# answers,diagnostic test

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diagnostic testtext



**please answer all questions**

## PROBLEM 0

**find all global maxima of the following maximization problem or show that none exist.**

| maximization problem | |
|---|---|
| Objective function | $\mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = 4x - x^2$ |
| Constraints | none $(x \in \mathbb{R})$ |
| Variables | $x$ |

## PROBLEM 1

**find all global maxima of the following maximization problem, or show that none exist**

| maximization problem | |
|---|---|
| Objective function | $\mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = 12x - x^3$ |
| Constraints | none $(x \in \mathbb{R})$ |
| Variables | $x$ |

(1)

## PROBLEM 2

**find all global maxima of the following maximization problem, or show that none exist**

| maximization problem | |
|---|---|
| Objective function | $\mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = 12x - x^3$ |
| Constraints | $x \geq 0$ |
| Variables | $x$ |

(2)

## PROBLEM 3

**find all global maxima of the following maximization problem or show that none exist.**

$$\boxed{\begin{array}{ll} \boxed{\text{maximization problem}} & \\ \text{Objective function} & \mathbb{R}^2 \xrightarrow{f} \mathbb{R}, f(x) = x_1x_2 - 2x_1 - 3x_2 \\ \text{Constraints} & \text{none } \left(x \in \mathbb{R}^2\right) \\ \text{Variables} & x_1, x_2 \end{array}} \tag{3}$$

## PROBLEM 4

**find all global maxima of the following maximization problem or show that none exist.**

$$\boxed{\begin{array}{ll} \boxed{\text{maximization problem}} & \\ \text{Objective function} & \mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = x \\ \text{Constraints} & 0 \le x < 1 \\ \text{Variables} & x \end{array}} \tag{4}$$

## PROBLEM 5

**For all allowed values of the parameters, find all global maxima of the following maximization problem or show that none exist.**

$$\boxed{\begin{array}{ll} \boxed{\text{maximization problem}} & \\ \text{Objective function} & \mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = px^2 - wx \\ \text{Constraints} & x \ge 0 \\ \text{Variables} & x \\ \text{Parameters} & p, w \\ \text{conditions on parameters} & p > 0, w > 0 \end{array}} \tag{5}$$

## PROBLEM 6

**For all allowed values of the parameters, find all global maxima of the following maximization problem or show that none exist.**

| maximization problem | |
|---|---|
| Objective function | $\mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = px - wx$ |
| Constraints | $x \geq 0$ |
| Variables | $x$ |
| Parameters | $p, w$ |
| conditions on parameters | $p > 0, w > 0$ |

(6)

## PROBLEM 7

**For all allowed values of the parameters, find all global maxima of the following maximization problem or show that none exist.**

| maximization problem | |
|---|---|
| Objective function | $\mathbb{R} \xrightarrow{f} \mathbb{R}, f(x) = p\sqrt{x} - wx$ |
| Constraints | $x \geq 0$ |
| Variables | $x$ |
| Parameters | $p, w$ |
| conditions on parameters | $p > 0, w > 0$ |

(7)

## PROBLEM 8

**Compute all pareto efficient points in the following**

| pareto efficency problem | |
|---|---|
| Objective functions | |
| $\mathbb{R}^2 \xrightarrow{f} \mathbb{R}, f(x, y) = x$ | |
| $\mathbb{R}^2 \xrightarrow{g} \mathbb{R}, g(x, y) = y$ | |
| Constraints | $x \geq 0, y \geq 0, \sqrt{x} + y \leq 1$ |
| Variables | $x, y$ |

(8)



PROBLEM 0

$x = 2$ IS GLOBAL MAX, BY FRITZ-JOHN AND ARROW-ENTHOVEN.

PROBLEM 1

THERE IS NO GLOBAL MAX, BECAUSE

$$f(-t) = t^3 - 12t \longrightarrow \infty \quad \text{AS} \quad t \longrightarrow \infty$$

PROBLEM 2

$x = 2$ IS GLOBAL MAX, BY FRITZ JOHN AND ARROW-ENTHOVEN

PROBLEM 3

THERE IS NO GLOBAL MAXIMUM, BECAUSE

$$f(t,t) = t^2 - 5t \longrightarrow \infty \quad \text{AS} \quad t \longrightarrow \infty$$

PROBLEM 4

THERE IS NO GLOBAL MAXIMUM, BECAUSE

$\forall\, b \in [0,1)$, $\frac{b+1}{2} \in [0,1)$ AND

$$f\left(\frac{b+1}{2}\right) > f(b)$$

PROBLEM 5

THERE IS NO GLOBAL MAX, BECAUSE

$$f(x) = px\left(x - \frac{w}{p}\right) \longrightarrow \infty \quad \text{AS} \quad x \longrightarrow \infty$$

PROBLEM 6

$$\text{GLOBAL MAX} = \begin{cases} \text{NONE} & \text{IF } \frac{w}{p} < 1 \\ \text{ALL } x \geq 0 & \text{IF } \frac{w}{p} = 1 \\ 0 & \end{cases}$$

MAX $L_0$ $+\frac{w}{p} < 1$

BY FRITZ JOHN AND ARROW-ENTHOVEN

PROBLEM 7

GLOBAL MAX $x = \frac{p^2}{4w^2}$

BY FRITZ JOHN AND ARROW-ENTHOVEN

PROBLEM 8 MAY BE LATER

## 5.2 A note of caution

In this chapter we will discuss various *necessary* optimality conditions for a given point to be a local minimum to a nonlinear programming model. If the NLP is a convex program, any point satisfying these necessary optimality conditions is not only a local minimum, but actually a global minimum (see Section 5.8). Arguably, most NLP models that arise in real world applications tend to be nonconvex, and for such a problem, a point satisfying the necessary optimality conditions may not even be a local minimum. Algorithms for NLP are usually designed to converge to a point satisfying the necessary optimality conditions, and as mentioned earlier, one should not blindly accept such a point as an optimum solution to the problem without checking (e.g., using the second order necessary optimality conditions, see [BSS93, Section 4.4], or by means of some local search in the vicinity of the point) that it is at least better than all the other nearby points. Also, the system of necessary optimality conditions may have many solutions. Finding alternate solutions of this system, and selecting the best among them, usually leads to a good

We will illustrate the importance of this with the story of the US Air Force's controversial B-2 Stealth bomber program in the Reagan era of the 1980s. There were many design variables, such as the various dimensions, the distribution of volume between the wing and the fuselage, flying speed, thrust, fuel consumption, drag, lift, air density, etc., that could be manipulated for obtaining the best range (i.e., the distance it can fly starting with full tank, without refueling). The problem of maximizing the range subject to all the constraints was modeled as an NLP in a secret Air Force study going back to the 1940s. A solution to the necessary optimality conditions of this problem was found; it specified values for the design variables that put almost all of the total volume in the wing, leading to the *flying wing design* for the B-2 bomber. After spending billions of dollars, building test planes, etc., it was found that the design solution implemented works, but that its range was too low in comparison with other bomber designs being experimented subsequently in the US and abroad.

A careful review of the model was then carried out. The review indicated that all the formulas used, and the model itself, are perfectly valid. However, the model was a nonconvex NLP, and the review revealed a second solution to the system of necessary optimality conditions for it, besides the one found and implemented as a result of earlier studies. The second solution makes the wing volume much less than the total volume, and seems to maximize the range; while the first solution that is implemented for the B-2 bomber seems to actually minimize the range. (The second solution also looked like an aircraft should, while the flying wing design was counter-intuitive.) In other words, the design implemented was the aerodynamically *worst* possible choice of configuration, leading to a very costly error. The aircraft does fly, but apparently, then, has the only advantage that it is a "stealth" plane.

For an account, see "Skeleton Alleged in the Stealth Bomber's Closet," *Science*, vol. 244, 12 May 1989 issue, pages 650–651.

The short answer: beyond the B-2 story, there are essentially *families* of real-world examples where "optimal" solutions from first-order conditions / gradient-based NLP are either (i) clearly wrong in hindsight or (ii) physically or economically meaningless. A few representative sources:

### 1. The B-2 stealth bomber / flying-wing (the canonical case)

You already have this; here are the key sources that spell out the optimization issue explicitly.

1. **Andréasson–Evgrafov–Patriksson, "An Introduction to Optimization: Foundations and Fundamental Algorithms" (draft, 2005)**
   Section 5.3 uses the B-2 as a cautionary example. They describe a nonconvex NLP for maximizing range subject to aerodynamic constraints; one stationary point puts almost all volume into the wing (flying wing), which **turned out to *minimize* range** relative to other feasible designs, while another stationary point (more conventional layout) actually maximizes range.
   Point stressed: *solving the necessary optimality conditions of a nonconvex NLP and implementing the resulting stationary point can give you the worst feasible design.*
2. **Biddle, "Skeleton Alleged in the Stealth Bomber's Closet", Science 244 (1989), 650–651**
   Original journalistic account of the Air Force study; explains that a calculus-based optimization study from the 1940s led

to the flying-wing configuration, and only later was it realized that this was aerodynamically poor compared with alternative configurations.

A number of optimization texts, lecture notes, and even pedagogy papers on "math mistakes that make the news" simply repeat or paraphrase this story and use it to hammer home "first-order ≠ optimum" in nonconvex problems.

## 2. Structural / topology optimization: "optimal" designs that are non-physical

Here the pathology is very similar conceptually: if you take the standard SIMP / density-based formulations literally and just drive a gradient-based method to a stationary point, you routinely get *mathematically optimal* layouts that no engineer would accept as real structures.

### 2.1 Checkerboard patterns and mesh-dependent "optimal" layouts

- **Rozvany, "A critical review of established methods of structural topology optimization", Struct. Multidisc. Optim. 37 (2009) 217–237**
  This review is basically a long list of ways in which standard topology-optimization formulations fail in practice. It emphasizes that straight SIMP+finite element formulations produce:
  *checkerboard patterns*, mesh-dependent designs, and other layouts that are **artificially stiff but not manufacturable**.
  Message: the numerical optimizer happily converges to KKT points that exploit discretization pathologies; they satisfy first-order conditions but violate the *real* engineering problem ("not realistic solutions to manufacture").
- **Takahashi et al., "Convolutional Neural Network-based Topology Optimization", 2019**
  The paper explicitly shows checkerboard patterns as "locally discontinuous elements" and notes that such an "overall structure is not realistic"; filtering is then used to remove these patterns.
- **Antonietti et al., "On the virtual element method for topology optimization on polygonal meshes", 2016**
  Discusses how standard formulations can steer the optimizer to **sub-optimal, non-physical layouts**, and how modified discretizations (VEM, special meshes) help avoid such spurious stationary points.
- **Wu, "Topology optimization of multi-scale structures: a review", Struct. Multidisc. Optim. 60 (2019)**
  Notes explicitly that some commonly used SIMP-based macro-models are "non-physical" and that naive use leads to designs whose predicted performance "does not make mechanical sense".

All of these are saying, in effect:

> If you treat the FEM+SIMP model as "the problem" and accept any KKT point it gives you as an "optimal design", you can be badly misled. The stationary point reflects discretization artefacts, not real structural behaviour.

This is the same logical failure as in the B-2 story: **necessary conditions + an over-simplified model + nonconvexity ⇒**

**“optimal” nonsense.**

### 3. Photonic inverse design: naive formulations give non-physical devices

Inverse design in photonics is another area where the first, naïve optimization model routinely produces “solutions” that satisfy the optimality system but are physically impossible or meaningless.

- **Christiansen & Sigmund, “Inverse design in photonics by topology optimization: tutorial”, JOSA B 38 (2021) 496–509**
  They set up a basic PDE-constrained optimization problem for designing a photonic device and show that solving this **naive formulation** (with unconstrained material interpolation, insufficient regularization, etc.) “leads to non-physical solutions,” which then motivates additional constraints and filters.

Again, the point is not that the optimizer is wrong: it has found a stationary point of *the* mathematical problem. The issue is that in a nonconvex, badly posed formulation, the necessary conditions happily point you toward designs with unphysical permittivity distributions or tiny, unmanufacturable features.

### 4. Finance: Markowitz mean-variance optimization and “unrealistic” portfolios

In portfolio theory the pathology is not geometric shape but asset weights. The classical Markowitz mean–variance problem is convex for given inputs, so KKT is *also* sufficient; the problem is that practitioners plug in noisy estimates and then accept the optimizer’s solution at face value.

What the finance literature is very explicit about is:

> The mathematically optimal mean–variance portfolio is often *economically absurd* (extreme, unstable weights, corner solutions), because the model and inputs are not trustworthy.

Representative sources:

- **Michaud, “Estimation Error and Portfolio Optimization: A Resampling Solution”**
  Shows that standard mean–variance optimization tends to produce **extreme and unstable portfolio weights** and “unrealistic portfolios” because the optimizer treats sample estimates of means and covariances as exact.
- **Black & Litterman-type critiques; e.g. real-estate mixed-asset portfolio analysis**
  One paper notes that naively applying Markowitz formulas leads to “corner solutions” with some assets at zero weight and others with very large allocations, and that such “unrealistic portfolio combinations” are unacceptable in practice.

Here the “same point” is slightly shifted: the math problem *is* convex, but the culture of treating “solution of the optimization model” as gospel, without interrogating the model’s realism, leads to systematically bad real-world recommendations.

### 5. What you actually get from this search

1. **Genuinely B-2-style stories are rare.** The stealth-bomber/flying-wing example is the textbook case where a misused system of necessary conditions for a nonconvex engineering NLP drove a major, visible design choice.
2. **In most other fields the pathology shows up as "optimal but non-physical / non-usable designs", not a single famous bad product.**
   - Structural and topology optimization: checkerboards, mesh-dependent junk, free-floating or disconnected material regions, etc.
   - Photonics: non-physical material distributions.
   - Portfolio optimization: portfolios that are "optimal" only because the model ignores estimation error and practical constraints.
3. **All of these are routinely used in the literature to make your point:**
   – First-order (or KKT) conditions are at best *necessary* in nonconvex engineering problems.
   – Even when they are sufficient mathematically, the optimization model is an abstraction; blindly trusting the optimizer's stationary point without checking convexity, second-order conditions, globality and model realism is a good way to get either a flying wing with lousy range or a structure/portfolio/device that no sane engineer or investor wants.

If you want, I can next:

- Pull out precise theorem statements / geometric optimality conditions from Andréasson–Evgrafov–Patriksson right after the B-2 anecdote, and
- Parallel that with a short "lecture-note style" section collecting these real-world examples as standard counterexamples to "KKT ⇒ good design".