Question 1

THE ECONOMY

- N+1 consumers, N>4.
- Two goods: A and X.
- One firm, with production function: $A = 2\sqrt{L}$

  where L is the quantity of good X used in the production of good A.

- Consumer 0 has no endowment of either good. He is the sole owner of the firm. His preferences are given by the utility function $U_0 = X_0$.
- Consumers i=1...N have preferences described by the utility functions $U_i = \log A_i + \log X_i$.Each owns one unit of good X.
- Profits Π are taxed at a rate $0 \le t < 1$.Tax revenue $R = t\Pi$ is returned to consumers 1,2..,N with lump sum transfers $T_i = R / N$

1. Compute all Pareto efficient points

2. Compute all competitive equilibria with profit taxation.

3. Is profit taxation efficient?

ANSWERS

1.PARETO EFFICIENT POINTS

are the solutions of the maximization problem

$$\begin{array}{|l|}\hline \max\ U_0 = X_0 \\ \text{subject to} \\ U_i = \log A_i + \log X_i \ge \theta_i, i = 1..N \\ \sum_{i=1}^{N} A_i \le A = 2\sqrt{L} \\ \sum_{i=0}^{N} X_i + L \le N \\ A_i \ge 0, i = 1..N, X_i \ge 0, i = 0..N, L \ge 0 \\ \hline \end{array} \qquad (1)$$

for all values of the parameters $\theta_i, i = 1..N$

PARETO EFFICIENT POINTS

$$X_i = A_i\sqrt{L}, i = 1..N$$
$$\sum_{i=1}^{N} A_i = 2\sqrt{L}$$
$$X_0 = N - 3L$$
$$0 \le L \le \frac{N}{3}, A_i \ge 0, i = 1..N \qquad (2)$$

2.COMPETITIVE EQUILIBRIA WITH PROFIT TAXATION

1. NAME THE PRICE OF EACH GOOD

$p = \text{price of } A, w = \text{price of } x$

2. NORMALIZE PRICES (OPTIONAL)

$w = 1$

3. SOLVE THE OPTIMIZATION PROBLEMS OF FIRMS

Post-tax profit $F = (1-t)\Pi = (1-t)(pA - wL) = (1-t)(2p\sqrt{L} - L)$ is maximized at

$$\begin{bmatrix} A \\ L \\ \Pi \end{bmatrix} = \begin{bmatrix} 2p \\ p^2 \\ p^2 \end{bmatrix} \qquad (3)$$

4. SOLVE THE OPTIMIZATION PROBLEMS OF CONSUMERS

$$\max\ U_0 = X_0, \text{subject to } X_0 \le (1-t)\Pi$$

$$\max\ U_i = \log X_i + \log A_i, \text{subject to } X_i + pA_i \le 1 + t\frac{\Pi}{N}, i = 1..N$$

The solutions are

$$\begin{bmatrix} X_0 \\ A_0 \end{bmatrix} = \begin{bmatrix} (1-t)\Pi \\ 0 \end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} X_i \\ A_i \end{bmatrix} = \begin{bmatrix} M/2 \\ M/2p \end{bmatrix}, M = 1 + t\frac{\Pi}{N} \quad (5)$$

5. SOLVE THE EQUILIBRIUM CONDITIONS

$$\sum_{i=0}^{N} X_i + L = N, \sum_{i=0}^{N} A_i = A \quad (6)$$

There is a unique solution, described by

EQUILIBRIUM WITH PROFIT TAXATION

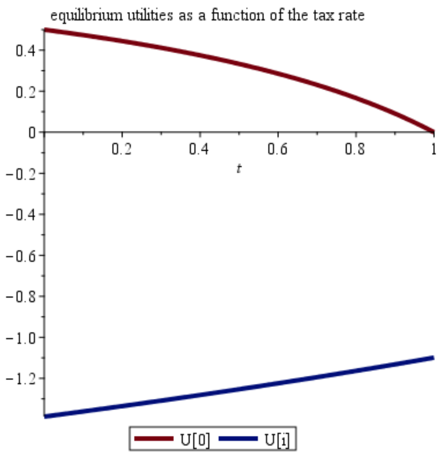
$$p = \sqrt{\frac{N}{4-t}}, M = \frac{4}{4-t}$$

$$L = \Pi = \frac{N}{4-t}, A = 2\sqrt{\frac{N}{4-t}}$$

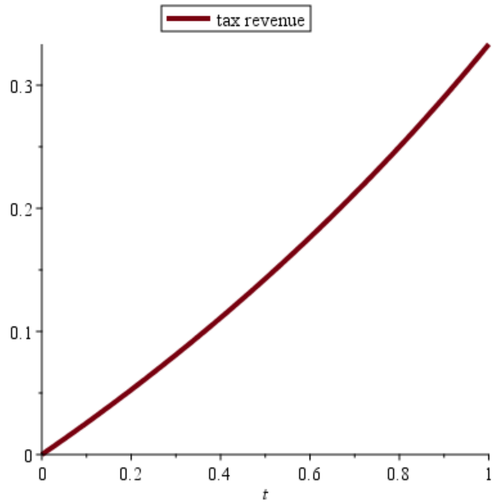
$$X_i = \frac{2}{4-t}, A_i = \frac{2}{\sqrt{N(4-t)}}$$

$$U_i^E = 2\log 2 - \frac{1}{2}\log N - \frac{3}{2}\log(4-t)$$

$$U_0^E = X_0 = \frac{N(1-t)}{4-t}, A_0 = 0$$



(7)



3.EFFICIENCY OF PROFIT TAXATION

Comparing (7) with (2) we conclude that profit taxation is efficient

Question 2

THE ECONOMY

- Consumers A, B
- Goods 1, 2.
- Preferences

$$u_A = 2\sqrt{A_1 A_2}, u_B = 2\sqrt{B_1 B_2}$$

- Endowments $e_A = [1, 0], e_B = [0, 1]$
- Consumers pay a tax $-1 < t$ for each unit of good 1 they buy. Tax revenue R is distributed to consumers with lump-sum transfers $T_A = \alpha R, T_B = (1-\alpha) R, 0 \le \alpha \le 1$.

1. Compute competitive equilibria as a function of the policy parameters t, α

2. Plot the equilibrium values of all variables as a function of the tax rate

ANSWERS

NAME THE PRICE OF EACH GOOD

$p_i = \text{price of good i}$. *Normalize* $p_1 = 1$. *Let* $p = p_2$

2. DEFINE CONSUMER INCOMES

$$M_A = 1 + T_A, M_B = p + T_B \tag{8}$$

3. SOLVE THE OPTIMIZATION PROBLEMS OF CONSUMERS

$$\max U_A = \log A_1 + \log A_2, \text{subject to } A_1 + pA_2 \le M_A$$
$$\max U_B = \log B_1 + \log B_2, \text{subject to } (1+t)B_1 + pB_2 \le M_B$$

The solutions are

$$(A_1, A_2) = \left(\frac{M_A}{2}, \frac{M_A}{2p} \right) \tag{9}$$

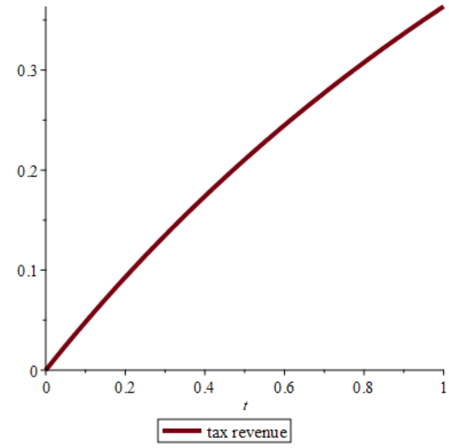
$$(B_1, B_2) = \left(\frac{M_B}{2(1+t)}, \frac{M_B}{2p} \right) \tag{10}$$

5. SOLVE THE EQUILIBRIUM CONDITIONS

$$1 = A_1 + B_1, 1 = A_2 + B_2, T_A + T_B = tB_1 \tag{11}$$

There is a unique solution, described by

$$\boxed{\begin{array}{l} \text{competitive equilibria with unit taxation} \\ p = \frac{\alpha t + t + 2}{\alpha t + 2}, T_A = \frac{\alpha t}{\alpha t + 2}, T_B = \frac{t(1-\alpha)}{\alpha t + 2} \\ A = \left[\frac{\alpha t + 1}{\alpha t + 2}, \frac{\alpha t + 1}{\alpha t + t + 2} \right] \\ B = \left[\frac{1}{\alpha t + 2}, \frac{1+t}{\alpha t + t + 2} \right] \\ R = \frac{t}{\alpha t + 2} \end{array}} \tag{12}$$


0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
t
tax revenue