method: ML – ARCH
Sample: 1 1500
Included observations: 1500
Convergence achieved after 15 iterations
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.143512	0.048491	2.959540	0.0031
		Variance Equation		
C	0.141469	0.040472	3.495508	0.0005
RESID(-1)^2	0.091900	0.012350	7.441156	0.0000
GARCH(-1)	0.881804	0.016829	52.39736	0.0000

R-squared	-0.000542	Mean dependent var	0.092885
Adjusted R-squared	-0.002548	S.D. dependent var	2.176245
S.E. of regression	2.179016	Akaike info criterion	4.295719
Sum squared resid	7103.173	Schwarz criterion	4.309887
Log likelihood	-3217.789	Durbin-Watson stat	2.122506

6. **What is meant by "positivity restrictions"? State the positivity restrictions for this model. Do the coefficients comply with these restrictions? What is the value of the variance persistence?**

Table 4 (EGARCH (1,1) Estimation)

Dependent Variable: DJ
Method: ML – ARCH
Sample: 1 1500
Included observations: 1500
Convergence achieved after 14 iterations
Variance backcast: ON
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.070371	0.049544	1.420371	0.1555
	Variance Equation			
C(2)	-0.057136	0.020938	-2.728801	0.0064
C(3)	0.171804	0.023353	7.356780	0.0000
C(4)	-0.095301	0.014983	-6.360542	0.0000
C(5)	0.947901	0.012094	78.37473	0.0000

R-squared	-0.000107	Mean dependent var	0.092885
Adjusted R-squared	-0.002783	S.D. dependent var	2.176245
S.E. of regression	2.179271	Akaike info criterion	4.273782
Sum squared resid	7100.088	Schwarz criterion	4.291493
Log likelihood	-3200.336	Durbin-Watson stat	2.123428

7. **Is the model second order stationary? Identify the asymmetry parameter(s). Is it (are they) significant?**
8. **Compare the GARCH(1,1) with the EGARC(1,1) models (Tables 3 and 4)**

You could use the following:
Critical Value of the Standard Normal leaving 10% at the <u>right</u> tail is 1.285, leaving 5% is 1.645, and 2.5% is 1.960.

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