1.THE ECONOMY

1.consumers:1,2

2.goods:A,L

3.Preferences/endowments

$$U_1 = A_1 - \frac{1}{2}L^2{}_1, e_1 = (0,1)$$
$$U_2 = A_2, e_2 = (0,0)$$

Player 2 receives all profit income.

4.firms:1

5.producion function

$$A = \begin{cases} 0 & \text{if} \quad L \le f \\ L - f & \text{if} \quad L \le f \end{cases} \qquad 0 < f < \frac{1}{2} \tag{1}$$

2. EFFICIENT POINTS

Pareto efficient points are the solutions of

$$\max U_1 = A_1 - \frac{1}{2}L^2{}_1, \text{subject to}$$
$$U_2 = A_2 \ge u, A_1 + A_2 \le A, L \le L_1, L_1 \le 1$$
$$A = \max(L - f, 0), A_1 \ge 0, A_2 \ge 0, L_1 \ge 0, L \ge 0$$

efficient points

$$L = L_1 = 1, A_1 + A_2 = 1 - f, U_1 + U_2 = \frac{1}{2} - f, 0 \le U_2 \le 1 - f \tag{2}$$

3.MONOPOLY EQUILIBRIUM

1.prices

p for A,w for L.

2.incomes

$$M_1 = wL_1, M_2 = \Pi \tag{3}$$

3.consumer optimization problems

$$\begin{aligned} &\max\ U_1 = A_1 - \frac{1}{2} L^2{}_1 \\ &subject\ to\ \ pA_1 \le wL_1 0 \le L_1 \le 1, A_1 \ge 0 \end{aligned} \tag{4}$$

$$\begin{aligned} &\max\ U_2 = A_2 \\ &subject\ to\ \ pA_2 \le \Pi, A_2 \ge 0 \end{aligned} \tag{5}$$

$$(A_1, L_1) = \begin{cases} (\left(\frac{w}{p}\right)^2, \frac{w}{p}) & \text{if} \quad \frac{w}{p} \le 1 \\ (\frac{w}{p}, 1) & \text{if} \quad \frac{w}{p} \ge 1 \end{cases} \tag{6}$$

$$A_2 = \frac{\Pi}{p} \tag{7}$$

4.equilibrium conditions

$$\boxed{\begin{array}{ccc} \text{demand} & = & \text{supply} \\ A_1 + A_2 & = & A \\ L & = & L_1 \end{array}} \tag{8}$$

5.normalization rule

$$p = 1 \tag{9}$$

6.firm optimization problem.

The firm chooses the allocation of resources $\left(A_1, A_2, L, L_1\right)$ and the price w so as to maximize profits $\Pi = pA - wL$ subject to the equilibrium constraints (9),(8),(7),(6)

Eliminating redundant variables and constraints we end up with

$$\underbrace{\max}_{w}\ \Pi = \begin{cases} -w^2 & \text{if} \quad w \le f \\ (1-w)w - f & \text{if} \quad f \le w \le 1 \\ 1 - w - f & \text{if} \quad w \ge 1 \end{cases} \tag{10}$$

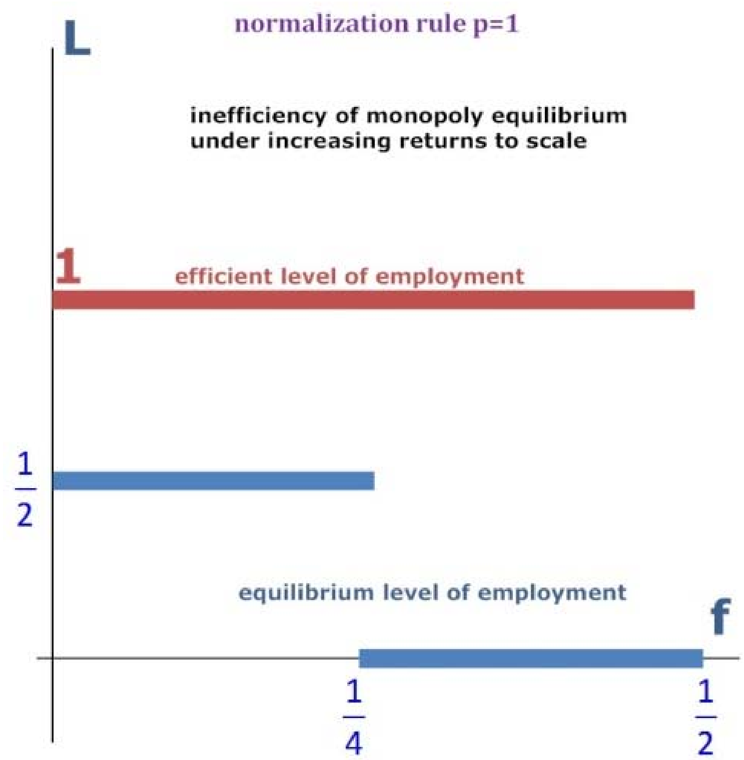
The solution is

$$\left(w, \Pi\right) = \begin{cases} \left(0,0\right) & \text{if} \quad f \ge \frac{1}{4} \\ \left(\frac{1}{2}, \frac{1}{4} - f\right) & \text{if} \quad f < \frac{1}{4} \end{cases} \tag{11}$$

The corresponding monopoly equilibria are

monopoly equilibria with normalization rule $p = 1$

$$\left(L, A, L_1, A_1, A_2, w, \Pi\right) = \begin{cases} (0,0,0,0,0,0,0) & \text{if} \quad f \geq \frac{1}{4} \\ \left(\frac{1}{2}, \frac{1}{2} - f, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} - f, \frac{1}{2}, \frac{1}{4} - f\right) & \text{if} \quad f < \frac{1}{4} \end{cases} \tag{12}$$



4. THE NORMALIZATION RULE AFFECTS EXISTENCE OF EQUILIBRIA

Steps 1-4 remain the same. We only examine the case f=0.

5.normalization rule

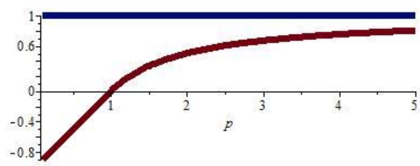
$$w = 1 \tag{13}$$

6.firm optimization problem.

The firm chooses the allocation of resources $\left(A_1, A_2, L, L_1\right)$ and the price p so as to maximize profits $\Pi = pA - wL$ subject to the equilibrium constraints (13),(8),(7),(6)
Eliminating redundant variables and constraints we end up with

$$\underbrace{\max}_{p} \ \Pi = \begin{cases} \dfrac{p-1}{p} & \text{if} \quad p \geq 1 \\ p-1 & \text{if} \quad p \leq 1 \end{cases} \tag{14}$$

No solution exists



5.MONOPOLY EQUILIBRIA DEPEND ON THE NORMALIZATION RULE

Steps 1-4 remain the same. We only examine the case f=0.

5.normalization rule

$$p^{\frac{1}{a}} w^{1-\frac{1}{a}} = 1 \tag{15}$$

where a is a positive parameter.

6.firm optimization problem.

The firm chooses the allocation of resources $\left(A_1, A_2, L, L_1\right)$ and the prices w, p so as to maximize profits $\Pi = pA - wL$ subject to the equilibrium constraints (15),(8),(7),(6)
Eliminating redundant variables and constraints we end up with

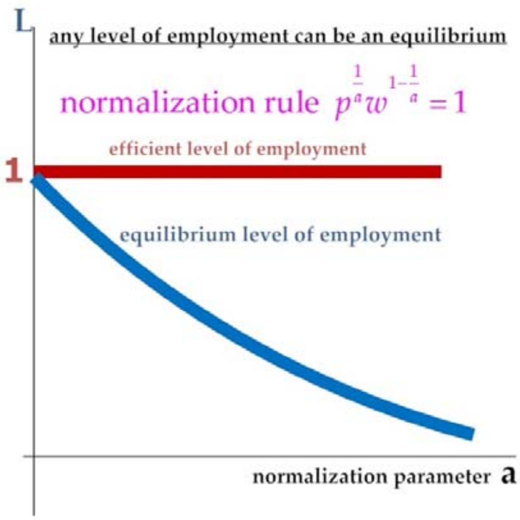
$$\underbrace{\max}_{w} \ \Pi = \begin{cases} w - w^{1+a} & if \quad w \leq 1 \\ w^{1-a} - w & if \quad w \geq 1 \end{cases} \tag{16}$$

The solution is

$$\left(w, \Pi\right) = \left(\frac{1}{\left(a+1\right)^{\frac{1}{a}}}, \left(a+1\right)^{-\frac{1}{a}} - \left(a+1\right)^{-\frac{a+1}{a}} \right) \tag{17}$$

The corresponding monopoly equilibria are

$$
\boxed{
\begin{array}{l}
\text{monopoly equilibria with normalization rule } p^{\frac{1}{a}}w^{1-\frac{1}{a}}=1 \\
w=\dfrac{1}{(a+1)^{\frac{1}{a}}},\ p=(a+1)^{1-\frac{1}{a}},\ \dfrac{w}{p}=\dfrac{1}{1+a} \\
\Pi=(a+1)^{-\frac{1}{a}}-(a+1)^{-\frac{a+1}{a}},\ A_2=\dfrac{a}{(a+1)^2} \\
L_1=L=A=\dfrac{1}{1+a},\ A_1=\dfrac{1}{(1+a)^2}
\end{array}
}
\tag{18}
$$



6.MARGINAL COST-PRICING EQUILIBRIA

We examine the case $f \geq 0$,with the normalization rule p=1.

The authorities dictate to the monopolist relative prices so that the first-order necessary conditions for profit maximization are satisfied at the chosen pareto optimal point. Any losses incurred by the monopolist are exactly covered by a lump-sum subsidy T,paid for by lump-sum taxes T_1, T_2 on consumers.

$$
\begin{gathered}
\Pi=(1-w)L-f+T \\
\left.\frac{d\Pi}{dL}\right|_{L=1}=0
\end{gathered}
\tag{19}
$$

Rule (19) implies that

$$
w=1, T=f, T_1+T_2+T=0 \tag{20}
$$

The resulting equilibrium is then computed as before

1.prices

p for A,w for L.

2.incomes

$$M_1 = wL_1 + T_1, M_2 = \Pi + T_2 \tag{21}$$

3.consumer optimization problems

$$\begin{aligned}&\max\ U_1 = A_1 - \frac{1}{2}L^2{}_1 \\ &subject\ to\ \ pA_1 \le wL_1 + T_1, 0 \le L_1 \le 1, A_1 \ge 0\end{aligned} \tag{22}$$

$$\begin{aligned}&\max\ U_2 = A_2 \\ &subject\ to\ \ pA_2 \le \Pi + T_2, A_2 \ge 0\end{aligned} \tag{23}$$

$$(A_1, L_1) = \begin{cases} (\left(\frac{w}{p}\right)^2 + \frac{T_1}{p}, \frac{w}{p}) & \text{if} \quad \frac{w}{p} \le 1 \\ (\frac{w}{p} + \frac{T_1}{p}, 1) & \text{if} \quad \frac{w}{p} \ge 1 \end{cases} \tag{24}$$

$$A_2 = \frac{\Pi + T_2}{p} \tag{25}$$

4.firm optimization problem.

The firm chooses its input-output vector (A, L) so as to maximize profits $\Pi = pA - wL + T$

subject to (1),(20).The solution is

$$L \ge 0, A = \max(L - f, 0), \Pi = 0 \tag{26}$$

5.equilibrium conditions

$$\boxed{\begin{array}{ccc} \text{demand} & = & \text{supply} \\ A_1 + A_2 & = & A \\ L & = & L_1 \end{array}} \tag{27}$$

The resulting equilibrium allocation is then, by (27),(26),(25),(24),(20),(9)

$$\boxed{\begin{array}{l} \text{marginal cost pricing equilibrium with normalization rule p=1} \\ w = 1, T = f, \Pi = 0, T_2 + T_1 = -f, 0 \le T_2 \le 1 - f \\ L = L_1 = 1, A_1 = 1 - f - T_2, A_2 = T_2 \end{array}} \tag{28}$$

7.TWO-PART TARIFFS

The monopolist charges consumers a price p per unit of consumption, and a tariff T that is independent of the level of consumption, and has to be paid if and only if consumption is positive. We only examine the case where $f \geq 0, p = 1$,and the tariff is paid by consumer 2.

1.prices

p for A,w for L.

2.incomes

$$M_1 = wL_1 - T, M_2 = \Pi \tag{29}$$

3.consumer optimization problems

$$\begin{aligned} &\max\ U_1 = A_1 - \frac{1}{2}L^2{}_1 \\ &subject\ to\ \ pA_1 \leq wL_1 - T \\ &A_1 = 0 \to T = 0 \\ &0 \leq L_1 \leq 1, A_1 \geq 0 \end{aligned} \tag{30}$$

$$\begin{aligned} &\max\ U_2 = A_2 \\ &subject\ to\ \ pA_2 \leq \Pi \\ &A_2 \geq 0 \end{aligned} \tag{31}$$

$$(A_1, L_1) = \begin{cases} (w^2 - T, w) & \text{if} \quad w \leq 1, T \leq \dfrac{w^2}{2} \\ (w - T, 1) & \text{if} \quad w \geq 1, T \leq w - \dfrac{1}{2} \\ (0,0) & \text{otherwise} \end{cases} \tag{32}$$

$$A_2 = \Pi \tag{33}$$

4.equilibrium conditions

$$\boxed{\begin{array}{ccc} \text{demand} & = & \text{supply} \\ A_1 + A_2 & = & A \\ L & = & L_1 \end{array}} \tag{34}$$

5.firm optimization problem.

The firm chooses the allocation of resources $\left(A_1, A_2, L, L_1\right)$ and the prices $\left(w, T\right)$ so as to maximize profits $\Pi = pA - wL + T$ subject to the equilibrium constraints (9),(34),(33),(32) Eliminating redundant variables and constraints we end up with

$$T = \begin{cases} \dfrac{w^2}{2} & \text{if} \quad w \le 1 \\ w - \dfrac{1}{2} & \text{if} \quad w \ge 1 \end{cases} \tag{35}$$

$$\underbrace{\max}_{w} \ \Pi = \begin{cases} -w^2 & \text{if} \quad w \le f \\ (1-w)w - f + \dfrac{w^2}{2} & \text{if} \quad f \le w \le 1 \\ 1 - w - f + w - \dfrac{1}{2} & \text{if} \quad w \ge 1 \end{cases} \tag{36}$$

The solution is

$$\left(w, \Pi, T\right) = \left(1, \frac{1}{2} - f, \frac{1}{2}\right) \tag{37}$$

The resulting equilibrium allocation is then

two-part tariff equilibrium with normalization rule p=1

$$w = 1, T = \frac{1}{2}, \Pi = \frac{1}{2} - f$$
$$L = L_1 = 1, A_1 = \frac{1}{2}, A_2 = \frac{1}{2} - f \tag{38}$$