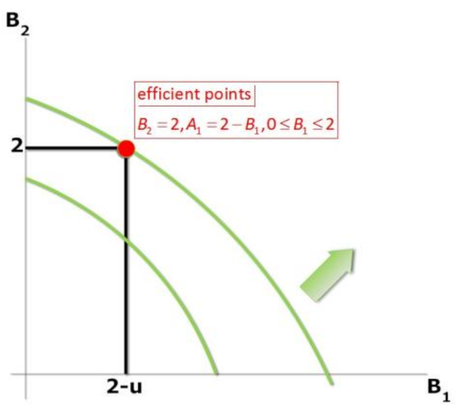
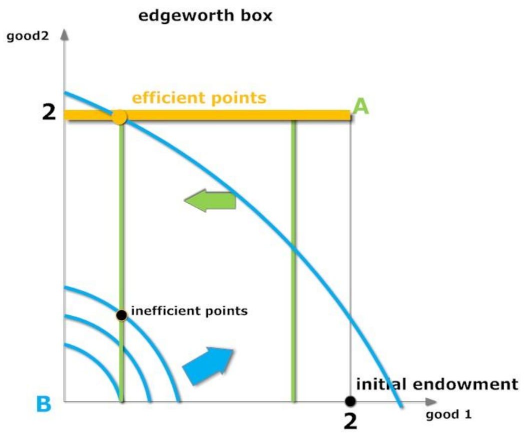
NONCONVEXITIES AND THE SECOND WELFARE THEOREM

- Consumers A,B
- Goods 1,2.
- Preferences $u_A = A_1, u_B = B_1^2 + B_2^2$
- Endowments $e_A = [0,2], e_B = [2,0]$

EFFICIENT POINTS

$$\max u_B = B_1^2 + B_2^2$$
$$\text{subject to } A_1 \geq u, A_1 + B_1 \leq 2, B_2 \leq 2, A_1 \geq 0, B_1 \geq 0, B_2 \geq 0$$





$$\boxed{\begin{array}{l} \text{pareto efficient points} \\ B_2 = 2, A_1 = 2 - B_1, 0 \leq B_1 \leq 2 \\ \text{pareto frontier} \\ u_B = 4 + (2 - u_A)^2, 0 \leq u_A \leq 2 \end{array}} \quad (1)$$

COMPETITIVE EQUILIBRIUM WITH LUMP SUM TRANSFERS

1. NAME THE PRICE OF EACH GOOD

$p_i = \text{price of good i}$

2.DEFINE CONSUMER INCOMES

$$M_A = 2p_2 + T_A, M_B = 2p_1 - T_A \quad (2)$$

2. SOLVE THE OPTIMIZATION PROBLEMS OF CONSUMERS

$\max U_A = A_1$, subject to $p_1A_1 + p_2A_2 \le M_A$
$\max U_B = B_1^2 + B_2^2$, subject to $p_1B_1 + p_2B_2 \le M_B$

The solutions are

$$(A_1, A_2) = \left(\frac{M_A}{p_1}, 0\right) \tag{3}$$

$$(B_1, B_2) = \begin{cases} (\frac{M_B}{p_1}, 0) & \text{if} \quad p_1 < p_2 \\ \left\{\left(\frac{M_B}{p_1}, 0\right), (0, \frac{M_B}{p_2})\right\} & \text{if} \quad p_1 = p_2 \\ (0, \frac{M_B}{p_2}) & \text{if} \quad p_1 > p_2 \end{cases} \tag{4}$$

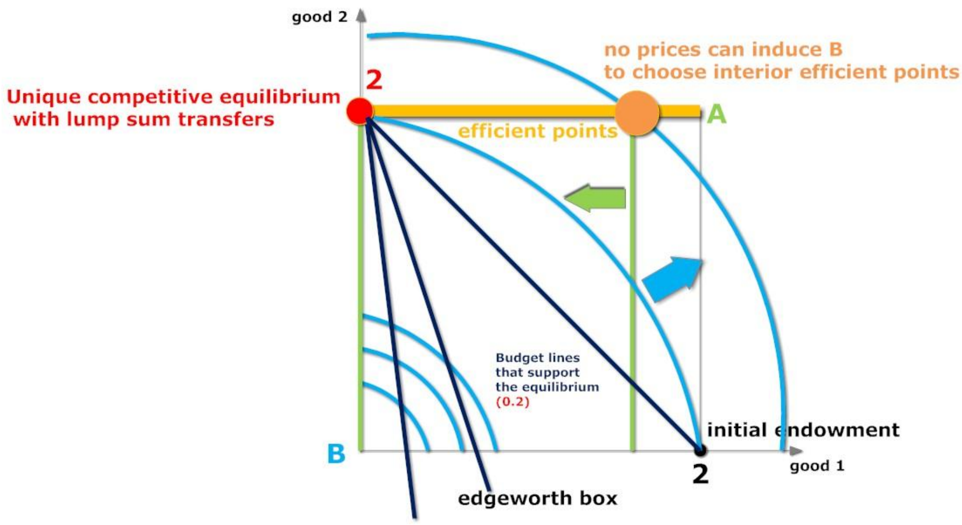
5. SOLVE THE EQUILIBRIUM CONDITIONS

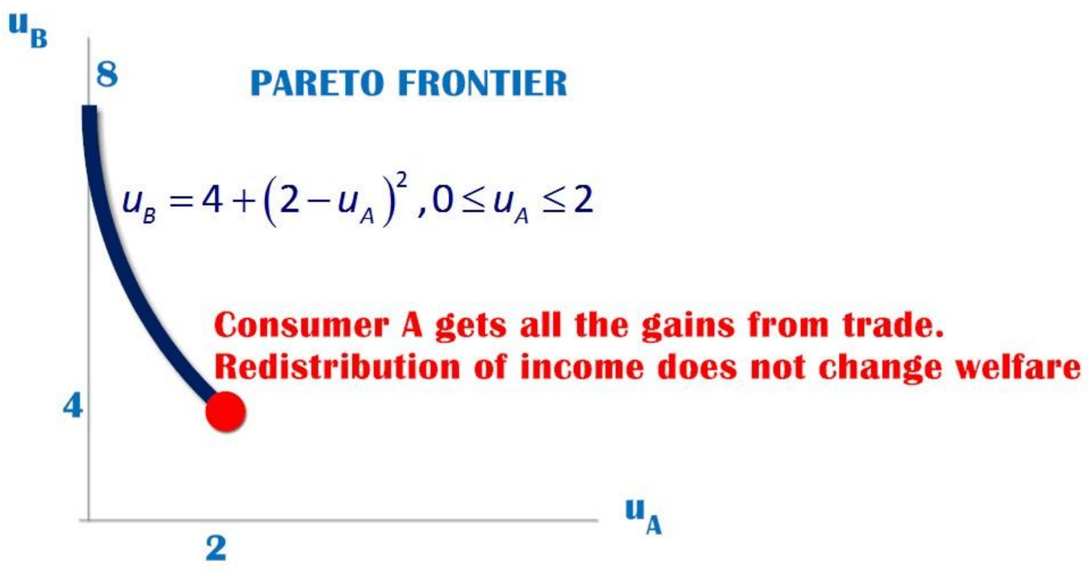
$$2 = A_1 + B_1, 2 = B_2 \tag{5}$$

There is a unique solution, described by

$$\boxed{\begin{array}{l} \text{competitive equilibria with lump-sum transfers} \\ \frac{p_2}{p_1} = 1 - \frac{T_A}{2p_1}, 0 \le \frac{T_A}{2p_1} < 1 \\ B_2 = 2, B_1 = 0, A_1 = 2 \end{array}} \tag{6}$$

Failure of the second welfare theorem due to non convexities.No efficient point other than (0,2) can be obtained as a competitive equilibrium with lump-sum transfers




u_B
8
PARETO FRONTIER
$u_B = 4 + (2 - u_A)^2 , 0 \le u_A \le 2$
Consumer A gets all the gains from trade.
Redistribution of income does not change welfare
4
u_A
2