Auctioneer game

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Consumers: A, B

Commodities: $1, .., N$

Consumption sets: $X_A = X_B = R^L_+$

Endowments: $\bar{A} = (\bar{A}_1, .., \bar{A}_L)$; $\bar{B} = (\bar{B}_1, .., \bar{B}_L)$

Consumption vectors (notation)

$A = (A_1, .., A_L)$, $B = (B_1, .., B_L)$

Utility functions $u_A(A, B)$, $u_B(A, B)$, assumed to be strictly increasing.

A competitive equilibrium is

- A consumption vector A^* for consumer A
- A consumption vector B^* for consumer B
- A price vector $p^* \in R^L$ such that

(MAX_A): A^* solves the maximization problem

$$\max u_A(A, B^*), \text{ subject to}$$
$$p^* A \leq p^* \bar{A}, \quad A \geq 0$$

(MAX_B) B^* solves the maximization problem

$$\max u_B(A^*, B), \text{ subject to}$$
$$p^* B \leq p^* \bar{B}, \quad B \geq 0$$

(EQ) $$A^* + B^* = \bar{A} + \bar{B}$$

The auctioneer game associated with the economy $\langle u_A, u_B, \bar{A}, \bar{B} \rangle$ is defined by

- Players: A, B, W

- Strategy spaces

$$S_W = \{ p \geq 0 : \; p_1 + \ldots + p_L = 1 \}$$

$$S_A(p) = \{ A \geq 0 : \; pA \leq p\bar{A} \}$$

$$S_B(p) = \{ B \geq 0 : \; pB \leq p\bar{B} \}$$

- Payoff functions

$$V_A : S_A(p) \times S_B(p) \times S_W \longrightarrow R$$
$$V_A(A, B, p) = u_A(A, B)$$

$$V_B : S_A(p) \times S_B(p) \times S_W \longrightarrow R$$
$$V_B(A, B, p) = u_B(A, B)$$

$$V_W : S_A(p) \times S_B(p) \times S_W \longrightarrow R$$
$$V_W(A, B, p) = p(A + B - \bar{A} - \bar{B})$$

A NASH EQUILIBRIUM of the auctioneer game is

- a consumption vector $\hat{A}$ for consumer A
- a consumption vector $\hat{B}$ for consumer B
- a price vector $\hat{p}$ for player W

, such that

MAX_A : $\hat{A}$ solves the maximization problem

$$\max u_A(A, \hat{B}), \text{ subject to}$$
$$A \in S_A(\hat{p})$$

MAX_B: $\hat{B}$ solves the maximization problem

$$\max u_B(\hat{A}, B), \text{ subject to } B \in S_B(\hat{p})$$

MAX_W: $\hat{p}$ solves the maximization problem

$$\max p(\hat{A} + \hat{B} - \bar{A} - \bar{B}), \text{ subject to } p \in S_W$$

Lemma: If $(\hat{A}, \hat{B}, \hat{p})$ is a Nash equilibrium of the auctioneer game then

(1) $\hat{p} \gg 0$

(2) $\hat{p}\hat{A} = \hat{p}\bar{A}$

(3) $\hat{p}\hat{B} = \hat{p}\bar{B}$

Proof: By monotonicity of u_A, u_B

Theorem: If $(\hat{A}, \hat{B}, \hat{p})$ is a Nash equilibrium of the auctioneer game associated with the economy $\langle u_A, u_B, \bar{A}, \bar{B} \rangle$, then it is also a competitive equilibrium of this economy.

Proof: We only need to show that $\hat{A} + \hat{B} = \bar{A} + \bar{B}$.

By the lemma

$$\hat{p}\,(\hat{A}+\hat{B}-\bar{A}-\bar{B})=0 \qquad (4)$$

By MAXw if $\hat{A}_i+\hat{B}_i-\bar{A}_i-\bar{B}_i < 0$ then $\hat{p}_i=0$, contradicting (1). Hence

$$\hat{A}+\hat{B}-\bar{A}-\bar{B}\geqslant 0 \qquad (5)$$

By (1), (4), (5)

$$\hat{A}+\hat{B}=\bar{A}+\bar{B}$$