micro problem set 1

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Please provide only the final answers to the following problems.No proofs needed.Try to give an answer even if you are not yet sure that it is correct.

Problem 1 *draw the indifference curves* $I_c^f = \{(x, y) \in R_+^2 : f(x, y) = c\}$ *and the upper contour sets(better-than sets)* $B_c^f = \{(x, y) \in R_+^2 : f(x, y) >= c\}$ *of the following functions* **.** *Which ones of these functions are concave and/or quasi-concave?In which cases are indifference curves representable by functions?*

- $f(x, y) = x + \sqrt{y}, c = 4$
- $f(x, y) = x + y^2, c = 4$
- $f(x, y) = x - \frac{1}{y}, c = 4$
- $f(x, y) = \min(x, y), c = 1$
- $f(x, y) = \max(x, y), c = 1$
- $f(x, y) = \min(x/4 + 1, y + 2), c = 3$
- $f(x, y) = \max(2x/3, 3y/2), c = 1$
- $f(x, y) = \min(x/4 + y - 1, x + y - 2, x, y), c = 1/2, 2, 4$
- $f(x, y) = \min(x, y, \frac{x^2+y^2}{8}), c = 1$
- $f(x, y) = \min(\max(x, y), \max(2x/3, 3y/2)), c = 1$
- $f(x, y) = -(x - 3)^2 - (y - 3)^2, c = -4$

Problem 2 *solve the following maximization problems in one variable*

1. - objective function: $f(x) = px - wx$
   - variables: $x \in R$
   - constraints: $x \geq 0$
   - parameters: p, w
   - conditions on parameters:all parameters are strictly positive

- objective function: $f(x) = 2p\sqrt{x} - wx$
- variables: $x \in R$
- constraints: $x \geq 0$
- parameters: p, w
- conditions on parameters:all parameters are strictly positive

- objective function: $f(x) = \frac{p}{2}x^2 - wx$
- variables: x
- constraints: $x \geq 0$
- parameters: p, w
- conditions on parameters:all parameters are strictly positive

- objective function: $f(x) = -(x-3)^2$
- variables: x
- constraints: $px \leq m, x \geq 0$
- parameters: p, m
- conditions on parameters:all parameters are strictly positive

Problem 3 *solve the following maximization problems in two variables*

1. - objective function: $f(K, L) = \frac{p}{4}\sqrt{KL} - wL - rK$
   - variables: K, L
   - constraints: $K \geq 0, L \geq 0$
   - parameters: p, w, r
   - conditions on parameters:all parameters are strictly positive

   - objective function: $f(K, L) = 4pK^{\frac{1}{4}}L^{\frac{1}{4}} - wL - rK$
   - variables: K, L
   - constraints: $K \geq 0, L \geq 0$
   - parameters: p, w, r
   - conditions on parameters:all parameters are strictly positive

   - objective function: $f(K, L) = pKL - wL - rK$
   - variables: K, L
   - constraints: $K \geq 0, L \geq 0$
   - parameters: p, w, r
   - conditions on parameters:all parameters are strictly positive

   - objective function: $f(x, y) = x + 2\sqrt{y}$

- variables: x, y
- constraints: $px + wy \leq m, x \geq 0, y \geq 0$
- parameters: p, w, m
- conditions on parameters:all parameters are strictly positive

- objective function: $f(x, y) = x - \frac{1}{y}$
- variables: x, y
- constraints: $px + wy \leq m, x \geq 0, y \geq 0$
- parameters: p, w, m
- conditions on parameters:all parameters are strictly positive

- objective function: $f(x, y) = x^2 + y^2$
- variables: x, y
- constraints: $px + wy \leq m, x \geq 0, y \geq 0$
- parameters: p, w,m
- conditions on parameters:all parameters are strictly positive

- objective function: $f(x, y) = \min(2x + y - 3, x + y + 1)$
- variables: x, y
- constraints: $x + 2y \leq m, x \geq 0, y \geq 0$
- parameters: m
- conditions on parameters:all parameters are strictly positive

Problem 4 *solve the following maximization problems in n variables*

1. - objective function: $f(x) = \sum_{i=1}^{n} \alpha_i \log x_i$
   - variables: $x_1, .., x_n$
   - constraints: $\sum_{i=1}^{n} p_i x_i \leq m, x_1 \geq 0, .., x_n \geq 0$
   - parameters: $\alpha_1, .., \alpha_n, p_1, .., p_n, m$
   - conditions on parameters:all parameters are strictly positive

   - objective function: $f(x) = \prod_{i=1}^{n} x_i - \sum_{i=1}^{n} w_i x_i$
   - variables: $x_1, .., x_n$
   - constraints: $x_1 \geq 0, .., x_n \geq 0$
   - parameters: $w_1, .., w_n$
   - conditions on parameters:all parameters are strictly positive

   - objective function: $f(x) = x_1 + \sum_{i=2}^{n} \alpha_i \log x_i$
   - variables: $x_1, .., x_n$
   - constraints: $\sum_{i=1}^{n} p_i x_i \leq m, x_1 \geq 0, .., x_n \geq 0$
   - parameters: $\alpha_1, .., \alpha_n, p_1, .., p_n, m$
   - conditions on parameters:all parameters are strictly positive