PROBLEM 1

Let $[a,b]$ be a basis of R^2 ,and let $\alpha \in R, \beta \in R$ be two real numbers. For which values of α, β is $[a+b, \alpha a]$ a basis of R^2 ? For which values of α, β is $[\alpha a, \beta b]$ a basis of R^2 ?

PROBLEM 2

Consider the following subset of R^5

$$W = \{x \in R^5 : x_1 - x_3 - x_5 = 0\} \tag{1}$$

1.show that W is closed under linear combinations, hence a subspace of R^5

2.Find a linear map $R^5 \xrightarrow{T} R^5$ such that $W = \text{nullspace}(T)$ .Can this linear map T be one-to-one? onto?

3.Find a linear map $R^5 \xrightarrow{T} R^5$ such that $W = \text{Range}(T)$ .Can this linear map T be one-to-one? onto?

4. Find a basis of W

PROBLEM 3

Consider the following subset of R^5

$$A = \{x \in R^5 : x_2 - x_5 = 4\} \tag{2}$$

1.show that A is closed under affine combinations, hence a flat in R^5 .Is it a hyperplane?

2.Find an affine map $R^5 \xrightarrow{T} R^5$ such that $A = \text{nullspace}(T)$ .Can this affine map T be one-to-one? onto?

3.Find an affine map $R^5 \xrightarrow{T} R^5$ such that $A = \text{Range}(T)$ .Can this affine map T be one-to-one? onto?

PROBLEM 4

Consider the following subset of R^5

$$C = \{x \in R^5 : x_2 - x_5 \geq 0, x_1 - x_2 \leq 0, x_3 \geq 0\} \tag{3}$$

1.show that C is closed under nonnegative linear combinations, hence a convex cone in R^5

2.Find a linear map $R^5 \xrightarrow{T} R^5$ such that $C = \{x \in R^5 : T(x) \geq 0\}$ .Can this linear map T be one-to-one? onto?

3.show that C is an intersection of half-spaces through the origin. Describe these half-spaces explicitly

PROBLEM 5

Consider the following subset of R^4

$$C = \{x \in R^4 : x_2 - x_4 \geq 6, x_1 - x_2 - x_3 \leq 7, x_3 \geq 0\} \quad (4)$$

1.show that C is closed under convex combinations, hence a convex set in R^5

2.Find an affine map $R^4 \xrightarrow{T} R^4$ such that $C = \{x \in R^4 : T(x) \geq 0\}$ .Can this affine map T be one-to-one? onto?

3.show that C is an intersection of half-spaces. Describe these half-spaces explicitly

PROBLEM 6

For each one of the following functions f ,and for each value of the real parameter c, compute and draw their better-than sets $B_c^f = \{(x, y) \in R_+^2 : f(x, y) \geq c\}$; state whether they are quasi-concave functions on R_+^2 ,or concave functions on R_+^2

- $f(x, y) = x + \sqrt{y}, c = 4$
- $f(x, y) = x + y^2, c = 4$
- $f(x, y) = x - \frac{1}{y}, c = 4$
- $f(x, y) = \min(x, y), c = 1$
- $f(x, y) = \max(x, y), c = 1$
- $f(x, y) = \min(x/4 + 1, y + 2), c = 3$
- $f(x, y) = \max(2x/3, 3y/2), c = 1$
- $f(x, y) = \min(x/4 + y - 1, x + y - 2, x, y), c = 1/2, 2, 4$
- $f(x, y) = \min(x, y, \frac{x^2 + y^2}{8}), c = 1$
- $f(x, y) = \min(\max(x, y), \max(2x/3, 3y/2)), c = 1$
- $f(x, y) = -(x-3)^2 - (y-3)^2, c = -4$

PROBLEM 7

Find all global maxima of the following maximization problem, or show that none exist

Objective function $f(x) = 3(2\sqrt{x+1} - 2) - 9x$

constraints $x \geq 0$

variables x

PROBLEM 8

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist

Objective function $f(x_1, x_2, x_3) = x_1 x_2 x_3 - w_1 x_1 - w_2 x_2 - w_3 x_3$

constraints $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

variables x_1, x_2, x_3

parameters w_1, w_2, w_3

conditions on parameters $w_1 > 0, w_2 > 0, w_3 > 0$

PROBLEM 9

For all allowed values of the parameters, find all global maxima of the following maximization problem, or show that none exist

Objective function $f(x_1, x_2) = \min(\frac{x_1}{4} + 1, x_2 + 2)$

constraints $x_1 + px_2 \leq 4, x_1 \geq 0, x_2 \geq 0$

variables x_1, x_2

parameters p

conditions on parameters $p > 0$