Consider an economy consisting of

- Consumers 1,2
- Goods A, X
- One firm, with production function $A = X$

Consumer 1

- Endowment $e_1 = \overset{A,X}{[0,1]}, \theta_1 = 0$
- preferences $U_1 = X_1 + 2\sqrt{A_1}$

Consumer 2

- Endowment $e_2 = \overset{A,X}{[0,0]}, \theta_2 = 1$
- preferences $U_2 = A_2$

1. Compute Pareto points.

2. Compute competitive equilibria when sellers of good X are taxed on the value of their sales at a rate $0 \le t < 1$, and any tax revenue is transferred to consumer 2 with a lump-sum subsidy.

3. Plot the equilibrium values of all variables as a function of the tax rate

ANSWERS

1.PARETO EFFICIENT POINTS

are the solutions of the following maximization problem for all parameter values $\zeta \in R$

$$\begin{aligned}
&\max\ U_1 = X_1 + 2\sqrt{A_1} \\
&\text{subject to } U_2 = A_2 \ge \zeta \\
&X + X_1 \le 1 \\
&A_1 + A_2 \le A = X \\
&A_1, A_2, A, X, X_1 \ge 0
\end{aligned} \qquad (1)$$

$$\boxed{\begin{array}{l}\text{pareto efficient points}\\ X_1=0, X=1, A_1+A_2=1, 0\le A_1\le 1\end{array}} \quad (2)$$

2.EQUILIBRIA WITH SALES TAX

1.NAME THE PRICE OF EACH GOOD

$p=$ price of A, w=price of X. *Normalize* $w=1$.

2. SOLVE THE OPTIMIZATION PROBLEMS OF CONSUMERS

$$\max U_1 = X_1 + 2\sqrt{A_1}, \text{subject to } pA_1 \le (1-t)(1-X_1), X_1\ge 0, A_1 \ge 0$$
$$\max U_2 = A_2, \text{subject to } pA_2 \le \Pi + R, A_2 \ge 0$$

The solutions are

$$(A_1, X_1) = \begin{cases} \left(\left(\frac{1-t}{p}\right)^2, 1-\frac{1-t}{p}\right) & \text{if} \quad p \ge 1-t \\ \\ \left(\frac{1-t}{p}, 0\right) & \text{if} \quad p \le 1-t \end{cases} \quad (3)$$

$$(A_2, X_2) = \left(\frac{\Pi + R}{p}, 0\right) \quad (4)$$

3. SOLVE THE OPTIMIZATION PROBLEMS OF FIRMS

$$\max \Pi = pA - wX = (p-1)A \quad (5)$$

$$A = X = \begin{cases} \infty & \text{if} \quad p > 1 \\ \ge 0 & \text{if} \quad p = 1 \\ 0 & \text{if} \quad p < 1 \end{cases} \quad (6)$$

5. SOLVE THE EQUILIBRIUM CONDITIONS

$$1 = X + X_1, A = X = A_1 + A_2, R = t(1 - X_1) \quad (7)$$

There is a unique solution, described by

$$\begin{array}{|l|}\hline \text{competitive equilibria with SALES TAX} \\ \hline p=1, R=t(1-t), \Pi=0 \\ (A_1, X_1) = \left[(1-t)^2, t\right] \\ (A_2, X_2) = \left[t(1-t), 0\right] \\ A = X = 1-t \\ \hline \end{array} \quad (8)$$