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An Analysis of a Strategic Decision in the Sport of Curling

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We apply decision analysis to an important decision in the sport of curling. In particular, we examine the choice between taking a single point or blanking an end in the latter stages of a curling game. There are benefits and drawbacks associated with each alternative. Taking a single point provides the team with an additional point but transfers the last-shot advantage to the opposition. Blanking an end foregoes an additional point but retains the last-shot advantage. Based on the observation of world-class competitions, North American curlers will always attempt to blank an end, while their European counterparts have been known to opt for the single point. We develop a decision tree to conceptualize the choices. Then, we use data from over 900 national championship curling games to empirically determine the expected values of each alternative. Our results indicate that blanking the end is the better alternative.

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Introduction

Sports offer scenarios suitable for the application of decision analysis. Feinstein (1990) developed a decision tree for a proposed drug-testing program of student athletes at Santa Clara University. His model prompted subsequent deliberations that led to a unanimous decision not to implement the program. Hurley (1998) assessed the timing and sequencing of decisions made during crucial junctures in football. Porter (1967) and Sackrowitz (2000) discussed extra point strategy in American football, determining when a more risky two-point conversion ought to be attempted.

In a previous paper, Willoughby and Kostuk (2004) discussed the evaluation of two different scenarios in the final end of a curling game. Specifically, these scenarios involved a team either ahead by one point without the last-shot advantage, or trailing by one point with the benefit of having the final shot. Within the curling community, significant debate exists regarding the preferred scenario. Using empirical data from national championship games, we determined the probability of victory under either scenario and discovered that, contrary to the opinions of many curlers, the better scenario is to be ahead by one point without the last-shot advantage as the 10th end begins. To incorporate expert judgment within our analysis, we conducted a survey of over 100 world-class curlers, including several former Canadian or world champions.

In this paper, we explore a very specific *decision* faced by curling teams. The Willoughby and Kostuk (2004) paper examined two disparate scenarios that a team rarely faces. This paper explores whether a team, playing its final rock in the 9th end should opt to blank the end (thereby retaining the last-shot advantage for the next end) or take one point (thus giving the last-shot advantage to its opponent). We will thoroughly explore this interesting decision over a range of possible point differentials.

We shall begin by providing a brief overview of the sport (a more complete illustration is provided in Willoughby and Kostuk 2004). This is followed by a description of our decision tree model. We then use data from 13 years of national championship games to empirically determine the best alternatives.

The Sport of Curling: An Overview

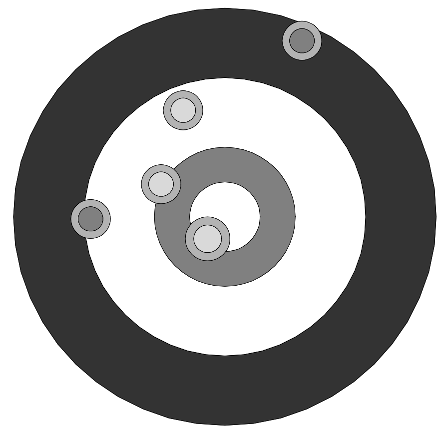
Curling is a winter team sport with a rich history. The earliest record of this sport is a sixteenth-century painting by Flemish artist, Peter Bruegel. His illustration depicted scenes similar to modern curling. In the seventeenth century, Scotland developed and formalized the modern game. North America's first curling club was founded in Montreal in 1807. In 1832, the Orchard Lake Curling Club (near Detroit) was established, the first such club in the United States.

Curling is a very popular sport in Canada and several European countries (e.g., Norway, Scotland, Sweden, and Switzerland). Participation is growing in the Pacific Rim region, and Japan has begun to field world-class teams. In the United States, there are over 15,000 curlers in 135 active curling clubs. A detailed historical discussion of curling is provided in Lukowich et al. (1990), Murray (1981), Smith (1981), and Weeks (1995).

Curling is played with circular disks of polished granite ("rocks" or "stones") weighing approximately 45 pounds each. The teams (four players a side) take alternate turns sliding these rocks down a sheet of ice. While one player is sliding a rock, two other team members shuffle alongside the rock, using a broom with brush heads to sweep the ice in front of the curling stone. Brushing or sweeping clears debris and helps to reduce friction. It has a two-pronged effect, namely, permitting the rock to travel both further and straighter. The remaining member of the team stands in the "house" (a set of four concentric circles of various diameters) at the opposite end of the curling sheet. He/she indicates a target for the person sliding the rock and instructs the two brushers as to the appropriate amount of brushing.

An "end" is completed when each team has played its eight rocks (respective players on a team will each slide two rocks). Scoring takes place at alternating ends of the ice sheet. Upon the completion of an "end," the score is tallied. A team scores a point for each stone closer to the middle of the house than their opposition's closest stone. For example, suppose Team A (playing the dark-colored stones) has two rocks in the house and Team B (the light-colored stones) has three. If each of Team B's stones are closer to the middle of the house than Team A's, Team B would score three points (Figure 1). A "blank" end (by no means a rare occurrence) happens when there are no rocks in the house after all 16 stones have been played. At most, one team can score in each end. A standard curling game consists of 10 ends. If the game is tied after 10 ends, extra ends are played until one of the teams scores at least one point.

Figure 1 The Team Playing the Lighter-Shaded Rocks Scores Three Points Because Each of Their Rocks Is Closer to the Center of the House Than Both of the Opposition's (Darker-Shaded) Rocks



As one may expect, there is a certain strategic advantage to being able to play the last rock in an end. In the jargon of curling, this last-shot advantage is known as "having the hammer." At the beginning of a game, teams will flip a coin to determine who will have the last rock in the first end. As the game progresses, the team that was scored on in the previous end receives the hammer in the next end. Should a blank end occur, the team that had the last shot retains the hammer for the subsequent end.

Decision Analysis Model

Based on our personal experience in playing the game (one of the authors is a member of the World Curling Tour, an association that organizes a top-notch championship for elite curlers), a crucial decision-making scenario can occur late in a game. During the 9th end of a play, teams may adopt relatively "conservative" strategies in which very few, if any, rocks are situated in the house. Curling teams may choose to "keep things clean" so as to limit the likelihood of a large number of points being scored (by their opponent) at such a late stage in the game.

Consider Team A "having the hammer" in the 9th end and facing a house that is completely empty just prior to their final shot. Team A is faced with two possible alternatives. First, they could attempt to position their final rock in the house (this shot is called a "draw"). For most curling teams (especially those that participate in national and world championships), the likelihood of such a shot coming to rest in the house is nearly guaranteed. The benefit of such a strategy for Team A is they gain one point. The drawback is that because Team A scored in the 9th end, Team B would have the hammer in the 10th and final end. We shall refer to this alternative as TAKE because Team A is taking a single point in the 9th end.

Team A's other alternative is to slide their final rock with sufficient force for it to completely glide through the house. No points are then scored in the 9th end because the house would still be entirely empty after Team A's final shot. By selecting this alternative, Team A foregoes a single point but retains the hammer for the 10th end of play. Because Team A is blanking the 9th end, this alternative is designated as BLANK. As an aside, we note that there is nothing in the rules preventing teams from sliding their rock with such little force that it comes to a halt before reaching the house. However, teams always adopt the former approach. It permits them the opportunity to observe the behavior of a curling stone in a particular path down the ice sheet's entire length.

Essentially, the decision (TAKE or BLANK) becomes one of *points* versus *hammer* (last-shot advantage). Our anecdotal evidence based on participation in several World Curling Tour events and from observations of international competitions suggests that North American elite curlers will always prefer blanking an end; European participants will occasionally opt to take a single point. This difference in choice is the key motivation for this study.

The optimal alternative will have the larger likelihood of eventual victory. We note that the probabilities of victory depend on the current score differential in the 9th end (just prior to the TAKE versus BLANK decision). For example, the likelihood of Team A eventually emerging victorious is far greater if they were currently leading by two points in the 9th end than if they were trailing by three. To recognize this, we shall apply our decision tree model over a range of score differentials.

To represent the probability of scoring a certain number of points in a particular end, with or without the hammer, we shall use the following general notation: $\mathrm{P}\{X = k \mid e, h\}$. We define:

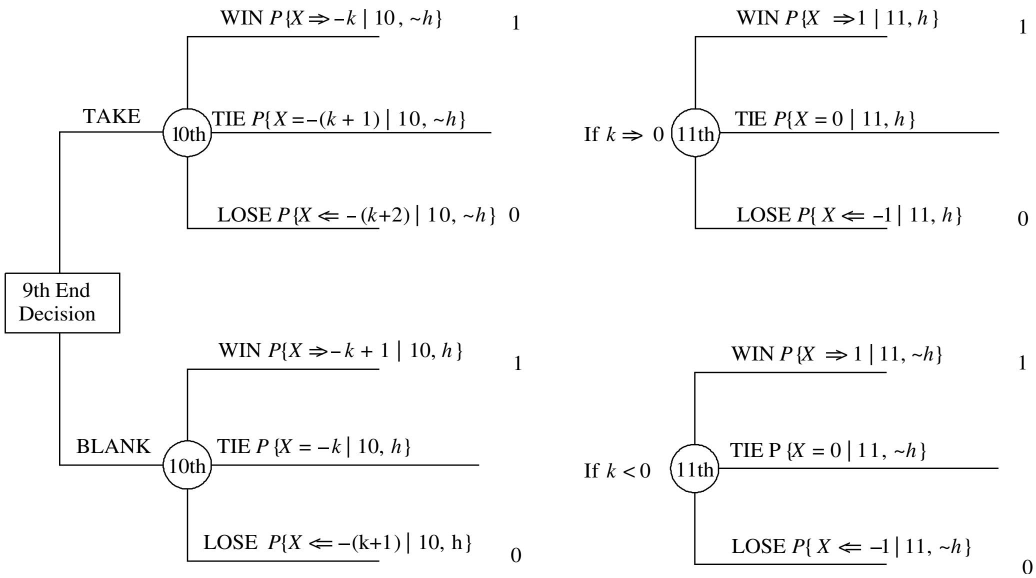
$X =$ a random variable representing the scoring during one end of a curling game
$k =$ the particular number of points scored
$e =$ the specific end under consideration
$h =$ whether or not Team A has the hammer ($h =$ hammer; $\sim h =$ does not have hammer).

This notation implicitly assumes that the teams are of relatively equal caliber. The probabilities of respective scores depend only on the particular end and possession of the hammer. Incorporating the relative skill of a specific team would have led to modeling complexities.

Our decision is depicted in Figure 2. We should remark here that it *is* possible for the 11th end to be blanked. This event suggests that the team with the hammer was unable to place their final rock in the house. As one might expect, this situation is extremely rare (especially for elite curlers). Incorporating the possibility of an eleventh-end blank to our model would have contributed little to our overall analysis; therefore, we shall ignore it in our decision tree.

Consider the case of Team A leading by $k = 2$ points immediately prior to making the ninth-end decision (Figure 2 is also applicable for the case of Team A trailing Team B; in that instance, use a negative value for k). Should they decide to take a single point (the upper portion of the decision tree), they would then lead by three points entering the 10th end. Because Team A scored in the 9th end, they would not have the last-shot advantage in the next end. Team A would win the game by scoring *any* number of points (because they are ahead by three and if they score, Team B gets zero). Indeed, Team A could permit their opposition to score as many as two points in the 10th end and still emerge victorious. This is indicated in the top branch of the tenth-end chance node. The game is tied if Team B scores three points (middle branch), whereupon an 11th end would be played. The bottom branch of the tenth-end node illustrates that Team A loses the game by allowing four points or more. The endpoint values on the decision tree

Figure 2 **The Alternatives and Subsequent Outcomes Associated with a k-point Differential Between Teams A and B**



suggest that a win is worth one point while a loss has no value.

The team that scores in the extra end wins the game; this is indicated in the branches emanating from the eleventh-end chance node. Note the two halves of the eleventh-end chance node. The top half is used for those instances in which the game was tied, or Team A led ($k \geq 0$), prior to the ninth-end decision. The bottom half refers to those cases in which Team A trailed Team B ($k < 0$).

Essentially, the only difference between the two scenarios is that in the first case ($k \geq 0$), Team A would have the hammer in the extra end because either Team B would have scored in the 10th end to tie the game, or the 10th end would have been blanked. In the second case ($k < 0$), Team A would have scored in the 10th end to force an extra end, thereby providing Team B with the last-shot advantage.

If Team A decides to blank the 9th end (the lower portion of the decision tree), they retain the hammer for the subsequent end. They win by allowing, at worst, 1 point in the 10th end. The game is tied if Team B scores two points. Any tally of three points or more for Team B results in a loss for Team A.

Results

The optimal alternative can be determined by selecting the option (TAKE versus BLANK) resulting in the higher likelihood of victory. To empirically calculate the best alternative, we gathered statistical information recorded from 1985 to 1997 (a total of 902 games) at the Canadian Men's Curling Championships (also known as the Brier). The uncertainties refer to tenth-end scoring (and 11th end, if required); consequently, we needed to generate conditional probability distributions of tenth-end scoring for a range of point differentials. In curling, a team may concede defeat at any point in the game if they feel their opponent's lead is insurmountable. As a result, not every contest in our 902-game data set featured a tally in the 10th end. In fact, less than half the games (410) went the full 10 ends.

The tenth-end scoring information is provided in Table 1. The rows represent the specific point differentials; the bottom row is the sum of each column. Each column represents the number of points scored in the 10th end with the hammer (negative values indicate that the team without the hammer scored). For example, should there be a one-point differential in the score after the 9th end of play, there is a $113/221 = 51.13\%$ chance that the team with the hammer will score a single point in the 10th end. To obtain the probability of scoring without the hammer, we simply note that $P\{X = k \text{ for Team A} \mid e, h\} = P\{X = -k \text{ for Team B} \mid e, \sim h\}$, and that $P\{X = k \text{ for Team A} \mid e, \sim h\} = P\{X = -k \text{ for Team B} \mid e, h\}$. There were no games that made

Table 1 Frequency Table for a Team with the Hammer in the 10th End for Various Point Differentials After Nine Ends of Play

Point differential k after 9th end	−4	−3	−2	−1	0	1	2	3	4	5	Totals
0			3	15	8	70	12	2			110
1	1	5	4	39	12	113	34	8	4	1	221
2		1	1	20	1	16	34	1			74
3			1	1	1	1		1			5
Totals	1	6	9	75	22	200	80	12	4	1	410

it to the 10th end where the difference after the 9th end was more than three points. If the point differential is so extreme after nine ends of play, historically the losing team has always conceded.

Table 2 gives the distributions for eleventh-end (and if required, twelfth-end) scoring. We observe that 118 games in our data set were tied after 10 ends of play. Teams with the hammer have a definite advantage during an 11th end; note that tallies of +1 point are very common. In only four games was a 12th end required (these games refer to the four instances in which no points were scored in the 11th end). We will lump these four games into the eleventh-end distribution.

In using our decision tree to determine the likelihood of victory, we did not partition the eleventh-end scoring distribution into various point differentials. Essentially, we are assuming that the likelihood of scoring k points in the 11th end is unaffected by the point differential that may have existed after nine ends of the curling game. Now, someone could argue that anxiety levels may increase, or one's concentration may suffer, especially if a team blew a big advantage in the 10th end and is forced to play an extra end. However, such considerations are beyond the scope of our model. From our personal experience in playing the game, we felt that using the overall scoring distribution in the extra end was valid.

Now that we have the scoring distribution for the 10th end (Table 1) and the extra end (Table 2), we can compute the expected values of each alternative. Because our decision tree featured utilities of a win or a loss of 1 and 0, respectively, the expected value of an alternative is simply the probability of victory. Thus, E(TAKE) = P(Win in 10th end) + P(Tie in 10th end) × P(Win in 11th end). Table 3 presents these results over a range of ninth-end situations. These situations refer to the circumstances of the game immediately prior to the TAKE versus BLANK decision.

As a sample calculation, we shall consider the situation in which Team A trails by a single point in the 9th end. If they choose to take the single point, they enter the 10th end tied with Team B but without the hammer. Using the "zero-point differential" row of Table 1, the chance of victory in the 10th end is $(3+15)/110 = 0.1636$. The possibility of the game entering the extra end is $8/110 = 0.0727$. Without the hammer in the extra end, Table 2 suggests that the probability that Team A wins is $19/118 = 0.1610$. Thus, E(TAKE) when Team A is losing by one point is $0.1636 + 0.0727(0.1610) = 0.1753$.

Should Team A blank the 9th end, they will retain the last-shot advantage but trail by one point in the 10th end. They win with probability $(34+8+4+1)/221 = 0.2127$. A tie game results if Team A scores a single point; this occurs with a likelihood of $113/221 =$

Table 2 The Frequency Table for the Number of Points Scored in an Extra End by a Team with the Hammer

	−2	−1	0	1	2	3	4	
11	4	15	4	85	7	2	1	118
12				3	1			4
Totals	4	15	4	88	8	2	1	122

Table 3 The Expected Values of Each Alternative Indicate that Taking a Point in the 9th End Is Never the Preferred Choice

Ninth-end situation prior to deciding to TAKE or BLANK	E(TAKE)	E(BLANK)
Winning by 3	1.0000	1.0000
Winning by 2	0.9678	0.9843
Winning by 1	0.9125	0.9263
Tie game	0.7050	0.8247
Losing by 1	0.1753	0.2950
Losing by 2	0.0737	0.0875
Losing by 3	0.0157	0.0322

0.5113. As in the TAKE alternative, Team A would not have the hammer in the extra end. They win the game in the extra end with a probability of $19/118 = 0.1610$. Using these values, E(BLANK) = $0.2127 + 0.5113\ (0.1610) = 0.2950$.

Willoughby and Kostuk's (2004) paper determined E(UP), the expected value when beginning the 10th end ahead by a single point without the hammer and E(DN), the expected value when starting the 10th end trailing by a single point with the last-shot advantage. These values from the earlier paper were, respectively, 0.7133 and 0.2867. We note here that E(UP) does not exactly equal our calculation of E(TAKE | Tie game) = 0.7050. The former expected value only considered tallies from games involving a one-point differential after nine ends of play. This involved a total of 221 matches, with 76 of these games leading to extra-end competition. While the current paper also analyzes 221 games that had a one-point difference in overall score after nine ends (see Table 1), we consider *all* extra-end games regardless of ninth-end score differential. As we claimed earlier, we are assuming that the likelihood of scoring a certain number of points in the 11th end is unaffected by the outcome after nine ends. This results in 118 extra-end games in this paper. Similar logic also applies for our calculations of E(DN) = 0.2867 and E(BLANK | Losing by 1) = 0.2950.

As an aside, Willoughby and Kostuk (2004) suggest that in examining two disparate scenarios, being up by one point without the hammer is the preferred situation. Curlers should favor that scenario over trailing by one point with the hammer. Here, we state that no matter what the situation is, when a team is faced with blanking the end or taking a single point, they ought to blank the 9th end and retain the last-shot advantage. It is the better choice. This maintains possession of the hammer, thereby permitting a team the opportunity to "have the final say" in the 10th end. As our analysis illustrates, the benefit of the last-shot advantage outweighs the value of one extra point. The largest advantage of following the BLANK strategy (over the TAKE decision) occurs in the cases of a tie game or Team A losing by a single point.

Granted, there is one instance (winning by three points) in which a curler would be indifferent between the two alternatives. However, note the likelihood of victory under both options. If Team A is leading by such a margin in the 9th end, *any* decision will result in certain victory. According to the data provided by 13 years of national championship games, the point differential is such that a comeback is essentially impossible.

As one would imagine, the expected values drop as the ninth-end situation becomes more precarious for Team A. For larger losing margins, the chance of "pulling victory from the jaws of defeat" is small, but the BLANK alternative marginally increases the probability of winning the game. The strategic choice favored by North American curlers may be preferable to the alternative often selected by world-class European competitors.

Conclusions

Faced with alternatives and a set of probabilistic outcomes, one may use decision analysis to optimize decision making. Such has been the focus of our paper. We applied decision analysis to an important decision in the sport of curling. Using data from over 900 national championship games, we were able to determine expected values of each alternative. Our results maintain that, given the choice between blanking the 9th end or taking a single point with the final rock, the former is the better alternative.

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